Discussion 7

Chebyshev Inequality, Markov Inequality and Weak Law of Large Numbers

Elita Lobo

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University of Massachusetts Amherst

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Quiz 5 Review

Problem #5

Problem Statement

Suppose the PDF of a random variable X is given below. What is $P(a < X < b), a \in \{0.1, 0.2, 0.3\}, b \in \{0.7, 0.8, 0.9\}$?

$$f(X) = \begin{cases} cX, 0 < X < 1 \\ 0 \text{ otherwise} \end{cases}$$

Problem #8

Problem Statement

Suppose the PDF of a random variable X is $f(X) = ce^{|x|}$. What is $P(1 < X < 1)$?

(a) $1 - \frac{1}{2}e^{-1}$ (b) $1 - e^{-1}$ (c) $\frac{1}{2}(1 - \frac{1}{2}e^{-1})$ (d) $\frac{1}{2}(1 - e^{-1})$

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Problem #9

Problem Statement

Suppose a certain electrical component has different breakdown rate in different voltage ranges, as listed inthe table below. The voltage, however, is a random variable $X \sim N(220, 25^2)$. Then what is the probability that the component breaks down?

Voltage	< 200	200-240	> 240
Breakdown Rate	0.1	0.001	0.2

(a) 0.064 (b) 0.056 (c) 0.060 (d) 0.052

Problem #11

Problem Statement

Let X be an exponential random variable with E[X] =c. Then what is $\mathbf{E}[X^2]$?

(a) $2c^2$ (b) c^2 (c) c (d) c^3

Problem #12

Problem Statement

Suppose that the weights of adult males are normally distributed with a mean of 172 lbs and a standard deviation of 29 lbs. What is the probability that one randomly selected adult male will weigh more than 180lbs ?

(a) 0.39 (b) 0.084 (c) 0.61 (d) 0.916

Practice Problems

Question #1

Problem Statement

A statistician wants to estimate the mean height h (in meters) of a population, based on n independent samples $X_1, X_2 \dots X_n$, chosen uniformly from the entire population. He uses the sample mean $M_n = (X_l + \cdots + X_n)/n$ as the estimate of h, and a rough guess of 1 .0 meters for the standard deviation of the samples X$_t$

(a) How large should n be so that the standard deviation of Mn is at most 1 centimeter?

(b) How large should n be so that Chebyshev's inequality guarantees that the estimate is within 5 centimeters from h, with probability at least 0.99?

Question #2

Problem Statement

In order to estimate f - the true fraction of smokers in a large population, Alvin selects n people at random. His estimator M_n is obtained by dividing S_n. the number of smokers in his sample, by n, i.e. , $M_n = \frac{S_n}{n}$. Alvin chooses the sample size n to be the smallest possible number for which the Chebyshev inequality yields a guarantee that

$P(|M_n - f| \geq \epsilon) \leq \delta$

where ϵ and δ are some pre-specified tolerances. Determine how the value of n recommended by the Chebyshev inequality changes in the following cases.

(a) The value of ϵ is reduced to half its original value.

(b) The probability δ is reduced to half its original value.

Helpful Formulas

Chebyshev Inequality, Markov Inequality and Weak Law of Large Numbers

Markov Inequality:

If a random variable X can only take nonnegative values, then:

$$P(X >= a) <= \frac{\mathbf{E}[a]}{a} \; \forall a > 0$$

Chebyshev Inequality:

If X is a random variable with mean μ and variance σ^2, then

$$P(|x - \mu| >= c) \leq \frac{\sigma^2}{c^2} \; \forall c > 0$$

Chebyshev Inequality, Markov Inequality and Weak Law of Large Numbers

Weak Law of Large Numbers:

Let $X_I, X_2, \ldots$ be independent identically distributed random variables with μ. For every $\epsilon > 0$, we have

$$P(|M_n - \mu| \geq \epsilon) = P\left(\left|\frac{X_1 + X_2 \ldots X_n}{n} - \mu\right| \geq 0\right) \rightarrow 0 \text{ as } n \rightarrow \infty$$

where $M_n = \frac{X_1 + X_2 \ldots X_n}{n}$

FIN