COMPSCI 240: Reasoning Under Uncertainty

Nic Herndon and Andrew Lan

University of Massachusetts at Amherst

Spring 2019

Lecture 1: Intro and Sets

General Information

- Instructors:
  - Section 01 Nic Herndon (nherndon@cs.umass.edu)
  - Section 01 Andrew Lan (andrewlan@cs.umass.edu)
- Lectures: MWF 9:05 AM – 9:55 AM
  - Section 01 in Integrative Learning Center S131
  - Section 02 in Hasbrouck Laboratory 124
- Course Webpage: `https://www-edlab.cs.umass.edu/cs240/`
- Moodle: `www.moodle.umass.edu`

More Information

- Teaching Assistants:
  - Paul Crouther (pcrouther@cs.umass.edu)
  - Elita Lobo (karine@cs.umass.edu)
  - Qingyang Xue (qxue@cs.umass.edu)
- Undergraduate Course Assistants:
  - Collin Giguere (cdgiguere@umass.edu)
  - Jinhong Gan (jinhonggan@umass.edu)
  - Mingfang Chan (mingfangchan@umass.edu)
  - Long Le (lnle@umass.edu)
  - Renos Zabounidis (rzabounidis@umass.edu)
- Textbook: Introduction to Probability, 2nd Edition by Dimitri P. Bertsekas and John N. Tsitsiklis
- Incoming emails: No math questions to either instructors or TAs. Come to discussion sessions and office hours instead!

Topics

- Basic counting problems
- Probability
- Discrete random variables
- Midterm Exam #1
- Continuous random variables
- Central limit theorem
- Probabilistic reasoning
- Game theory
- Midterm Exam #2
- Markov chains
- Bayesian network
- Final Exam

Grade breakdown

There will be 10 weekly quizzes, 4 homework assignments, 2 in class midterms and 1 final exam.

- Weekly Quiz in Moodle (10 of them) 5%
- Homeworks (best 3 out of 4) 15%
- Midterm 1 20%
- Midterm 2 20%
- Final 40%

See the course webpage for dates of homeworks and exams

Submission Policy

- Homeworks must be submitted on Gradescope: `https://www.gradescope.com/courses/37854/`, entry code: MVD8KX.
- Graded homework will be returned at Thursday discussion sessions the next week. Once returned, students have exactly 1 week to let the instructor/TAs know about any grading dispute/concern. No change in grades will be recorded after one week of returning the homeworks. NO exceptions.
- Since we will take the best 3 out of 4 assignments, you can miss one homework. But you should use that with caution. You can use this for medical reason, personal loss etc.
- There will be weekly short online quizzes on Moodle.
  The quiz should take about 30 minutes but you are allowed 1 hour.
  Think about it as a learning/self assessment opportunity!

Exam Policy

- All exams are closed book. Pens only. Only non-graphic calculators will be allowed.
- Once graded Midterms are returned, the students have exactly 1 week to let the TAs/instructors know about any grading dispute/concern. No change in grades will be recorded after one week of returning the exams. NO exceptions.
- Concerns/dispute regarding midterms/homework must be reported during the instructors/TAs office hours.
- Missing exams: With doctor's note and/or legal documents that comply the University's Class Absence Policy, you will be able to take a make-up exam. See https://www.umass.edu/registrar/students/policies-and-practices/class-absence-policy.
- Formal UMass academic honesty policy applies to all homework, quizzes, and exams. See https://www.umass.edu/honesty/

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- **Success has a lot of uncertainty, but reasoning can help your success in CS 240!**
- Do your best :)

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  - The Route 9 has so much traffic right now (if you happen to have no Google map with you)
  - Winning a football game
- We often make decisions that maximize the benefit given uncertainty
  - Should I bring my umbrella?
  - Should I detour?
  - Should I go for it on 4th-and-1?

Why do we need reasoning under uncertainty?

A Pseudo Real-World Problem:
You study all night and wake up 15 minutes before your exam. You could take a 10 minute bus ride but you have to catch a bus first: each minute, a bus to school passes your house with probability 1/5. Your other choice is to bike to the University from home, but it will take you 20 minutes. If your goal is to minimize the time it takes (on average) to get in, should you wait for the bus or hop on your bike?

You will learn the Math behind it in this class!

Why do we need reasoning under uncertainty?

- Prediction (Weather! Recall the snow storm on Sunday)
- Estimation (Stock Market)
- Detection (Communication)
- Almost all branches of Computer Science - Theory and AI

Why do we need reasoning under uncertainty? What if I don't do research

You need this not only in Go, but literally wherever you go

- Ad placement (Facebook, Google)
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The power of thinking rationally

How do we reason under uncertainty?

- Using Probability Theory
- Main idea: Assign each event a measure between 0 to 1: to signify its likelihood
- Then proceed very carefully - or our intuitions and observations will not match

Back to basics: Set theory

- A set is a collection of objects, which are the elements of the set
- If S is a set and x is an element of S, we write $x \in S$.
- If x is not an element of S, we write $x \notin S$. A set can have no elements, in which case it is called the empty set, denoted by $\emptyset$.

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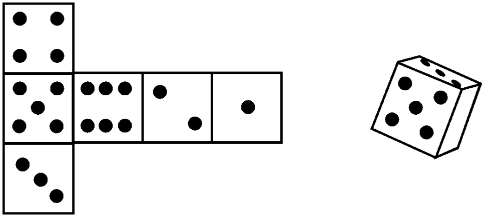
- Apple $\in$ { Orange, Apple, Pear }
  Strawberry $\notin$ { Orange, Apple, Pear }

Back to basics: Set theory



- Two ways of writing a set down:

$$S = \{1, 2, 3, 4, 5, 6\}$$

or

$$S = \{x | x \text{ is a possible outcome of a throw of a die}\}$$

"The collection of all elements that satisfy a certain condition is a set"

Set theory

- Size of a set S is denoted by $|S|$
- $|\{$ Orange, Apple, Pear $\}| = 3$
- S is a subset of T, $S \subset T$, means every element of S is also an element of T:
  $\forall x \in S, x \in T$

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$$S = T$$

Universal Set

- Ω: contains all objects that could conceivably be of interest in a particular context.
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- In the context of coin tossing, $\Omega = \{H, T\}$.
- In the context of dice, $\Omega = \{1, 2, 3, 4, 5, 6\}$.

Set operations

- Complement: $S^c = \{x \in \Omega | x \notin S\}$
  Example: $\Omega = \{1, 2, 3, 4, 5, 6\}; \quad S = \{2, 5\}$
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  $S^c = \{1, 3, 4, 6\}$
  Note that, $\Omega^c = \emptyset$
- The union of two sets S and T is the set of all elements that belong to S or T (or both), and is denoted by $S \cup T$.

$$S \cup T = \{x | x \in S \text{ or } x \in T\}$$

- The intersection of two sets S and T is the set of all elements that belong to both S and T , and is denoted by $S \cap T$.

$$S \cap T = \{x | x \in S \text{ and } x \in T\}$$

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Power Set

By default: $\emptyset \subset S \subset \Omega$.
Power Set: Set of all subsets

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Power Set: Set of all subsets

$$S = \{1, 2, 3\}$$

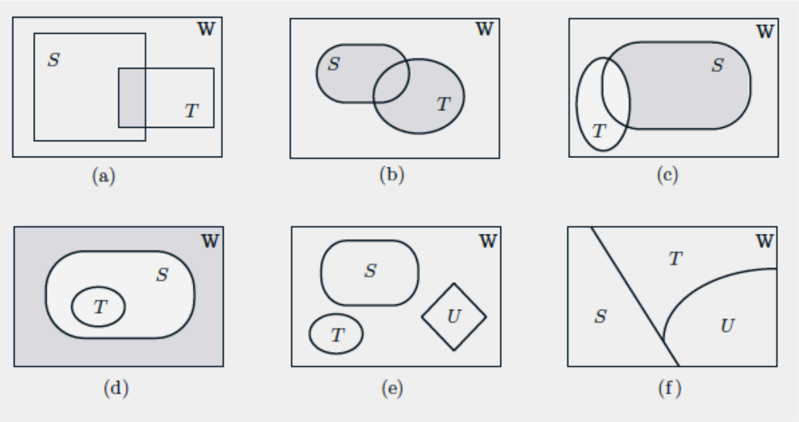
$$2^S = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

Disjoint Set

S and T are **disjoint** if $S \cap T = \emptyset$

$S_1, S_2, \ldots, S_n$ forms a **partition** of S if S_i and S_j are disjoint for any $i \neq j$ and $S_1 \cup S_2 \cup \cdots \cup S_n = S$.

Venn Diagram



Courtesy: Textbook

Venn Diagram - Partitions

- In (e), S, T, and U **do not** form partitions of W. However, S, T, U, and $(S \cup T \cup U)^C$ form partitions of W.
- In (f), S, T, and U form partitions of W.

Similarly,

- In (a),(b) and (c), S and T do not form partitions of W. However, $(S \cup T)$ and $(S \cup T)^C$ form partitions of W.
- In (d), T and $T^C \cap S$ form partitions of S. Furthermore, S and S^C form partitions of W.

Set Algebra

Using the above definitions, we can show that:

- Intersection Commutativity $S \cap T = T \cap S$
- Union Commutativity $S \cup T = T \cup S$
- Intersection Associativity $S \cap (T \cap U) = (S \cap T) \cap U$
- Union Associativity $S \cup (T \cup U) = (S \cup T) \cup U$
- Intersection Distributivity $S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$
- Union Distributivity $S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$