
CS 240: REASONING UNDER UNCERTAINTY

HOMEWORK #2

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[8 points] Problem 1. An ant starts from the origin of a 2-D grid, at the point (0,0). Then each time it moves either i) right by one unit distance, e.g., from (0,0) to (1,0), ii) up by one unit distance, e.g., from (0,0) to (0,1), or iii) stay still, with probabilities 0.3, 0.2, and 0.5, respectively. After 10 moves, what is the probability that the ant settles down at point (2,5)?

[2 + 4 + 4 = 10 points] Problem 2. Let X be a discrete random variable, and let $Y = |X|$.

(a) Assume that the PMF of X is

$$p_X(x) = \begin{cases} Kx^2 & \text{if } x = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

where K is a suitable constant. Determine the value of K .

(b) For the PMF of X given in part (a) calculate the PMF of Y .

(c) Give a general formula for the PMF of Y in terms of the PMF of X .

[4+4=8 points] Problem 3. You are offered to play the following game. You roll a fair 6-sided die once and observe the result which is shown by the random variable X . At this point, you can stop the game and win X dollars. Or, you can also choose to discard the X dollars you win in the first roll, and roll the die for a second time to observe the value Y . In this case, you will win Y dollars. Let W be the number of dollars that you win in this game.

a) What strategy do you use to maximize $E[W]$? What is the maximum $E[W]$ you can achieve using your strategy?

b) If you have to pay the price of 1 dollar to do a second roll, i.e., you win $Y - 1$ dollars if you choose to roll the die a second time. Will you change your strategy? Why?

[8 points] Problem 4. Let X be a discrete random variable that is uniformly distributed over the set of integers in the range $[a, b]$, where a and b are integers with $a < 0 < b$. Find the PMF of the random variables $\max(0, X)$ and $\min(0, X)$.

[8 points] Problem 5. You are taking a multiple choice test with 10 questions, with 4 different options per question. For each question, there's only one correct answer. You haven't studied for the test and you decide to choose the answers at random (each option equally likely). What is the probability that you get at least 7 of them right?

[8 points] Problem 6. You are visiting the rainforest, but unfortunately your insect repellent has run out. As a result, at each second, one mosquito lands on your neck with probability 0.5. If a mosquito lands, it will bite you with probability 0.2, or decide to find a meal elsewhere with probability 0.8, independently of other mosquitoes. What is the expected time between successive bites?