CS 240: Reasoning Under Uncertainty
Homework #4

University of Massachusetts Amherst

Student Name: ______________________________

Student ID Number: ______________________________

Instructor Name: (circle one): Herndon or Lan

Homeworks are due by 4pm. Late submissions are not accepted. Either upload to Gradescope or submit physical copies to the CS Main office. Uploaded submissions on Moodle will not be graded. You are not allowed to write your solution using the comment tool of PDF readers. They will not be graded. Please scan your work carefully; we cannot give you credit unless you show clear steps. Show all your work for each question. Only a single final correct answer will only get you a part of the points. Providing detailed, step-by-step solution is required.

[5+1+2=8 points] Problem 1**.** Consider the following variant of two-finger morra, where Alice picks an action $a \in \{1, 2\}$ and Bob picks an action $b \in \{1, 2\}$. Bob pays Alice \$$(a \times b)$ if $a + b$ is even, and Alice pays Bob \$$(a \times b)$ if $a + b$ is odd. Note that the payoff is different than that in the example we used in class.

1) If Alice plays 1 finger with probability p and 2 fingers with probability $1 - p$, what's the expected payoff that Bob can achieve if he knows p? How should Alice choose p such that Bob's payoff is indifferent of his own choices?
2) If Bob plays 1 finger with probability q and 2 fingers with probability $1 - q$, what's the expected payoff that Alice can achieve if she knows q? How should Bob choose q such that Alice's payoff is indifferent of her own choices?
3) What is the Nash equilibrium strategy for both players? And what is the expected payoff for each if they play the Nash equilibrium strategy?

[9 points] Problem 2**.** Consider a modified three-finger morra, where Alice picks an action $a \in \{1,2,3\}$ and Bob picks an action $b \in \{3,4,5\}$. Bob pays Alice $\$(2a+b)$ if $a+b$ is even, and Alice pays Bob $\$(2a+b)$ if $a+b$ is odd. If Bob plays 3 finger with probability r and 4 fingers with probability s and 5 fingers with probability $1-r-s$. What are the values of r and s that make Alice's choices indifferent in terms of her payoff?

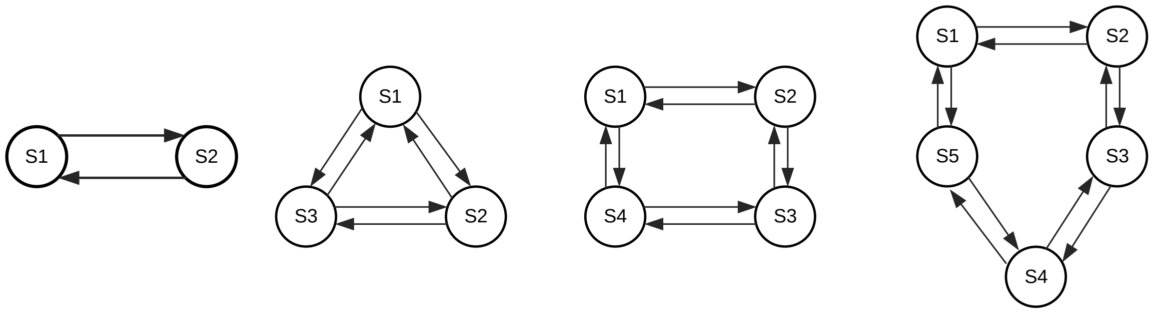
[3+4+4+2=13 points] Problem 3**.** Consider a Markov chain with three states $\{s_1, s_2, s_3\}$, and the following transition matrix:

$$\boldsymbol{A} = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/3 & 2/3 \\ 2/3 & 0 & 1/3 \end{pmatrix}$$

1) Draw the state diagram, annotated with transition probabilities for Markov chain.
2) Given that the initial distribution $v_0 = \langle 1, 0, 0 \rangle$, what is the probability that the chain is in state s_2 after 2 steps.
3) Find the steady state distribution for this Markov chain.
4) Suppose that for a single visit to state s, you receive $f(s)$ dollars, where $f(s_1) = 1, f(s_2) = 2$, and $f(s_3) = 3$. When the Markov chain reaches steady state, what's the value of $E[f(X_2)]$?

[3+3=6 points] Problem 4.

1) Given the following four Markov chains with 2, 3, 4, 5 states respectively, identify if the single recurrent class in each Markov chain is periodic or not. Directed arrows mean transition probability greater than zero.



(a) (b) (c) (d)

2) Now assume that you are given a N-state Markov chain, in which each state has bi-directional connections with its two neighboring states (i.e., the neighboring states of S_1 are S_2 and S_N; the neighboring states of S_2 are S_1 and S_3, ..., and the neighboring states of S_{N-1} is S_{N-2} and S_N). Identify under what conditions (i.e., what values of N) will this N-state Markov chain have a periodic recurrent class, and justify your answer.

[2.5+1.5+2×5=14 points] Problem 5**.** Suppose that most mornings you set your alarm for 8am so that you can catch the 8:30am bus to school which usually arrives just in time for your class at 9am. Unfortunately, sometimes you forget to set your alarm which might make you miss your bus and this increases the probability that you are late for class. Even if you catch the 8:30am bus, there's a chance you'll be late for class. To help analyze the situation we introduce some random variables. Let $A = 1$ if you remember to set your alarm and $A = 0$ otherwise. Let $B = 1$ if you catch the 8:30am bus and $B = 0$ otherwise. Let $L = 1$ if you are late for class and $L = 0$ otherwise. Suppose that $P(A = 1) = 3/4$, $P(B = 1|A = 1) = 4/5$, $P(B = 1|A = 0) = 2/5$, $P(L = 1|B = 1) = 1/4$, and $P(L = 1|B = 0) = 2/3$.

1) What are the values of:

$$P(A = 0)\ ,\ P(B = 0|A = 1)\ ,\ P(B = 0|A = 0)\ ,\ P(L = 0|B = 1)\ ,\ \text{and}\ \ P(L = 0|B = 0)\ ?$$

2) Draw the Bayesian network for the random variables A, B, and L.
3) What is the value of $P(A = 1, B = 1, L = 0)$?
4) What is the value of $P(B = 1)$?
5) What is the value of $P(L = 0)$?
6) What is the value of $P(L = 1, A = 0)$?
7) What's the value of $P(L = 1|A = 0)$?

Problem EC: **As we discussed - this is actual extra credit that can count for a maximum of 5% towards your total course grade.**
Let today's high temperature be T. Assume that T is a normal random variable with mean μ and variance $\sigma^2 = 1$. Let's say you don't know today's date, so your belief about μ follows a normal distribution with mean m and variance $\delta^2 = 1$, i.e., $P(\mu) = \mathcal{N}(m, 1)$. Now, at the end of the day, you observe that today's high temperature is actually t. Given this information, what is your new belief of μ, i.e., what is $P(\mu|T = t)$?
Hint: It has the same functional form as your initial belief, just different moments. You can use this hint as a fact.