Discussion 3

Binomial and Poisson PMF

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Table of Contents

Quiz 2 Review

Problem #11

Problem Statement

Suppose that a class has 60 students who were born in 1999. How many ways are there that no two students have the same birth date in that class? (a) 60! (b) $\frac{60!}{305!}$ (c) $\frac{365!}{305!}$ (d) $\frac{365!}{205! * 60!}$

Problem #9

Problem Statement

How many permutations of the letters A to H if the three letters A, B, and C must appear side-by-side but not necessarily in the order A,B,C?

(a) $6!$ (b) $6! * 3!$ (c) $8!$ (d) $\frac{8!}{3!}$

Problem #8

Problem Statement

If passwords may contain lower case letters and digits, how many 6-character passwords start with a lower case letter a or ends with a lower case letter z?

(a) $36^5 + 36^5 - 36^4$ (b) $36^5 + 36^5$ (c) 36^4 (d) $36^6 - 36^5 - 36^5$

Problem #7

Problem Statement

A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes contain at most three tails?

(a) 176 (b) 10^2 (c) 821 (d) $\frac{10!}{3!}$

Problem #4

Problem Statement

Franklin has three coins, two fair coins (head on one side and tail on the other side) and one two-headed coin. He randomly picks one, flips it and gets a head. What is the probability that the coin he picked is a fair one?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Practice Problems

Question #1

Problem Statement

The UMass football team has 2 games scheduled for one weekend. It has a 0.4 probability of not losing the first game. and a 0.7 probability of not losing the second game, independent of the first. If it does not lose a particular game, the team is equally likely to win or tie, independent of what happens in the other game. The UMass team will receive 2 points for a win, 1 for a tie. and 0 for a loss.

Find the PMF of the number of points that the team earns over the weekend.

Question #2

Problem Statement

You go to a party with 500 guests.

- What is the probability that exactly one other guest has the same birthday as you? Calculate this exactly.
- Also calculate the probability above, approximately, by using the Poisson PMF. (For simplicity. exclude birthdays on February 29.)

Helpful Formulas

Binomial and Poisson RV Formulas

Binomial RV: The Binomial probability of r successes in n independent Bernoulli trials (with probability of success p) is given as:

$$p(n) = P\{X = n\} = \binom{n}{r} p^r (1-p)^{n-r}$$

Properties: If X is a binomial random variable with parameters n and p, then

$$E[X] = np$$

Binomial and Poisson RV Formulas

Poisson RV: Poisson random variable: A random variable X that takes on one of the values 0,1,. . ., is said to be a *Poisson random variable* with parameter λ if for some $\lambda > 0$

$$p(i) = P\{X = i\} = \frac{\lambda^i}{i!}e^{-\lambda}$$

where $i = 0, 1, 2, \ldots$

Properties: If X is a Poisson random variable with parameter λ, then $E[X] = \lambda = np$

FIN