

Discussion 2

1.5: Independence

1.6: Counting

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Preliminaries

Reminders

1. Moodle quiz #2 is available now
2. Homework #1 is due on Friday, Feb 8 on Gradescope by 4PM

Questions from Last Week

1. Discussion slides will be posted on course website after class
 - Only solutions on slides will be for helpful quizzes over definitions, not for Moodle quizzes, homework problems or in-class practice problems.
2. Further practice problems are provided in the book
 - Solutions, corrections, and supplementary problems are provided at <http://athenasc.com/probbook.html>

Quiz 1 Review

Problem #4

Problem Statement

Under what conditions will the statement $(A \cap B) \cup C = A \cap (B \cup C)$ be true?

- (a) $C \subseteq A$ (b) $A \subseteq C$ (c) $C \subseteq B$ (d) $B \subseteq C$

Problem #7

Problem Statement

Caro is tossing three fair coins at the same time. What is the probability that at least two of them are heads?

(a) $\frac{1}{2}$

(b) $\frac{3}{8}$

(c) $\frac{3}{4}$

(d) $\frac{1}{4}$

Problem #9

Problem Statement

From the set $\{1, 2, \dots, 15\}$, Alice and Bob each choose a number (different from each other). We know that Alice's number can be divided by 5, then what is the probability that Alice's number is larger than Bob's?

(a) $\frac{8}{14}$

(b) $\frac{9}{14}$

(c) $\frac{10}{14}$

(d) $\frac{11}{14}$

Practice Problems

Subsetway - Question #1

Problem Statement

A branch of the sandwich shop Subsetway opens on campus. There are six sandwich fillings available:

{avocado, bacon, cheese, deli meat, egg, falafel}

A popular option is to order the Subsetway Special which is a sandwich with three random different fillings and each subset of three fillings is equally likely. For example, you could get the set of fillings {avocado,bacon, cheese} or {bacon, egg, falafel} or {avocado, bacon, egg} etc.

How many different combinations of 3 fillings are there?

Subsetway - Question #2

Problem Statement

A branch of the sandwich shop Subsetway opens on campus. There are six sandwich fillings available:

{avocado, bacon, cheese, deli meat, egg, falafel}

A popular option is to order the Subsetway Special which is a sandwich with three random different fillings and each subset of three fillings is equally likely. For example, you could get the set of fillings {avocado,bacon, cheese} or {bacon, egg, falafel} or {avocado, bacon, egg} etc.

How many different combinations of 3 fillings are there that include avocado?

Subsetway - Question #3

Problem Statement

A branch of the sandwich shop Subsetway opens on campus. There are six sandwich fillings available:

{avocado, bacon, cheese, deli meat, egg, falafel}

A popular option is to order the Subsetway Special which is a sandwich with three random different fillings and each subset of three fillings is equally likely. For example, you could get the set of fillings {avocado,bacon, cheese} or {bacon, egg, falafel} or {avocado, bacon, egg} etc.

Let A be the event that your three fillings includes avocado and let B be the event that your three fillings include bacon. What are the values for the following probabilities: $P(A)$, $P(B)$, and $P(A \cap B)$?

Poker Hands - Question #1

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

How many poker hands are there?

Poker Hands - Question #2

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

Four of a Kind: 4 cards of 1 rank; 1 card of a 2nd rank

How many hands are a “four of a kind”?

Poker Hands - Question #3

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

One Pair: 2 cards of same rank; others have different ranks

How many hands are a “one pair”?

Poker Hands - Question #4

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

Two Pairs: 2 of same rank; 2 of another rank; 1 of a 3rd rank

How many hands are a “two pairs”?

Poker Hands - Question #5

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

3 of a kind: 3 cards of 1 rank; others different ranks

How many hands are a “3 of a kind”?

Poker Hands - Question #6

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

Straight: 5 cards have consecutive rank (assuming the Ace can be the lowest and the highest value)

How many hands are a “straight”?

Poker Hands - Question #7

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

Flush: 5 cards have the same suit

How many hands are a “flush”?

Poker Hands - Question #8

Problem Statement

A deck of cards consists of 52 cards. Each card has 1 of 4 suits (**Clubs, Diamonds, Hearts, Spades**) and one of 13 ranks (**A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**). A poker hand consists of 5 cards.

Answer the following questions & show your work for each.

Full House: a pair and a 3 of a kind (3 cards of another rank)

How many hands are a “full house”?

Helpful Quiz (Time Permitting)

Counting Formulas

1. What is the formula for the number of permutations of n objects?

$$n!$$

2. What is the formula for the number of k -permutations of n objects?

$$\frac{n!}{(n-k)!}$$

3. What is the formula for the number of combinations of k out of n objects?

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

4. What is the formula for the number of partitions of n objects into r groups with the i th group having n_i objects?

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{n_1! n_2! \dots n_r!}$$

FIN