CS 240: REASONING UNDER UNCERTAINTY
EXAM I: OCT. 10, 2018

INSTRUCTORS
S.I. LEE AND J. XIONG
University of Massachusetts Amherst

STUDENT NAME: ______________________________

STUDENT ID NUMBER: ______________________________

INSTRUCTOR NAME: (CIRCLE ONE): LEE OR XIONG

TA NAME: (CIRCLE ONE): KARINE OR PAUL OR HIA OR ZACK

CLOSED BOOK. CALCULATORS OK. OTHER ELECTRONIC DEVICES NOT ALLOWED. PLEASE BE RIGOROUS AND PRECISE AND SHOW YOUR WORK. IF YOU NEED EXTRA SPACE, USE THE BACK OF A PAGE. IF YOU HAVE QUESTIONS DURING THE EXAM, RAISE YOUR HAND. THIS EXAM CONSISTS OF FIVE PROBLEMS THAT CARRY A TOTAL OF 50 POINTS, AS MARKED. DURATION: 60 MINUTES. BEST WISHES!

STANDARD RANDOM VARIABLES:

- **UNIFORM:** FOR $k = a, \ldots, b$:

$$P(X = k) = \frac{1}{b-a+1}; \qquad E[X] = \frac{a+b}{2}; \qquad V[X] = \frac{(b-a+1)^2 - 1}{12}$$

- **BERNOULLI:** FOR $k = 0$ OR 1:

$$P(X = k) = \begin{cases} 1-p & \text{IF } k = 0 \\ p & \text{IF } k = 1 \end{cases}; \qquad E[X] = p; \qquad V[X] = p(1-p)$$

- **BINOMIAL:** FOR $k = 0, \ldots, n$

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}; \qquad E[X] = np \qquad V[X] = np(1-p)$$

- **GEOMETRIC:** FOR $k = 1, 2, 3, \ldots$

$$P(X = k) = (1-p)^{k-1} \cdot p \qquad E[X] = \frac{1}{p} \qquad V[X] = \frac{1-p}{p^2}$$

- **POISSON:** FOR $k = 0, 1, 2, \ldots$

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}; \qquad E[X] = \lambda \qquad V[X] = \lambda$$

Problem 1: ($2 \times 5 = 10$ points) Write if the following statements are True or False beside them (no justification is necessary).

1) (2 points) If A and B are disjoint events then A and B are not independent.

2) (2 points) If A and B are disjoint events then $P(A \cup B) = P(A) + P(B)$.

3) (2 points) If S is a set containing exactly 6 elements, then the power-set of S contains exactly 36 elements.

4) (2 points) If $P(A) < P(B)$ then $P(A \cap B) < P(B)$.

5) (2 points) If $P(A) > 1/2$ and $P(B) > 1/2$ then $P(A \cap B) > 0$.

Problem 2: (5×2 points) Let us suppose that we have 10 men and 10 women to stand in a line.

1) (5 points) How many ways are there to arrange these people in the line if the men and women have to alternate (e.g., WMWM$\cdots$)?
2) (5 points) If the first and the last one are of different genders, how many ways are there to line up?

This page is intentionally left blank.

Problem 3: (2.5×4 points) Every Wednesday night after exciting CS240 lecture, a student goes to one of three dining halls: Worcester, Franklin, or Hampshire with probability 0.3, 0.6, and 0.1, respectively. If the student goes to Worcester, the student eats salad with probability 1. If the student goes to Franklin, the student eats salad with probability 0.3. If the student goes to Hampshire, the student eats salad with probability 0.4. Let us define the following events:

$$W = \text{"Go to Worcester"};\ F = \text{"Go to Franklin"};\ H = \text{"Go to Hampshire"}$$

$$S = \text{"Eat salad"}$$

1) (2.5 points) What's the probability the student goes to Franklin and eats salad on a Wednesday night?
2) (2.5 points) What's the probability that the student goes to Hampshire and not eat salad on a Wednesday night?
3) (2.5 points) What's the probability the student will eat salad on a Wednesday night?
4) (2.5 points) If you know that the student ate salad, what's the probability that the student went to Worcester?

This page is intentionally left blank.

Problem 4: (10 points) Let X be a random variable and $Y = aX + b$. Prove that $\text{var}[Y] = a^2 \cdot \text{var}[X]$

This page is intentionally left blank.

Problem 5: (10 points) Let a two-sided object (e.g., a coin) is tossed n times. At each toss, the object shows an outcome o_1 (e.g., head) with probability p_1 and another outcome o_2 (e.g., tail) with probability $p_2 = 1 - p_1$. Each toss is independent and identically distributed (i.i.d). Let X_1 and X_2 be the numbers of o_1 and o_2 in the n-sequence. Then, we learned that

$$P\big((X_1 = k_1) \cap (X_2 = k_2)\big) = \binom{n}{k_1} p_1^{k_1} p_2^{k_2} = \binom{n}{k_2} p_1^{k_1} p_2^{k_2},$$

where $k_1 + k_2 = n$. (**hint**: the above equation is equivalent to the PMF of a binomial random variable.)

Now let us assume that we have an m-sided object that is tossed n times. At each toss, outcomes $o_1, o_2, \cdots, o_m$ have probability of $p_1, p_2, \cdots, p_m$ respectively, where $\sum_{i=1}^{m} p_i = 1$. Each toss is again i.i.d. Let us suppose that $X_1, X_2, \cdots, X_m$ represent the numbers of $o_1, o_2, \cdots, o_m$ in any n-sequences, and $k_1, k_2, \cdots, k_m$ are any arbitrary constants where $\sum_{i=1}^{m} k_i = n$. What is the generic equation for $P\big((X_1 = k_1) \cap (X_2 = k_2) \cap \cdots \cap (X_m = k_m)\big)$?

This page is intentionally left blank.

	Points Available	Points Achieved
Problem 1 :	10	
Problem 2 :	10	
Problem 3 :	10	
Problem 4 :	10	
Problem 5 :	10	
Totals :	50	

This page is intentionally left blank.