CS 240: Reasoning Under Uncertainty
Homework #3

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Homeworks are due by 4pm. Late submissions are not accepted. Either upload to Gradescope or submit physical copies to the CS Main office. Uploaded submissions on Moodle will not be graded. You are not allowed to write your solution using the comment tool of PDF readers. They will not be graded. Please scan your work carefully; we cannot give you credit unless you show clear steps. Show all your work for each question. Only a single final correct answer will only get you a part of the points. Providing detailed, step-by-step solution is required.

- **The Standard Normal Table:**

$\Phi(x)$	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.5	.50399	.50798	.51197	.51595	.51994	.52392	.5279	.53188	.53586
.1	.53983	.5438	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
.3	.61791	.62172	.62552	.6293	.63307	.63683	.64058	.64431	.64803	.65173
.4	.65542	.6591	.66276	.6664	.67003	.67364	.67724	.68082	.68439	.68793
.5	.69146	.69497	.69847	.70194	.7054	.70884	.71226	.71566	.71904	.7224
.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.7549
.7	.75804	.76115	.76424	.7673	.77035	.77337	.77637	.77935	.7823	.78524
.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.8665	.86864	.87076	.87286	.87493	.87698	.879	.881	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.9032	.9049	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.9222	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.9452	.9463	.94738	.94845	.9495	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.9608	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.9732	.97381	.97441	.975	.97558	.97615	.9767
2	.97725	.97778	.97831	.97882	.97932	.97982	.9803	.98077	.98124	.98169
2.1	.98214	.98257	.983	.98341	.98382	.98422	.98461	.985	.98537	.98574
2.2	.9861	.98645	.98679	.98713	.98745	.98778	.98809	.9884	.9887	.98899
2.3	.98928	.98956	.98983	.9901	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.9918	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.9943	.99446	.99461	.99477	.99492	.99506	.9952
2.6	.99534	.99547	.9956	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.9972	.99728	.99736
2.8	.99744	.99752	.9976	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.999
3.1	.99903	.99906	.9991	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.9994	.99942	.99944	.99946	.99948	.9995
3.3	.99952	.99953	.99955	.99957	.99958	.9996	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.9997	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.9998	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.9999	.9999	.9999	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	0.99998

[5+5=10 points] Problem 1. Let X be a Geometric random variable, with success probability p.

1) Use the Markov bound to find an upper bound for $P(X \geq a)$, for a positive integer a.
2) If $p = 0.1$, use the Chebyshev bound to find an upper bound for $P(X \leq 1)$. Compare it with the actual value of $P(X \leq 1)$; is this bound tight or loose? Find the tightest bound possible.

[2+4+4=10 points] Problem 2. Let X be an Exponential random variable with rate $\lambda = 2$, and let Y be another random variable such that $Y = 3X + 2$.

1) Find $P(X > 2)$.
2) Find $E[Y]$ and $var[Y]$.
3) Find $P(X > 2|Y < 11)$.

[5+5=10 points] Problem 3. If two random variables are jointly normal, one consequence is that any linear combination of these random variables remains normal, even if they are not independent. Let X and Y be jointly normal random variables with means and variances given by $\mu_X = -1$, $\sigma_X^2 = 4$, $\mu_Y = 1$, and $\sigma_Y^2 = 1$, respectively. Moreover, their correlation is given by $\rho(X, Y) = -\frac{1}{2}$.

1) Find $P(X + 2Y \leq 3)$.
2) Find $cov(X - Y, X + 2Y)$.

[2+4+3+1=10 points] Problem 4. Let X and Y be continuous random variables with the following joint probability density function:

$$f_{X,Y}(x, y) = \begin{cases} cxy & \text{if } 0 \leq x, 0 \leq y, x + y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

1) Find out the value of c.
2) What is the probability that Y is greater than X?
3) Find the marginal probability density functions of X and Y.
4) Are X and Y independent? Justify your conclusion.

[10 points] Problem 5**.** A shipyard makes a container ship that can withstand the total amount of weight W, which is normally distributed with mean of 600 tons and standard deviation of 60 tons. Let us assume that the weight of a single container that will be loaded to the container ship is also normally distributed with mean of 4 tons and standard deviation of 0.4 tons. What is the maximum number of containers that the ship can load and still have at least a 90% chance to not exceed its weight limit? *Your answer has to be an integer.*