CS 240: REASONING UNDER UNCERTAINTY
HOMEWORK #1
ASSIGNED: FRIDAY FEB. 1ST, 2019
DUE: FRIDAY, FEB. 8TH, 2019 4:00 PM ET

INSTRUCTORS
NIC HERNDON AND ANDREW LAN
University of Massachusetts Amherst

NAME: _____

STUDENT ID: _____

INSTRUCTOR NAME: (CIRCLE ONE): HERNDON OR LAN

TA NAME (OPTIONAL): _____

Problem 1 $(3 + 3 + 3 = 9)$
Suppose you draw two cards from a deck of 52 cards without replacement.
1) What's the probability that the first draw is a heart and the second draw is not a heart?
2) What's the probability that exactly one of the cards are hearts?
3) If you draw two cards *with* replacement, what's the probability that none of the cards are hearts?

Problem 2 (4)
A factory produces 85 T-shirts and 10 sweatshirts each hour. If 3 shirts (either T-shirts or sweatshirts) are picked at random then what is the probability that all of them are T-shirts?

Problem 3 (3+4=7)
There are 4 bags each containing 100 marbles. Bag 1 has 40 red and 60 blue marbles. Bag 2 has 30 red and 70 blue marbles. Bag 3 has 75 red and 25 blue marbles. Bag 4 has 50 red and 50 blue marbles. Now a bag is chosen at random and a marble is also picked at random.
1) What is the probability that the marble is red?
2) What happens when the first two bags are chosen with probability 0.3 each and other two bags are chosen with probability 0.2 each?

Probem 4 (10)
The disc containing the only copy of your homework just got corrupted, and the disk got mixed up with two other corrupted discs that were lying around. It is equally likely that any of the three discs holds the corrupted remains of your homework. Your computer expert friend offers to have a look, and you know from past experience that his probability of finding your homework from any disc is 0.35 (assuming the homework is there). Given that he searches on disc 1 but cannot find your homework, what is the probability that your homework is on disc i, for $i = 1, 2, 3$?

Problem 5 (4+4+4=12)
We roll two fair 6-sided dice, A and B. Each one of the 36 possible outcomes is assumed to be equally likely.
1) Find the probability that dice A is larger than dice B.
2) Given that the roll resulted in a sum of 5 or less, find the conditional probability that the two dice were equal.
3) Given that the two dice land on different numbers, find the conditional probability that the two dice differed by 2.

Problem 6 (8)
For any events A, B, C, and $D = A \cap B \cap C$ prove the following equality:
$$P(D^c) = 1 - P(A) \cdot P(B \mid A) \cdot P(C \mid A \cap B)$$