CMPSCI 670: Computer Vision

Image formation

University of Massachusetts, Amherst
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Instructor: Subhransu Maji
• MATLAB setup and tutorial
  • Does everyone have access to MATLAB yet?
• EdLab accounts have been created
  • http://edlab-www.cs.umass.edu
• Homework 1 is up on the course webpage
  • Due September 22 before the start of the class
  • Submission instructions will the posted soon
• Lecture 1 slides posted
  • Do you also want 2 slides/page, 4 slides/page versions?
• Last day of class is December 3 (expect a mid-point report of your projects). Final project reports will be due on December 12.
Cameras

Albrecht Dürer early 1500s

Brunelleschi, early 1400s
Overview of the next two lectures

• The pinhole projection model
  • qualitative properties
  • perspective projection matrix

• Cameras with lenses
  • Depth of focus
  • Field of view
  • Lens aberrations

• Digital cameras
  • Sensors
  • Colors
  • Artifacts
Let's design a camera

**Object**

**Film**

**Idea 1:** Let's put a film in front of an object. Do we get a reasonable image?
Let's design a camera

Object

A

Film

Idea 1: Let's put a film in front of an object. Do we get a reasonable image?
Let's design a camera

Idea 1: Let's put a film in front of an object. Do we get a reasonable image?
Add a barrier to block of most rays
Add a barrier to block of most rays
Pinhole camera

Add a barrier to block of most rays
Pinhole camera

• Captures **pencil of rays** - all rays through a single point: aperture, center of projection, focal point, camera center
• The image is formed on the **image plane**
Camera obscura

• Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
• Drawing aids for artists: described by Leonardo Da Vinci (1452-1519 AD)

Gemma Frisius, 1558

“Camera obscure” Latin for “darkened room”
Pinhole cameras are everywhere

Tree shadow during a solar eclipse
photo credit: Nils van der Burg
http://www.physicstogo.org/index.cfm

Slide by Steve Seitz
Accidental pinhole cameras

My hotel room, contrast enhanced. The view from my window

Accidental pinholes produce images that are unnoticed or misinterpreted as shadows

A. Torralba and W. Freeman, Accidental Pinhole and Pinspeck Cameras, CVPR 2012
Home-made pinhole camera

http://www.pauldebevec.com/Pinhole
Dimensionality reduction: 3D to 2D

- What is preserved?
  - Straight lines, incidence
- What is not preserved?
  - Angles, lengths
Modeling projection
To compute the projection $P'$ of a scene point $P$, form a **visual ray** connection $P$ to the camera center $O$ and find where it intersects the image plane.
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- All scene points that lie on this visual ray have the same projection on the image.
To compute the projection $P'$ of a scene point $P$, form a **visual ray** connection $P$ to the camera center $O$ and find where it intersects the image plane.

- All scene points that lie on this visual ray have the same projection on the image.
- Are there points for which this projection is not defined?
Modeling projection
• The coordinate system
• **The coordinate system**
  
  • The optical center (\(O\)) is at the origin
Modeling projection

- The coordinate system
  - The optical center ($O$) is at the origin
  - The image plane is parallel to the xy-plane (perpendicular to the z axis)
- **The coordinate system**
  - The optical center \( O \) is at the origin
  - The image plane is parallel to the xy-plane (perpendicular to the z axis)

- **Projection equations**
Modeling projection

- The coordinate system
  - The optical center \( O \) is at the origin
  - The image plane is parallel to the xy-plane (perpendicular to the z axis)

- Projection equations
  - Derive using similar triangles

\[
(x, y, z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)
\]
Projection of a line

image plane

camera center

line in the scene
Projection of a line

image plane

camera center

line in the scene
Projection of a line

image plane

camera center

line in the scene
Projection of a line

image plane

camera center

line in the scene

Slide by Steve Seitz
Projection of a line

image plane

camera center

line in the scene
Projection of a line

image plane

camera center

line in the scene
Projection of a line

image plane

camera center

vanishing point

line in the scene
Projection of a line

- What if we add another line parallel to the first one?
Vanishing points

• Each direction in space has its own vanishing point
  • All lines going in the that direction converge at that point
Vanishing points

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  - All lines going in that direction converge at that point
  - **Exception**: directions that are parallel to the image plane
Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in the that direction converge at that point
  - **Exception**: directions that are parallel to the image plane
- What about the vanishing point of a plane?
Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - **Exception**: directions that are parallel to the image plane
- What about the vanishing point of a plane?
The horizon

- Vanishing line of the ground plane
  - All points at the same height of the camera project to the horizon
  - Points above the camera project above the horizon
  - Provides a way of comparing heights of objects
Is the person above or below the viewer?
Perspective cues
Perspective cues
Perspective cues
Comparing heights
Comparing heights
Comparing heights
Comparing heights

vanishing point
Measuring heights
Measuring heights
Measuring heights
Measuring heights
Measuring heights
Measuring heights
What is the height of the camera?
Measuring heights

What is the height of the camera?
Perspective in art

Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

One of the first consistent uses of perspective in Western art
(At least partial) Perspective projections in art well before the Renaissance

Several Pompei wallpaintings show the fragmentary use of linear perspective:

From ottobwiersma.nl

Also some Greek examples,
So apparently pre-renaissance…
What does a sphere project to?
What does a sphere project to?

Perspective distortion
Perspective distortion

- The exterior looks bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci
Modeling projection

- Projection equation
• Projection equation

\[(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z}\right)\]
Homogeneous coordinates

$$(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})$$
Homogeneous coordinates

\[(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})\]

- Is this a linear transformation?
Homogeneous coordinates

\[(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})\]

• Is this a linear transformation?
  • no — division by z is not linear
Homogeneous coordinates

\[(x, y, z) \mapsto \left( f \frac{x}{z}, f \frac{y}{z} \right) \]

• Is this a linear transformation?
  • no — division by z is not linear

• **Trick:** add one more coordinate

\[
(x, y) \mapsto \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{homogeneous image coordinates}
\]

\[
(x, y, z) \mapsto \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \text{homogeneous scene coordinates}
\]
Homogeneous coordinates

\[(x, y, z) \mapsto \left(f \frac{x}{z}, f \frac{y}{z}\right)\]

- Is this a linear transformation?
  - no — division by \(z\) is not linear

- **Trick:** add one more coordinate

\[
\begin{align*}
(x, y) &\Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} & (x, y, z) &\Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\end{align*}
\]

homogeneous image coordinates \hspace{2cm} homogeneous scene coordinates

- Converting from homogeneous coordinates

\[
\begin{align*}
\begin{bmatrix} x \\ y \\ w \end{bmatrix} &\Rightarrow (x/w, y/w) \\
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} &\Rightarrow (x/w, y/w, z/w)
\end{align*}
\]
Perspective projection matrix

- Projection is a matrix multiplication using homogeneous coordinates (scene and image)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/f & 0 \\
0 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
z/f \\
1
\end{bmatrix} \Rightarrow (f \frac{x}{z}, f \frac{y}{z})
\]

divide by the third coordinate

- In practice: lots of coordinate transforms

\[
\begin{bmatrix}
2D \\
\text{point} \\
(3x1)
\end{bmatrix}
= \begin{bmatrix}
\text{Camera to pixel coord.} \\
\text{trans. matrix} \\
(3x3)
\end{bmatrix}
\begin{bmatrix}
\text{Perspective} \\
\text{projection matrix} \\
(3x4)
\end{bmatrix}
\begin{bmatrix}
\text{World to} \\
\text{camera coord.} \\
\text{trans. matrix} \\
(4x4)
\end{bmatrix}
\begin{bmatrix}
3D \\
\text{point} \\
(4x1)
\end{bmatrix}
\]
Whole “pipeline”

\[
\begin{bmatrix}
w_P p_i \\
w_P p_j \\
w_P
\end{bmatrix} =
\begin{bmatrix}
s_x & k_1 & 0 \\
k_2 & s_y & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/f & 0
\end{bmatrix}
\begin{bmatrix}
r_{11} & r_{12} & r_{13} & t_x \\
r_{21} & r_{22} & r_{23} & t_y \\
r_{31} & r_{32} & r_{33} & t_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

2D point (3x1) = Camera to pixel coord. trans. matrix (3x3) \( \times \) Perspective projection matrix (3x4) \( \times \) World to camera coord. trans. matrix (4x4) \( \times \) 3D point (4x1)

• Just one matrix with a special structure
Orthographic projection

- Special case of perspective projection
  - Distance of the object from the image plane is infinite
  - Also called the “parallel projection”
Orthographic projection

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- What’s the projection matrix?
Orthographic projection

- Special case of perspective projection
  - Distance of the object from the image plane is infinite
  - Also called the “parallel projection”

- What’s the projection matrix?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\Rightarrow (x, y)
\]
More readings and thoughts

- **History of optics**, Wikipedia
- DIY [http://www.pauldebevec.com/Pinhole](http://www.pauldebevec.com/Pinhole)
- In MATLAB, compute the projection of a sphere using the perspective model and visualize the distortions