Discussion 8: Review for Midterm 2

Lectures 12 - 20

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Preliminaries

Reminders

1. Midterm 2 is in-class tomorrow: read Andrew's email about the logistics

Quiz 6 Review

Problem #2

Problem Statement

The PMF of two discrete random variables, X and Y, is given below. What is corr(X,Y)?

$x \setminus y$	12	15	20
12	a	0.05	0.1
15	0.05	0.15-a	0.35
20	0	0.20	0.10

where a $\in \{0.01, 0.02, 0.03, 0.04, 0.05\}$.

Problem #4

Problem Statement

The PDF of two continuous random variables, X and Y, is given below. What is corr(X,Y)?

$$f(X) = \begin{cases} 2 & \text{if } x, y \in (0, +\infty) \text{ and } x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Practice Exam Problems, Fall 2018

Problem #1.1

Problem Statement

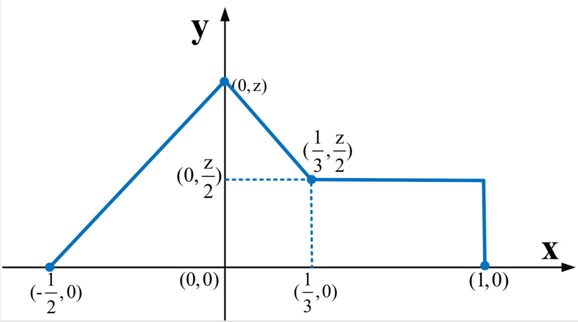
Let us assume that X is a random variable with the following probability density function. Find the value of a.

$$f_X(x) = \begin{cases} -ax, & -2 \leq x \leq 0 \\ 2ax^2, & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Problem #1.2

Problem Statement

Let us assume that X is a random variable whose probability density function is depicted in the following figure. Find the value of z.



Problem #2

Problem Statement

Suppose that we have two independent exponential random variables X and Y. The probability density functions of X and Y are:

$$f_X(x) = \begin{cases} \alpha e^{-\alpha x} & \text{if } 0 \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_Y(y) = \begin{cases} \beta e^{-\beta y} & \text{if } 0 \leq y \leq \infty \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability that X is greater than Y.

Problem #3.1

Problem Statement

Prove the following statement:

$\text{Var}(X+Y+Z) = \text{Var}(X)+\text{Var}(Y)+\text{Var}(Z) + 2\text{Cov}(X,Y) + 2\text{Cov}(X,Z) + 2\text{Cov}(Y,Z)$.

Problem #3.2

Problem Statement

Prove the following statement:

$$\text{cov}(X+Y, Z+K) =$$

$$\text{cov}(X, Z) + \text{cov}(X, K) + \text{cov}(Y, Z) + \text{cov}(Y, K)$$

Problem #4

Problem Statement

Consider a modified three-finger morra where Alice picks an action $a \in \{1,2,3\}$ and Bob picks an action $b \in \{3,4,5\}$. Bob pays Alice \$$(2a + b)$ if $a + b$ is even, and Alice pays Bob \$$(2a + b)$ if $a + b$ is odd. If Bob plays 3 finger with probability r and 4 fingers with probability s and 5 fingers with probability $1 - r - s$. What are the values of r and s that make Alice's choices indifferent in terms of her payoff?

Problem #5.1

Problem Statement

Suppose it is known that the number of items produced in a factory during a week is a random variable with mean of 100 and variance of 20. What are the **upper bounds** for the probability that this week's production will exceed 130? Find two upper bounds using both the Markov's inequality and the Chebyshev's inequality.

Problem #5.2

Problem Statement

Suppose again that it is known that the number of items produced in a factory during a week is a random variable with mean of 100 and variance of 20. Considering the probability that this week's production will exceed an arbitrary number N, what value range of N would make Markov's bound tighter than the Chebyshev's bound?

Problem #5.3

Problem Statement

The mean values of three normal random variables X, Y, Z are 1, 2, and 3, respectively. If $P(3 < X+3Y-Z < 5) = 0.4$, find $P(0.2X + 0.6Y - 0.2Z < 0.6)$.

FIN