

# **CMPSCI 670: Computer Vision**

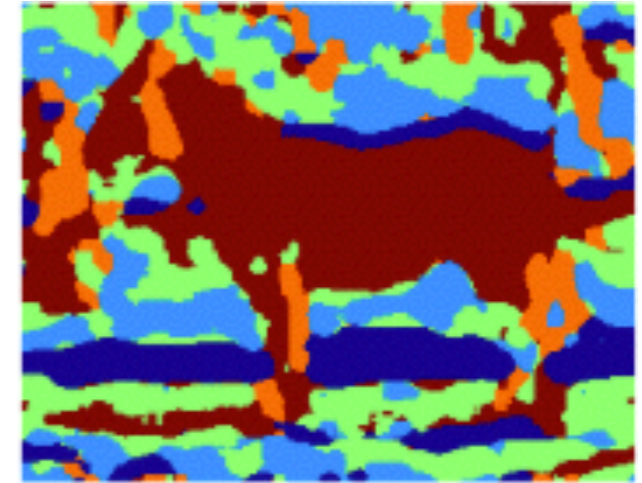
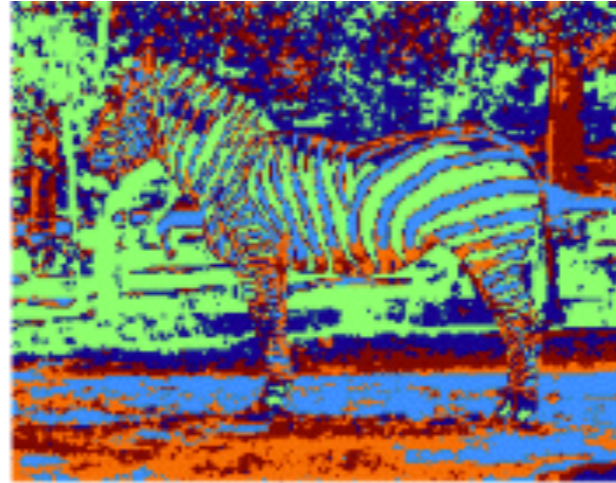
## Alignment continued ...

University of Massachusetts, Amherst  
October 22, 2014

Instructor: Subhransu Maji

# Administrivia

- Homework 4 posted (due Nov 5)
  - Grouping and texture



- Office hours ~~today~~ tomorrow 2:30 - 3:30 pm



## Distinguished Lecturer Series

Jennifer Chayes  
Microsoft Research

Wednesday, October 22, 2014  
4:00pm - 5:00pm  
Computer Science Building, Room 151  
Faculty Host: Andrew McGregor

### "Age of Networks"

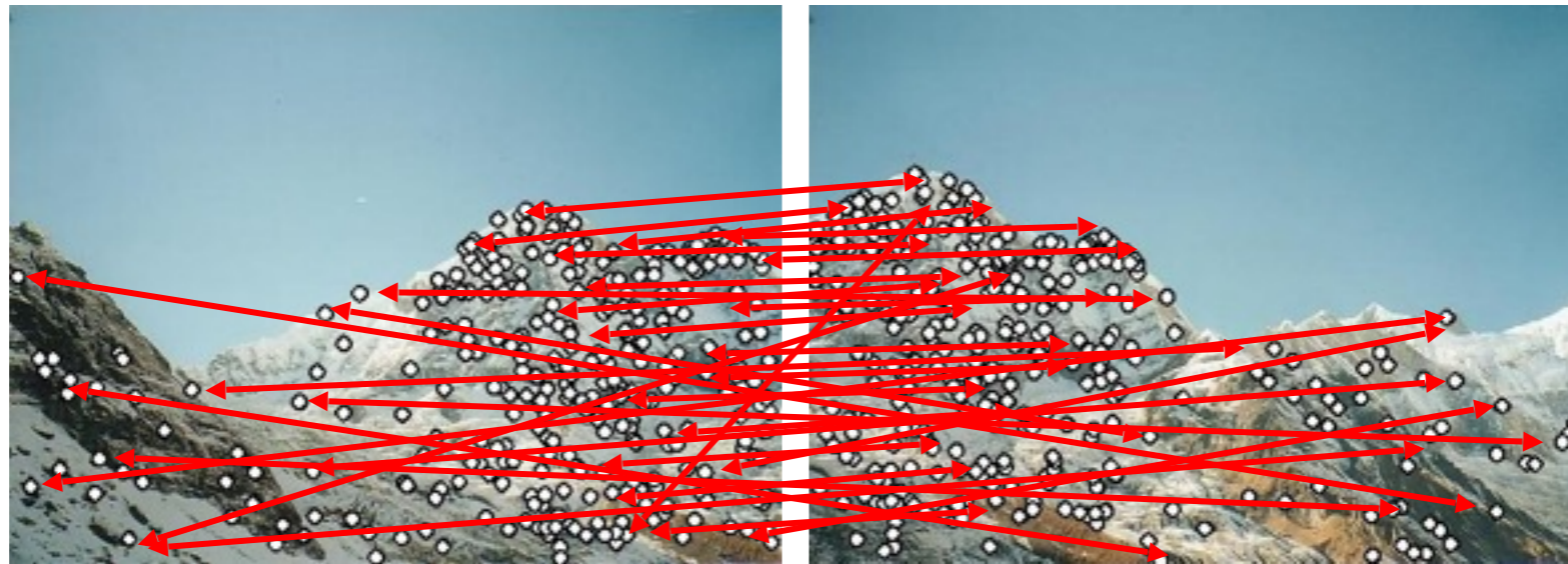
Everywhere we turn, we find that networks can be used to describe the World Wide Web, and a variety of online social networks. In economics, we see the negative effects of a global networked economy. In epidemiology, we see the spread of disease, complicated by mutation of the disease agents. In biomedical research,

# Alignment so far ...

- What are the alignment problems in computer vision?
  - rigid vs. deformable (non-rigid)
- Good features to match
  - Invariance properties
  - Local features: the SIFT descriptor
- Alignment algorithms
  - Rigid alignment: RANSAC
    - Application: panoramic photo stitching
  - Non-rigid alignment
    - Application: shape matching

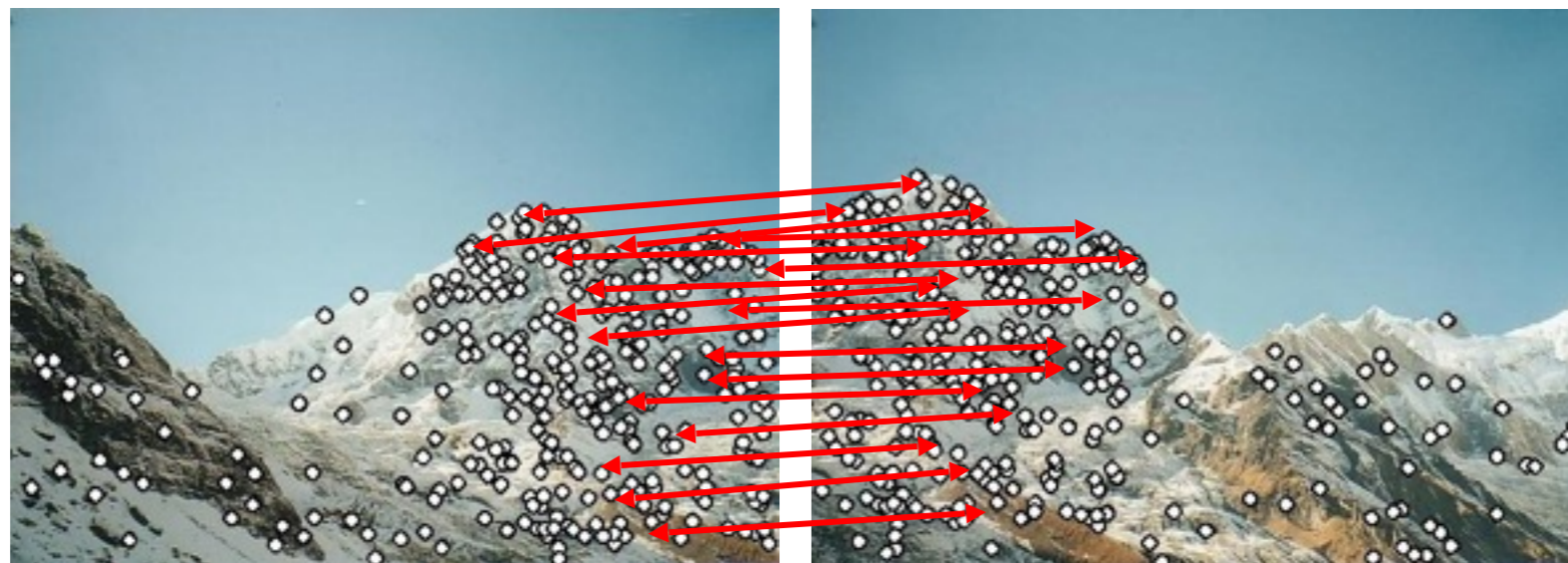
# A framework for alignment

- Matching local features
  - Local information used, can contain outliers
  - But hopefully enough of these matches are good

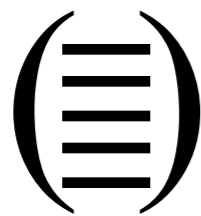
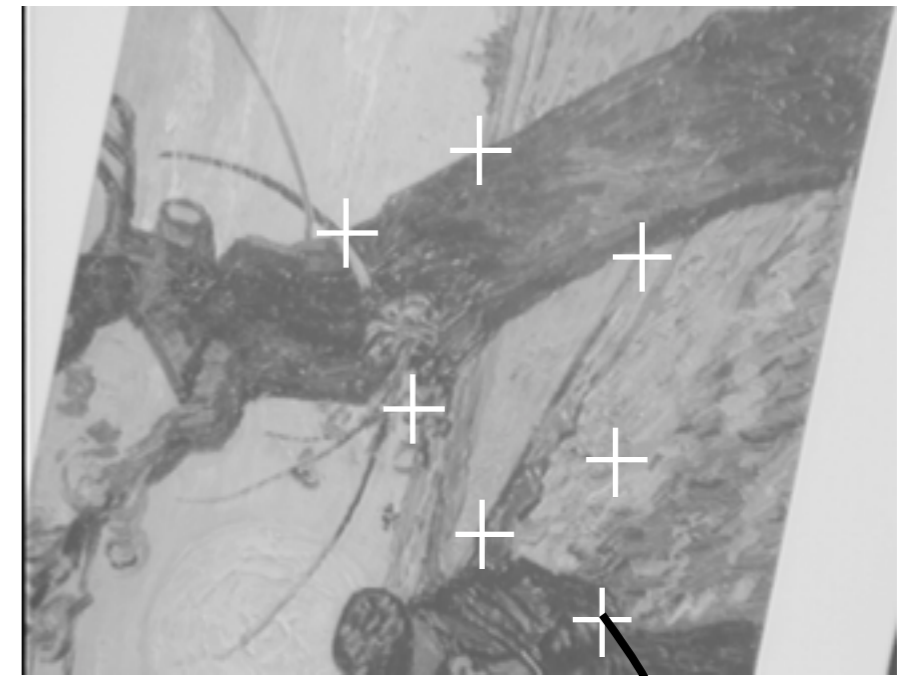
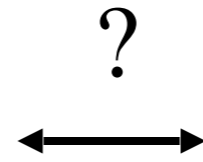
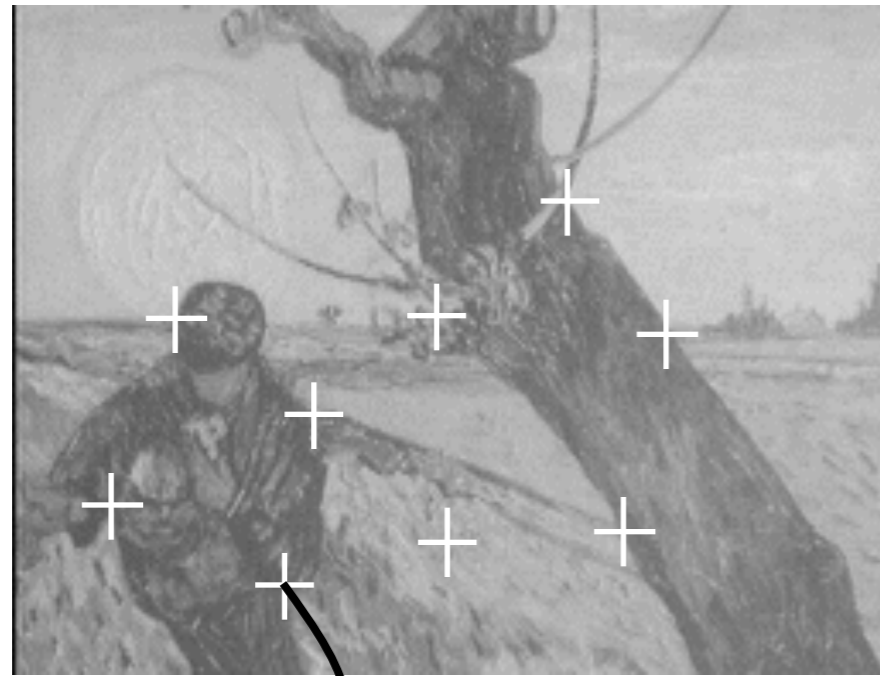


# A framework for alignment

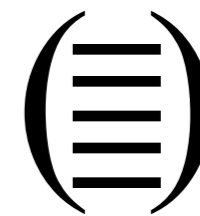
- Matching local features
  - Local information used, can contain outliers
  - But hopefully enough of these matches are good
- Consensus building
  - Aggregate the good matches and find a transformation that explains these matches



# Generating putative correspondences



feature  
descriptor

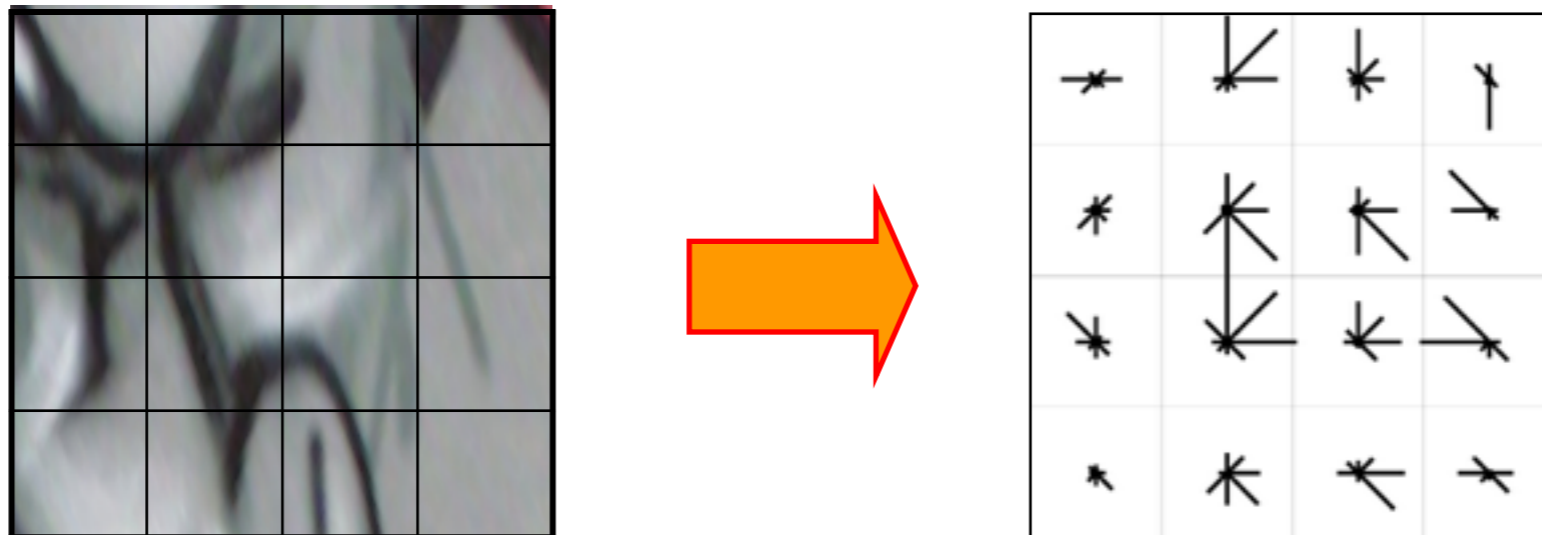


feature  
descriptor

- Need to compare *feature descriptors* of local patches surrounding interest points

# Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor:  $4 \times 4 \times 8 = 128$  dimensions



David G. Lowe. ["Distinctive image features from scale-invariant keypoints."](#) *IJCV* 60 (2), pp. 91-110, 2004.

# Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor:  $4 \times 4 \times 8 = 128$  dimensions
- Advantage over raw vectors of pixel values
  - Gradients less sensitive to illumination change
  - Pooling of gradients over the sub-patches achieves robustness to small shifts, but still preserves some spatial information

David G. Lowe. ["Distinctive image features from scale-invariant keypoints."](#) *IJCV* 60 (2), pp. 91-110, 2004.



# Alignment

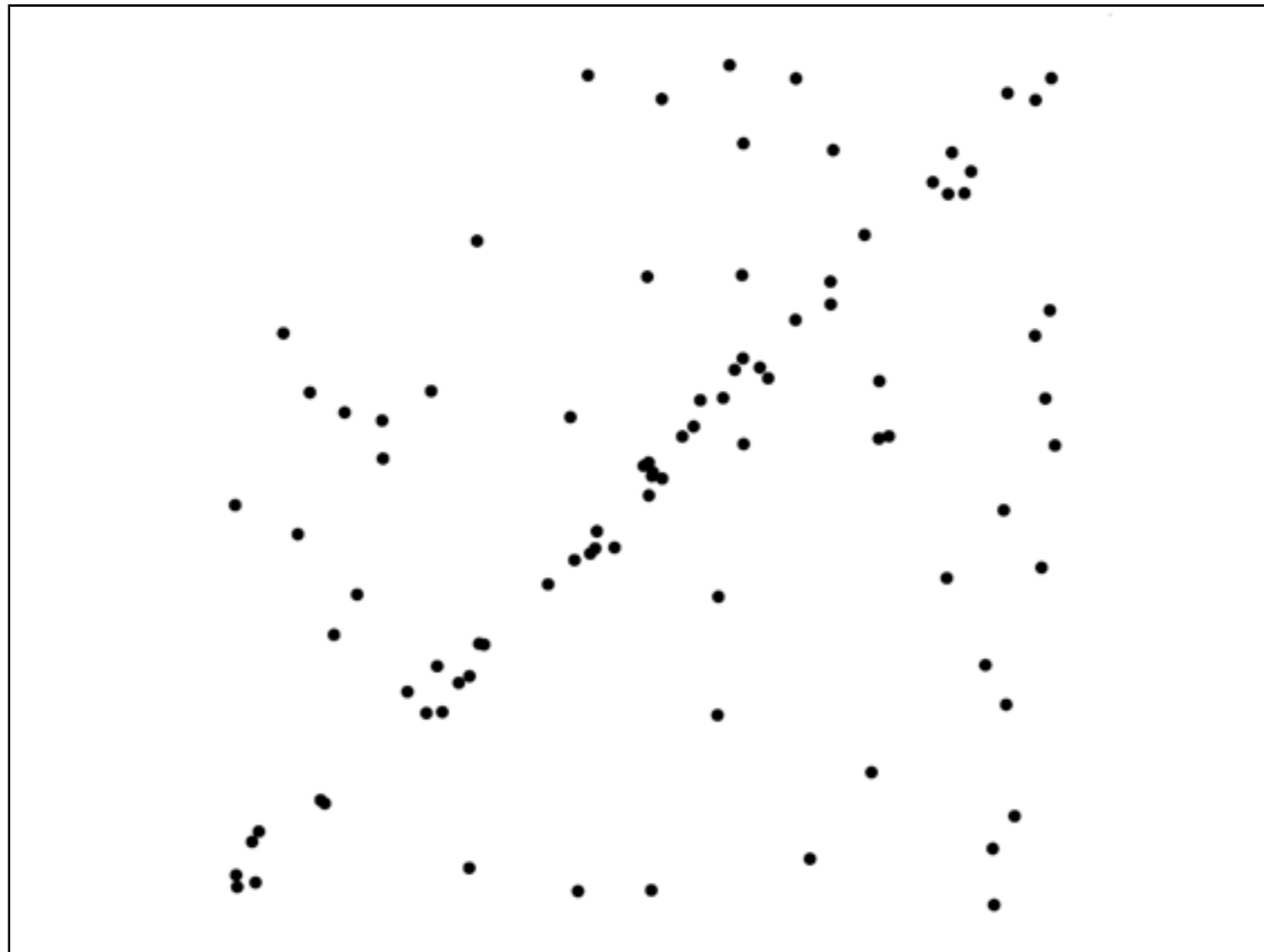
- What are the alignment problems in computer vision?
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    - Application: panoramic photo stitching
  - Non-rigid alignment
    - Application: shape matching

# RANSAC

- **R**andom **S**ample **C**onsensus
  - Choose a small subset of points uniformly at random
  - Fit a model to that subset
  - Find all remaining points that are “close” to the model and reject the rest as outliers
  - Do this many times and choose the best model

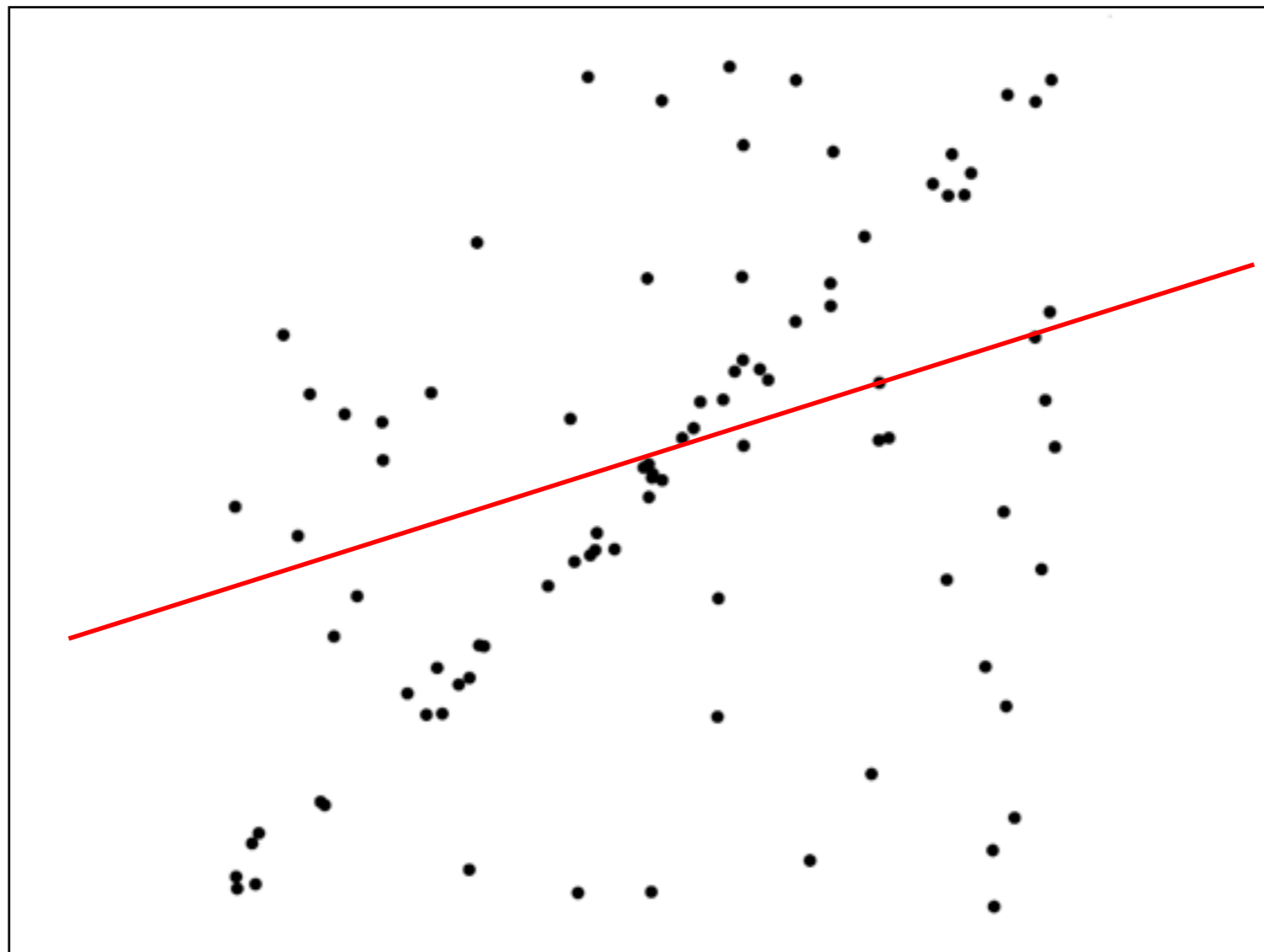
M. A. Fischler, R. C. Bolles. [Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography](#). Comm. of the ACM, Vol 24, pp 381-395, 1981.

# RANSAC for line fitting example



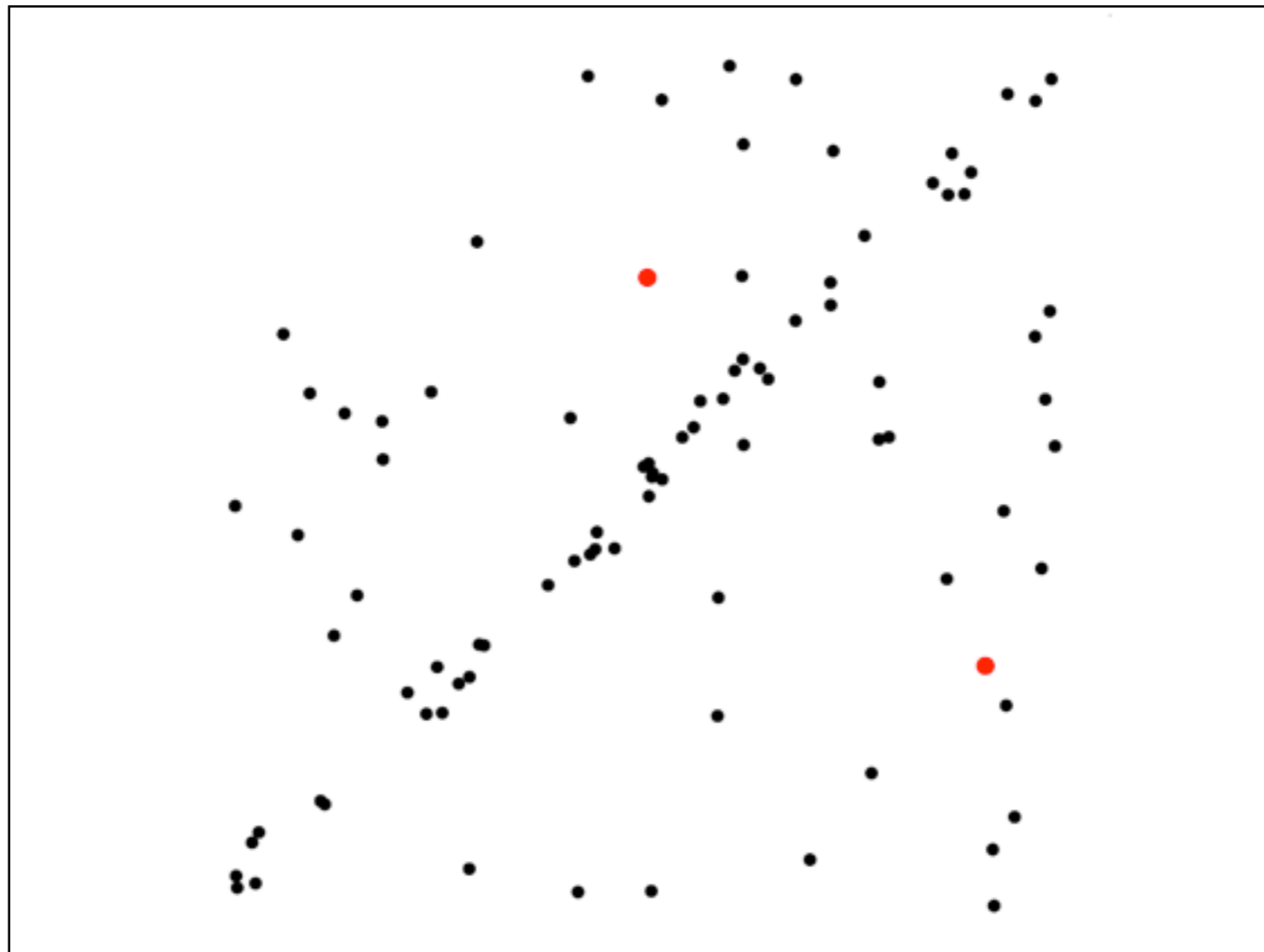
# RANSAC for line fitting example

$$\min_{a,b} \sum_i (ax_i + b - y_i)^2$$



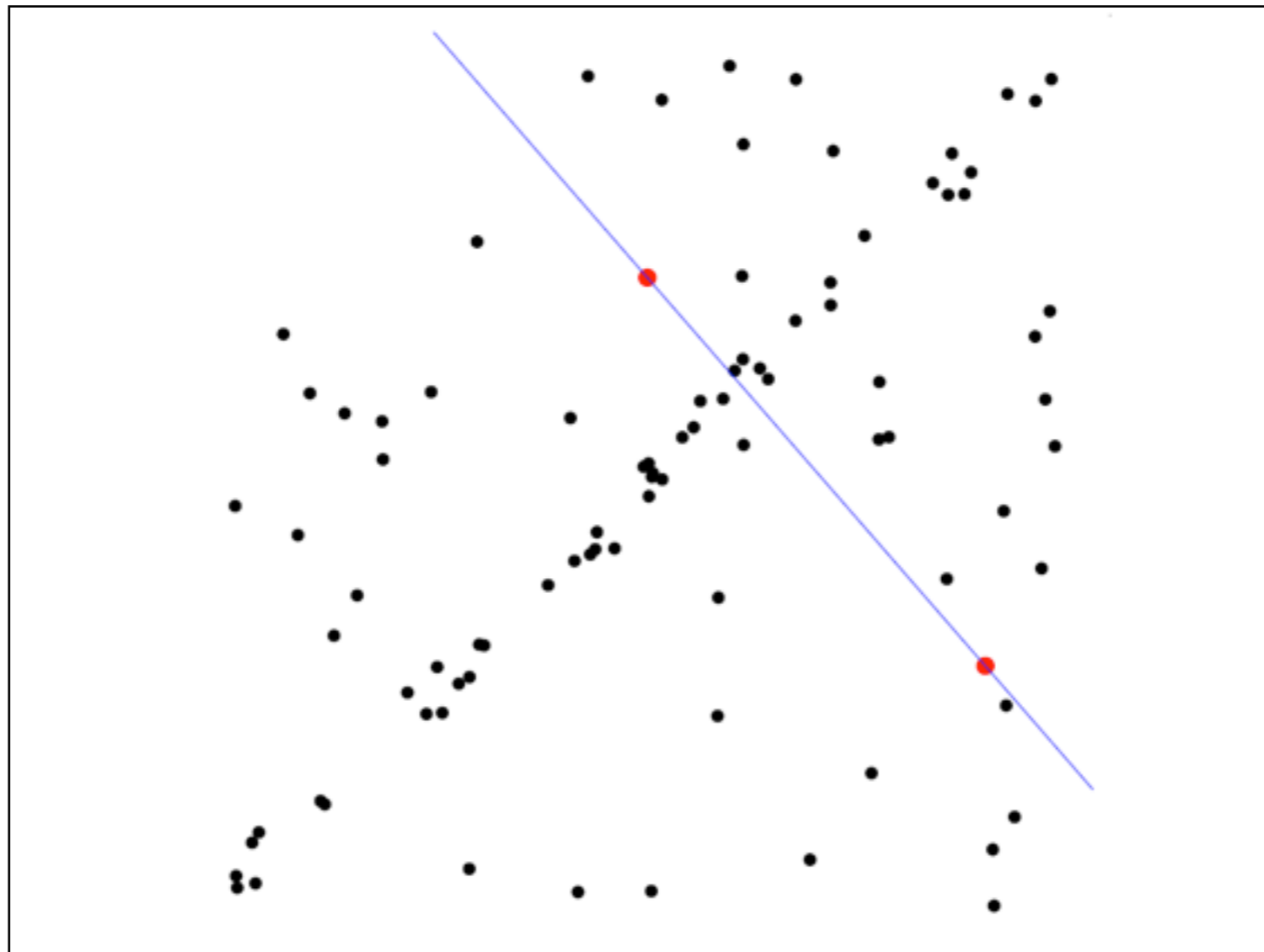
Least-squares fit

# RANSAC for line fitting example



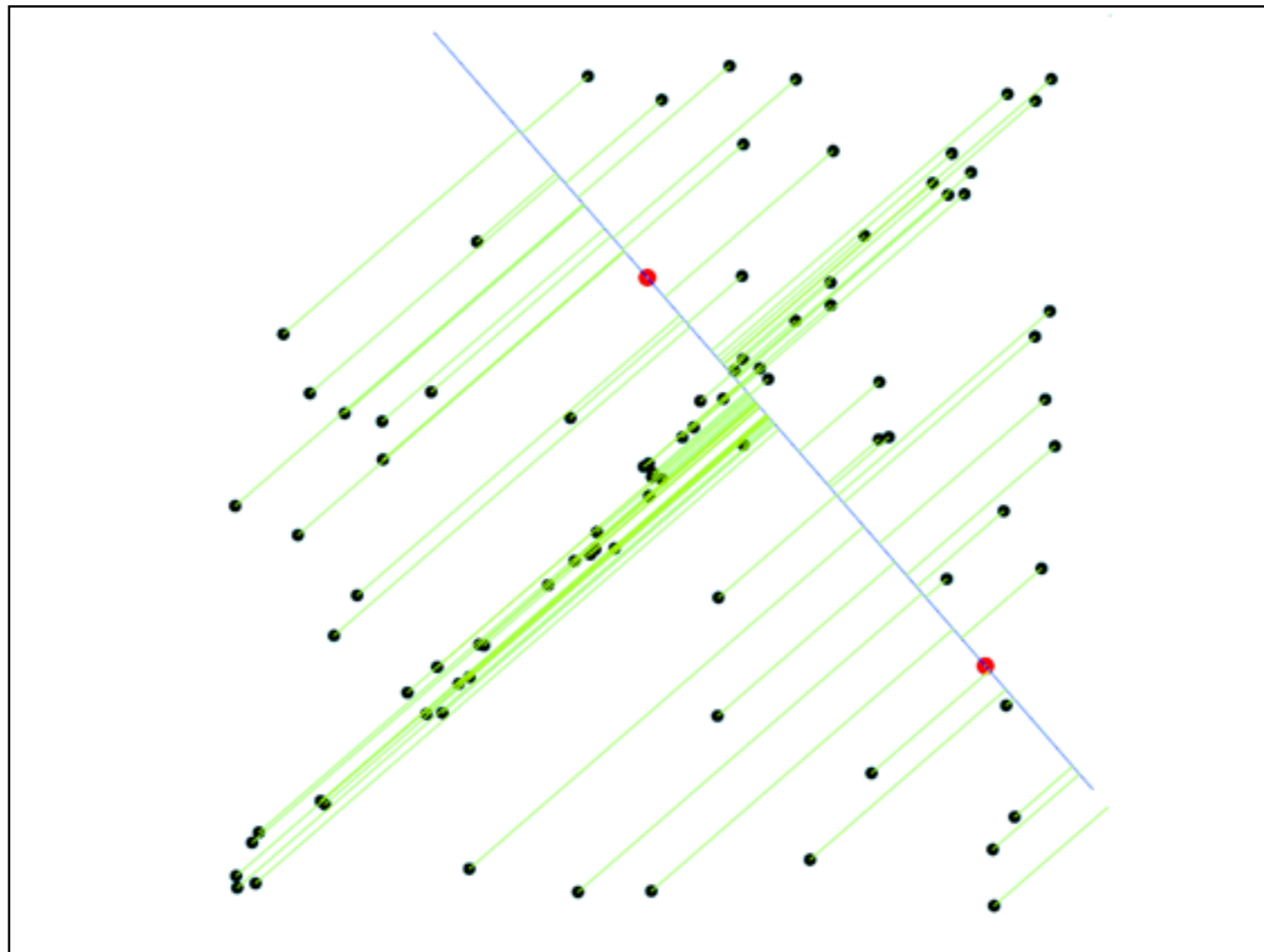
1. Randomly select minimal subset of points

# RANSAC for line fitting example



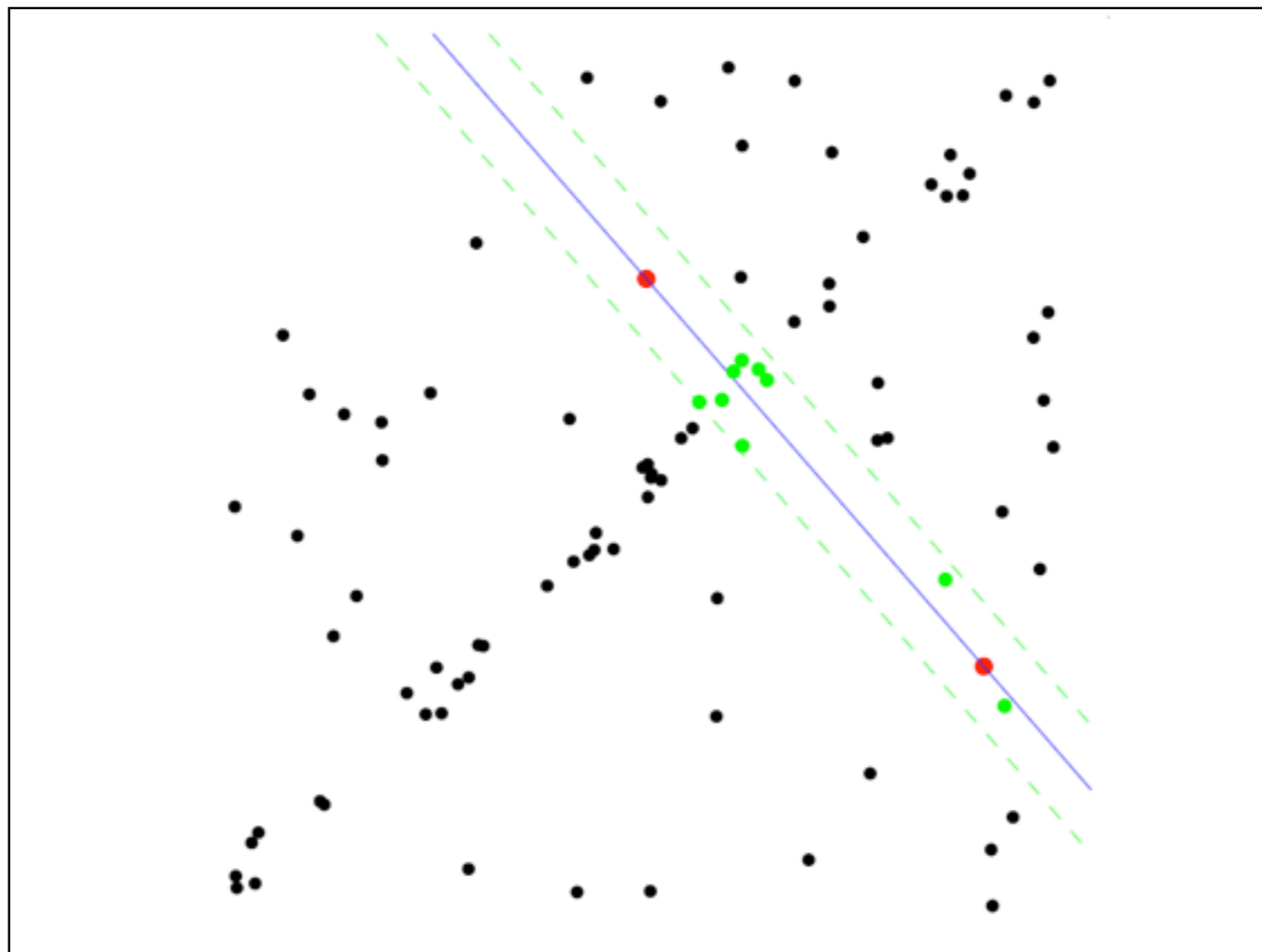
1. Randomly select minimal subset of points
2. Hypothesize a model

# RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

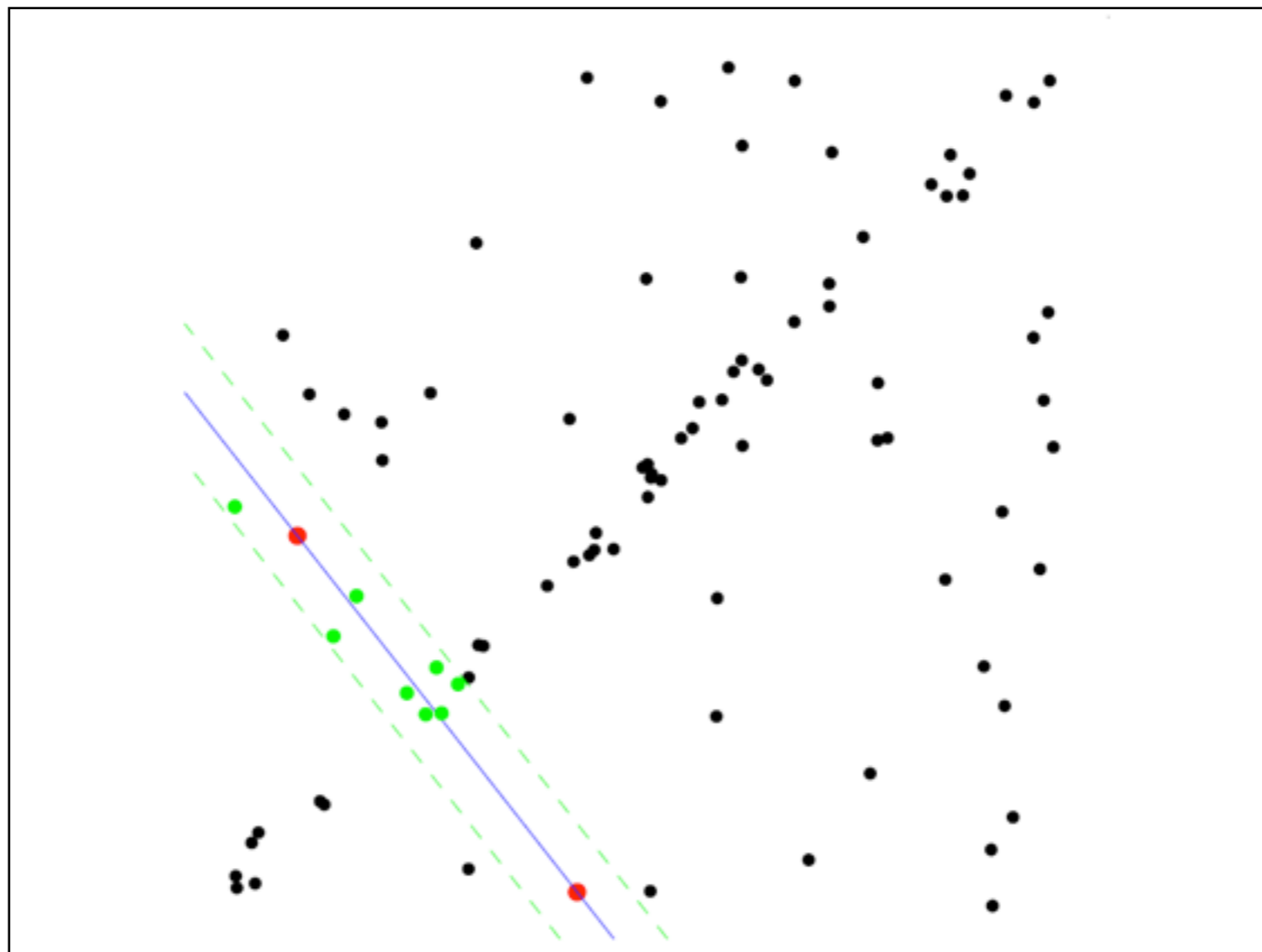
# RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. **Select points consistent with model**

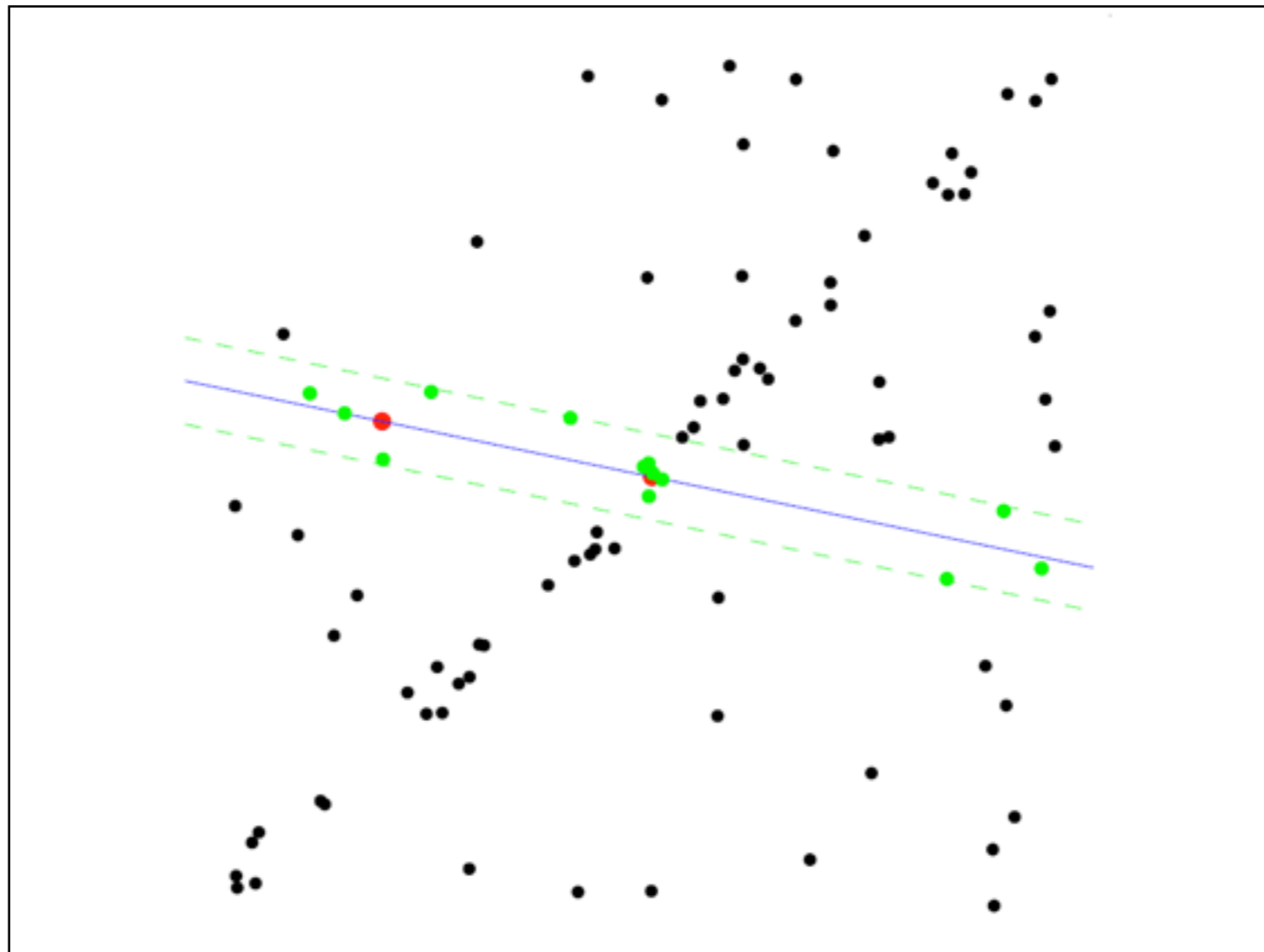


# RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. *Repeat hypothesize-and-verify loop*

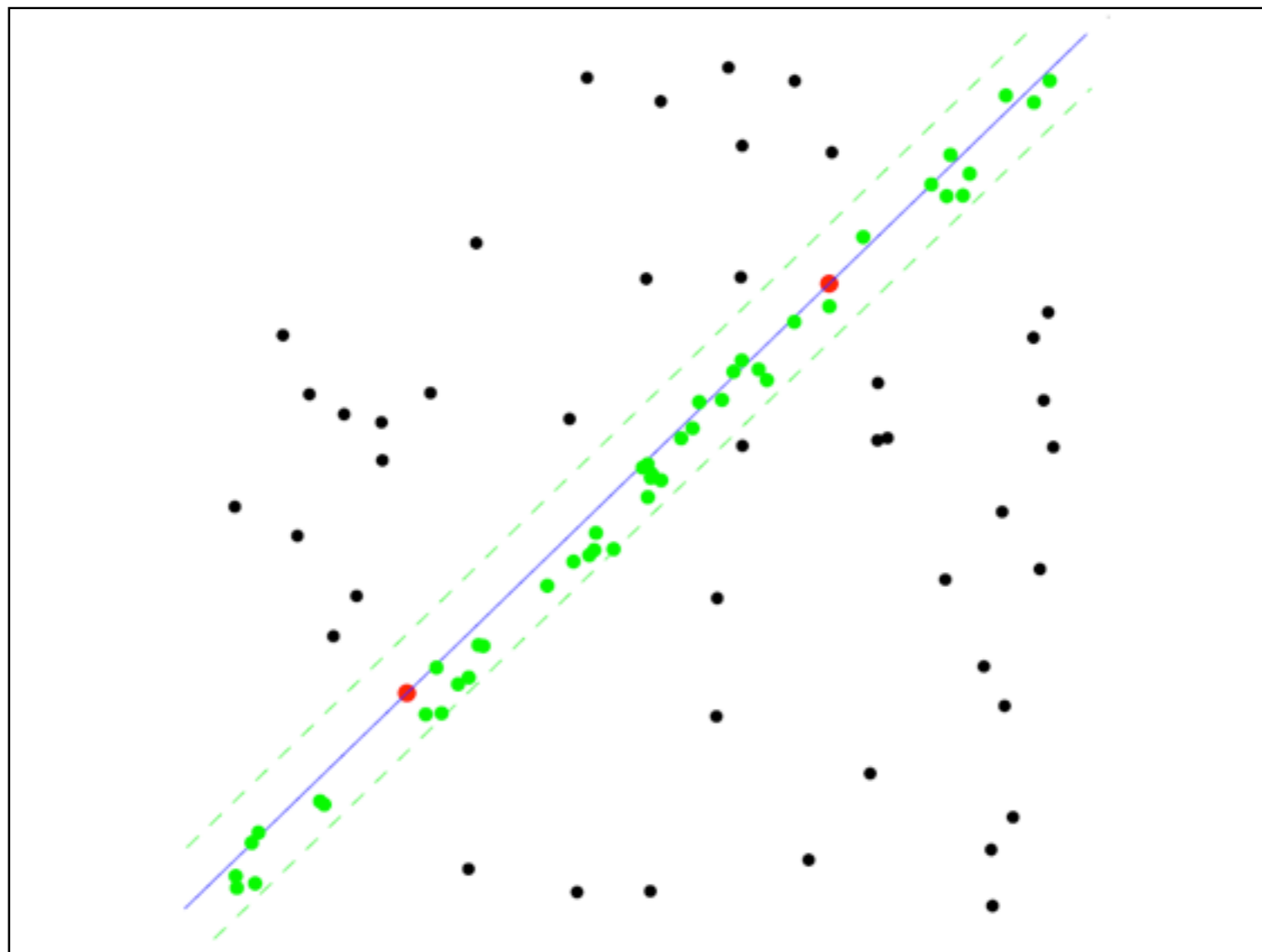
# RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

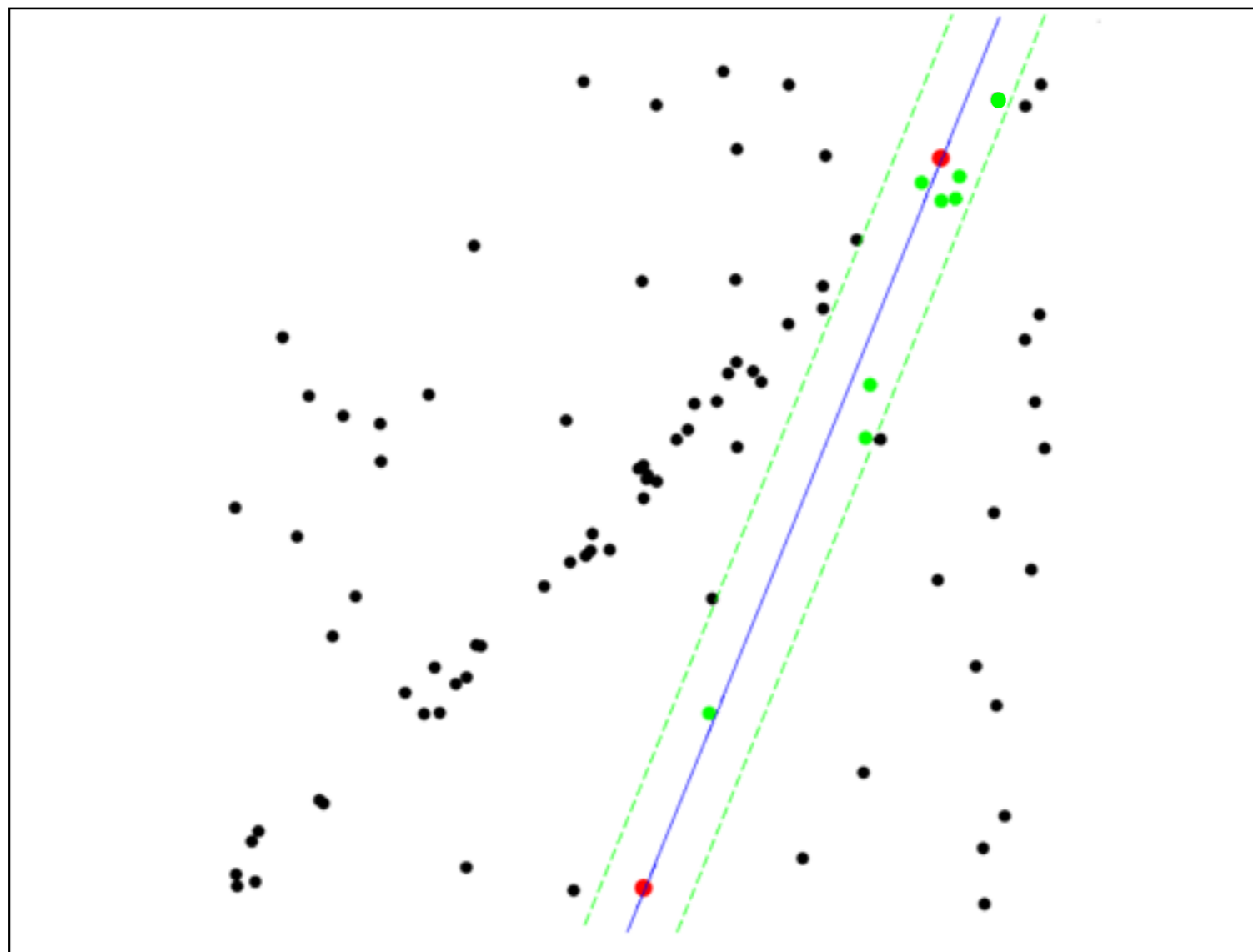
# RANSAC for line fitting example

## Uncontaminated sample



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

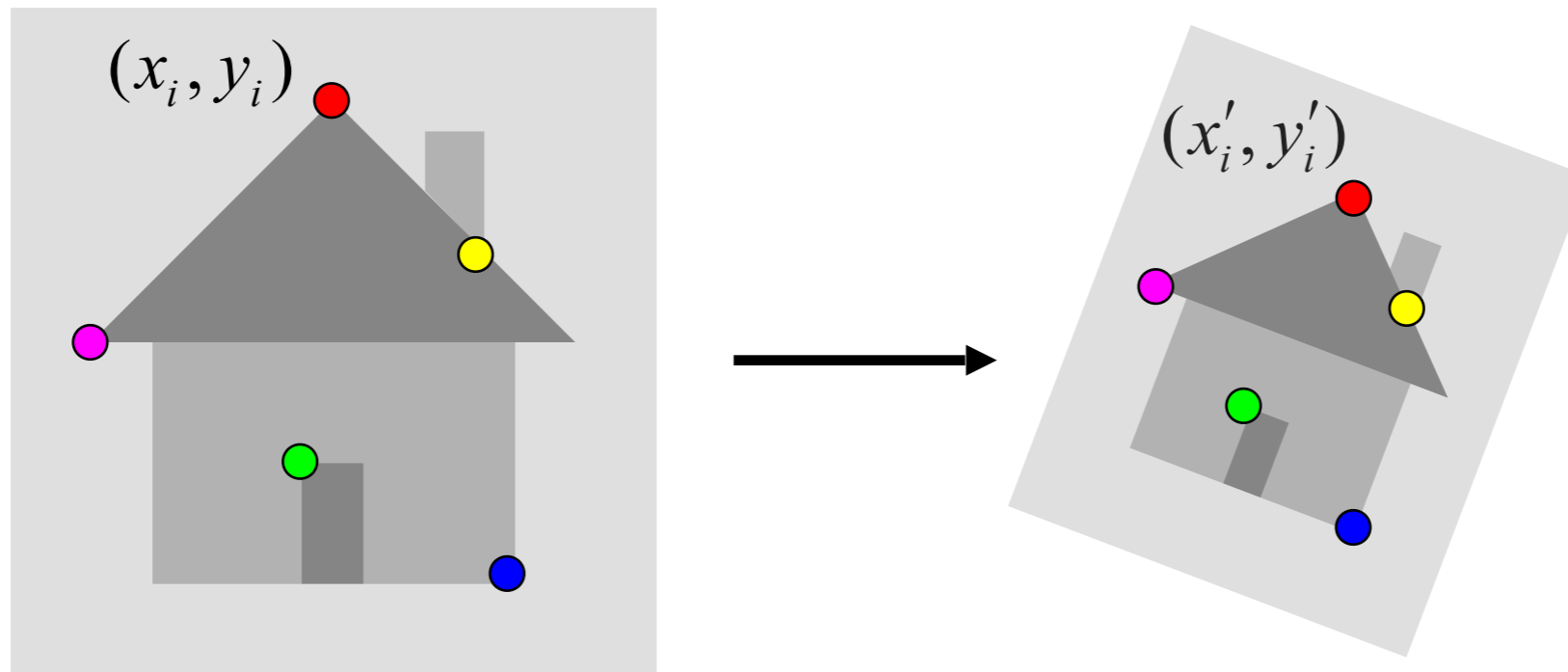
# RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

# Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?



$$\mathbf{x}'_i = \mathbf{M}\mathbf{x}_i + \mathbf{t}$$

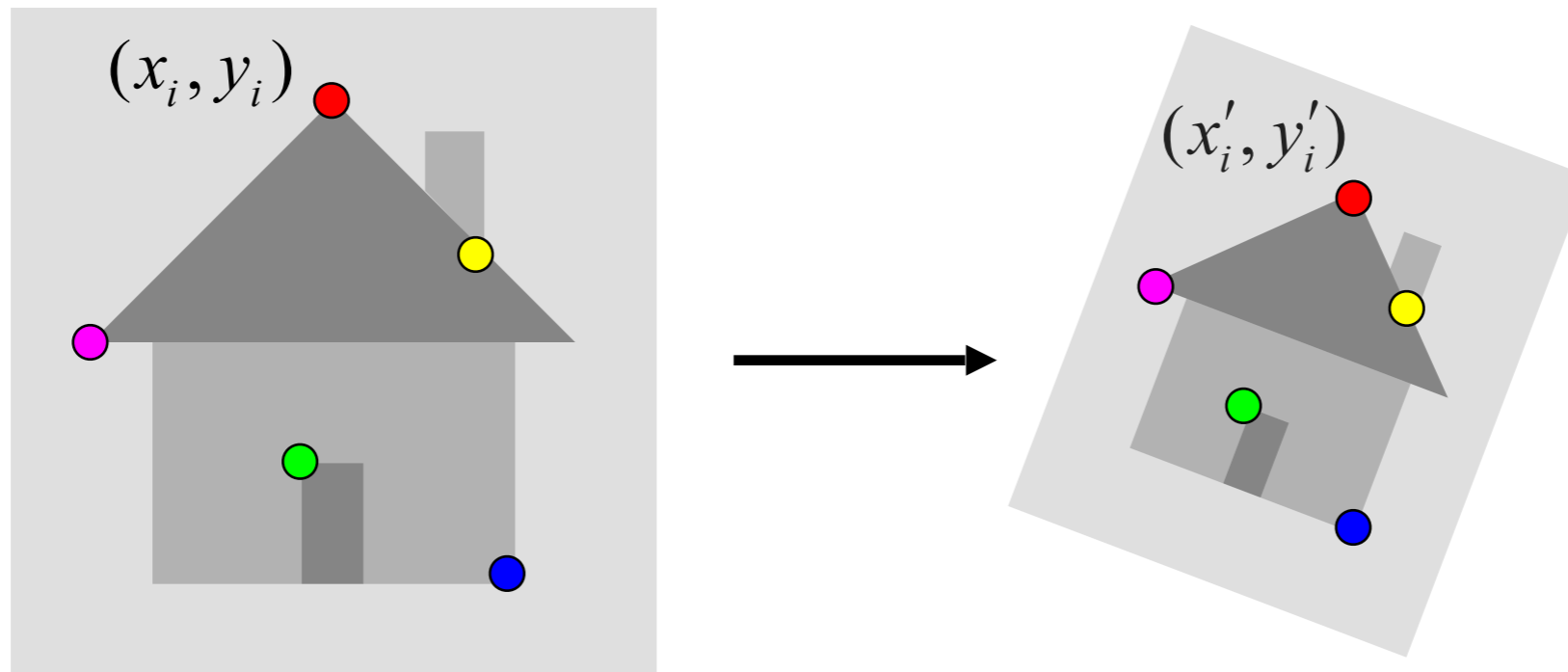
Want to find  $\mathbf{M}$ ,  $\mathbf{t}$  to minimize

$$\sum_{i=1}^n \|\mathbf{x}'_i - \mathbf{M}\mathbf{x}_i - \mathbf{t}\|^2$$

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

# Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

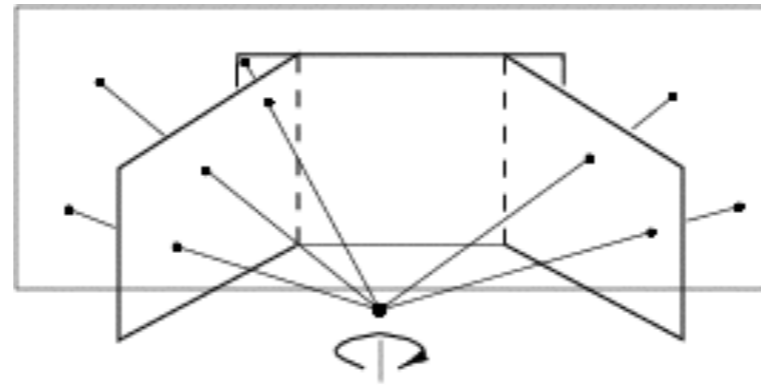
$$\begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix}$$

# Fitting an affine transformation

$$\begin{bmatrix} \dots & & & & & & \\ x_i & y_i & 0 & 0 & 1 & 0 & \\ 0 & 0 & x_i & y_i & 0 & 1 & \\ \dots & & & & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix}$$

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters

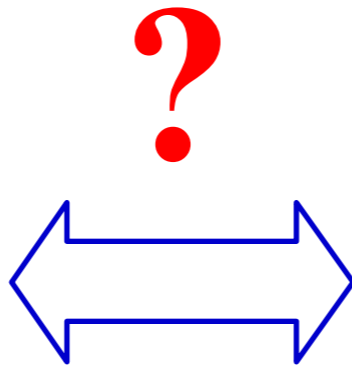
# Application: Panorama stitching





# Panoramic stitching

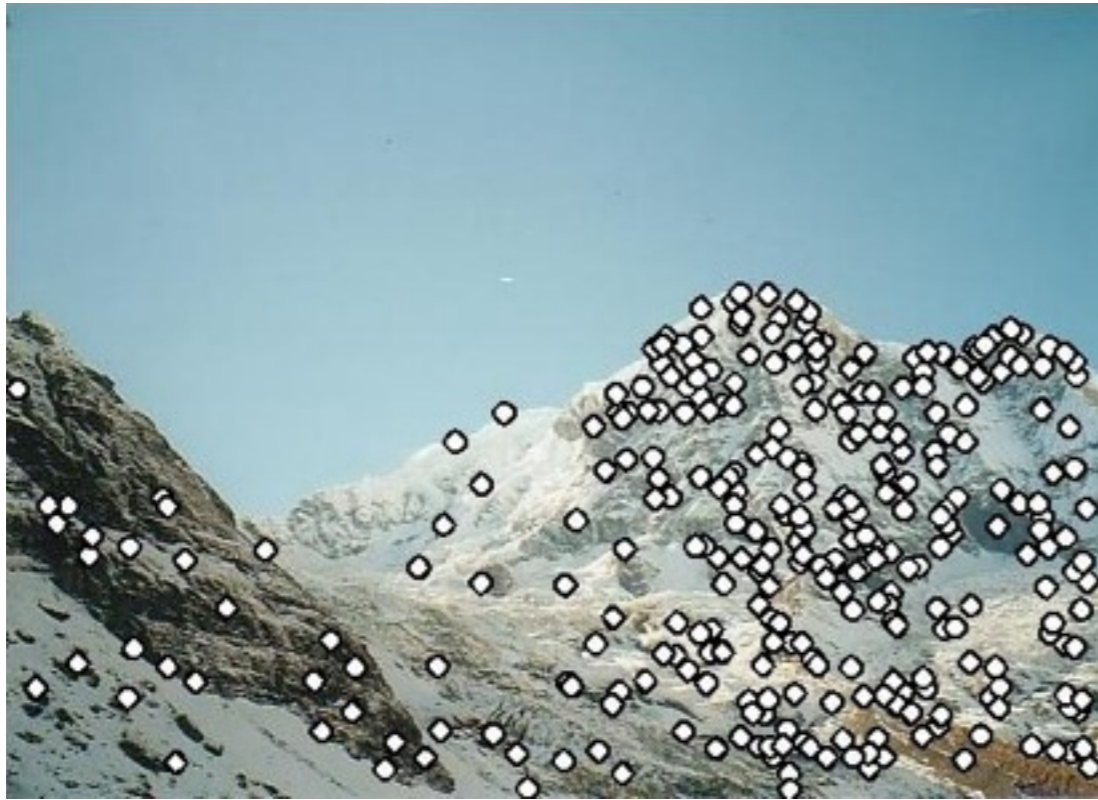
- **Approach**
  - Local feature matching
  - RANSAC for alignment



# Panoramic stitching



# Panoramic stitching



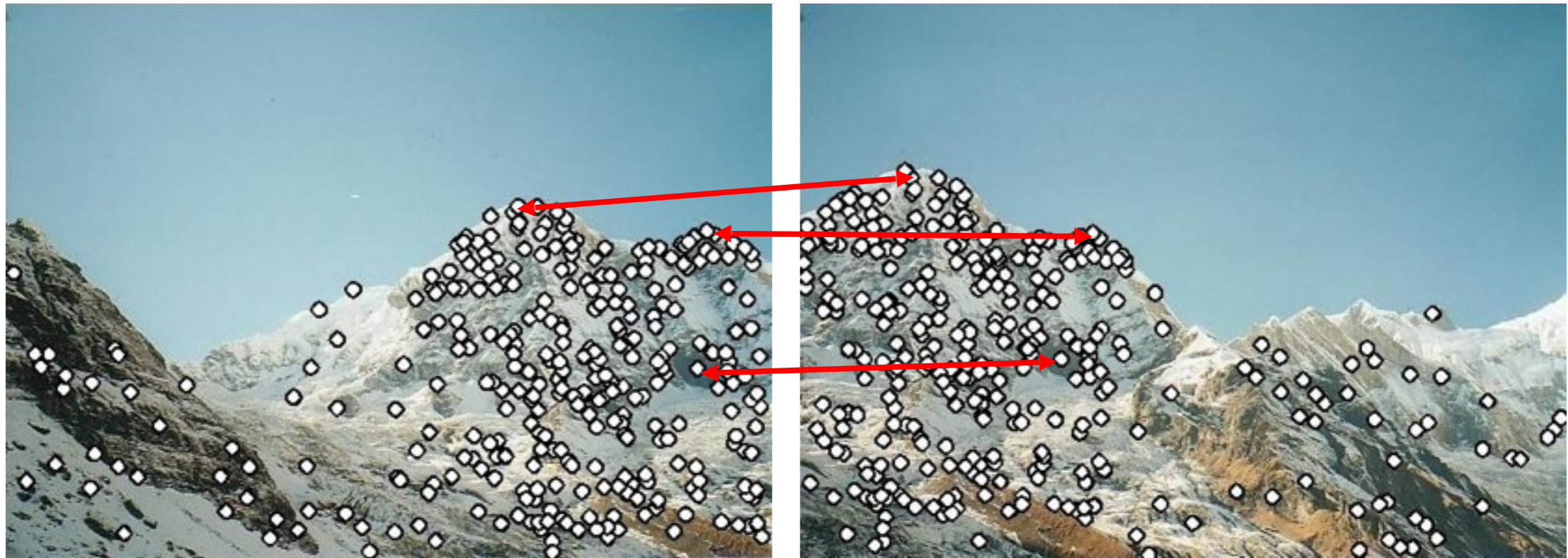
- Extract features
  - corner detector

# Panoramic stitching



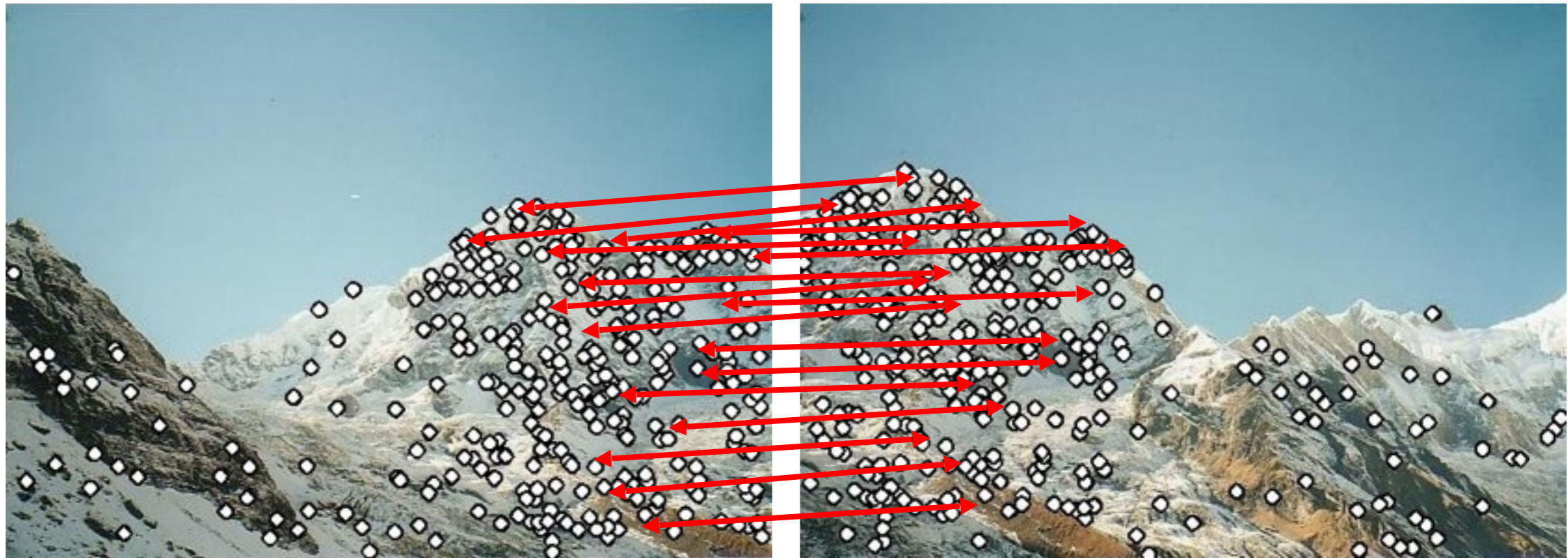
- Extract features
- Compute *putative matches*

# Panoramic stitching



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation  $T$

# Panoramic stitching



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation  $T$
  - *Verify* transformation (search for other matches consistent with  $T$ )

# Panoramic stitching



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation  $T$
  - *Verify* transformation (search for other matches consistent with  $T$ )

# City-scale alignment



Microsoft Photosynth

<https://www.youtube.com/watch?v=y9zF97JL30A>



# Alignment

- What are the alignment problems in computer vision?
  - rigid vs. deformable (non-rigid)
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    - Application: panoramic photo stitching
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# Non-rigid transformations

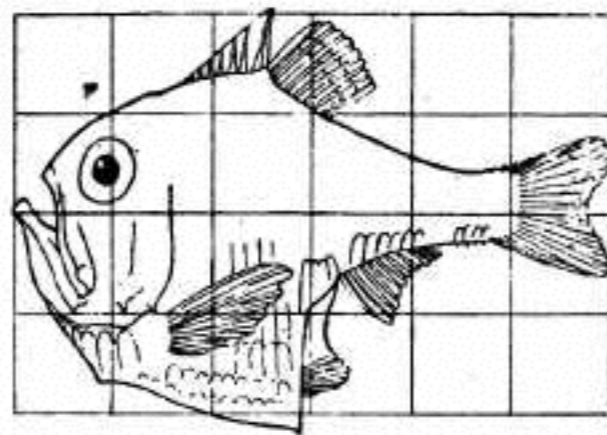
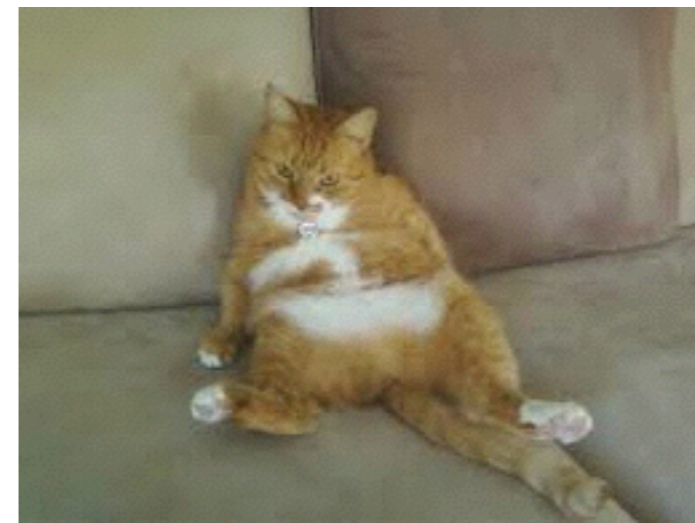
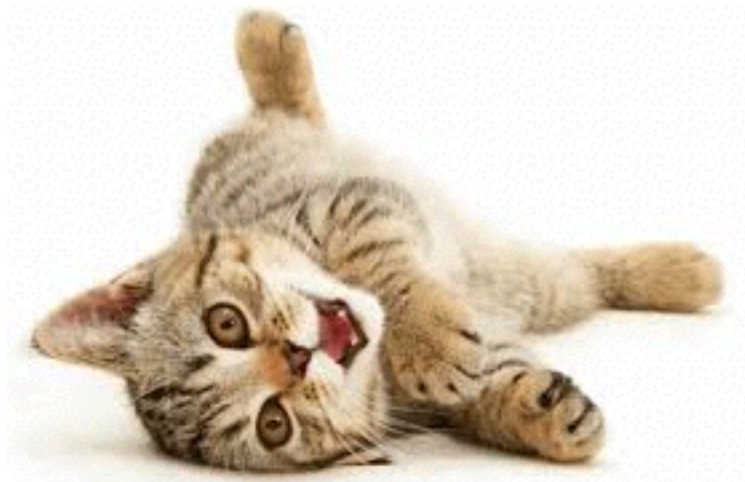
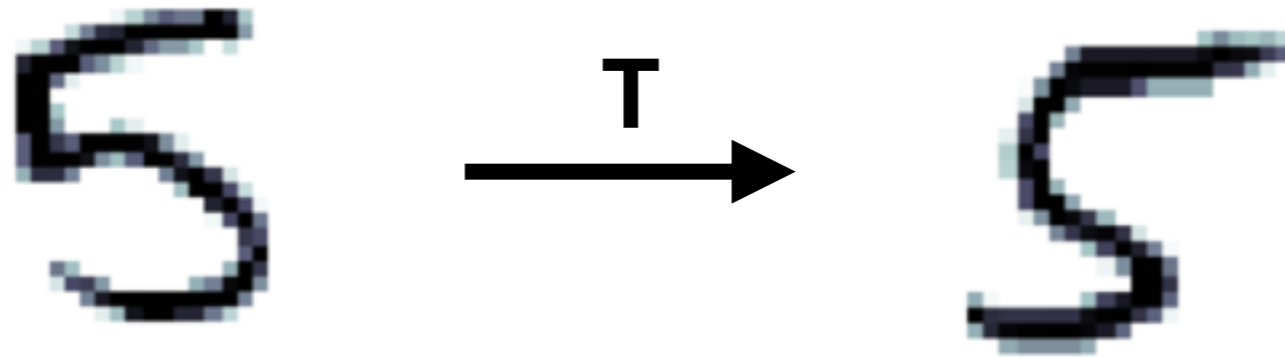


Fig. 517. *Argyropelecus Olfersi*.

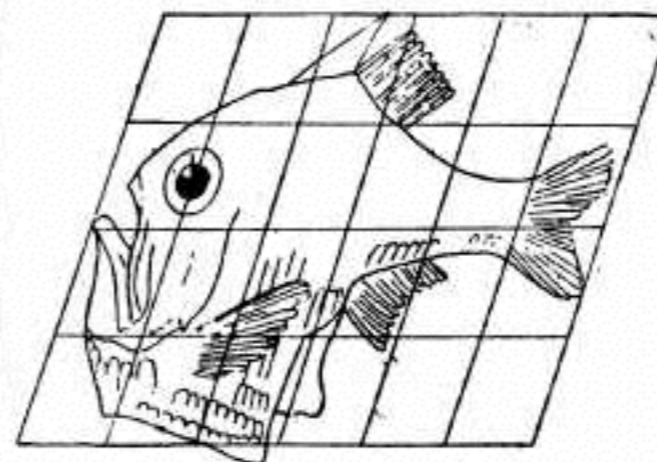
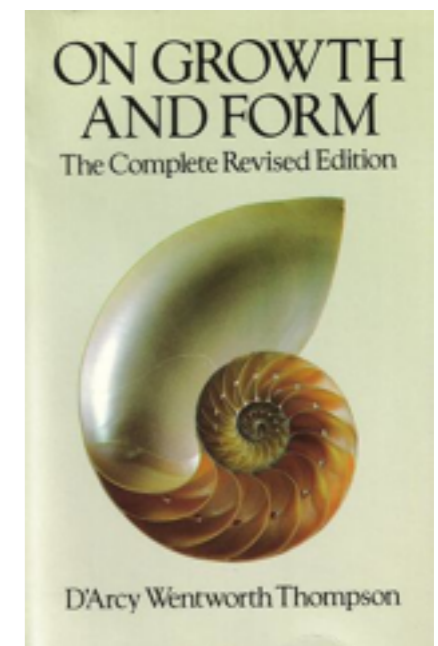


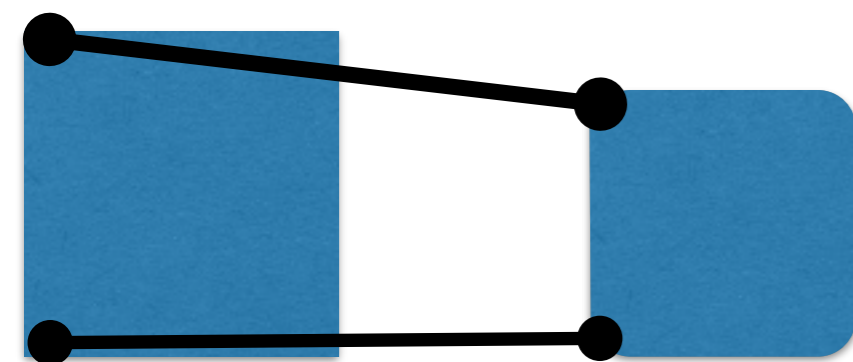
Fig. 518. *Sternoptyx diaphana*.

*On Growth and Form*, D'Arcy Thompson 1915

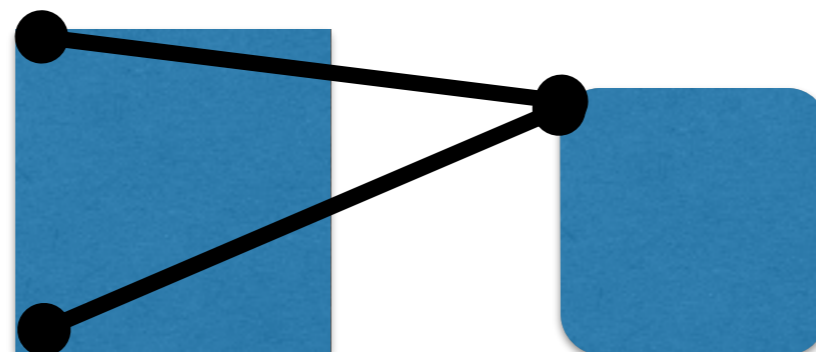


# Non-rigid transformations

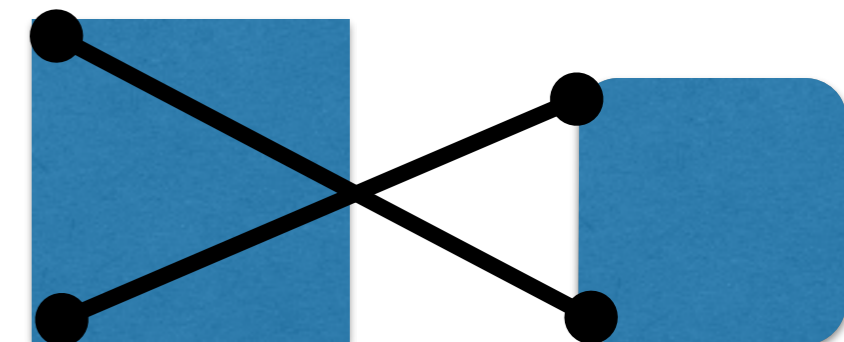
- How to characterize a non-rigid transformation?
- Global constraints
  - Should be from a family of constraints, e.g. Affine
- Local constraints
  - Low distortion, e.g., a good match should avoid
    - Many to one matches, one to many matches, criss crossings



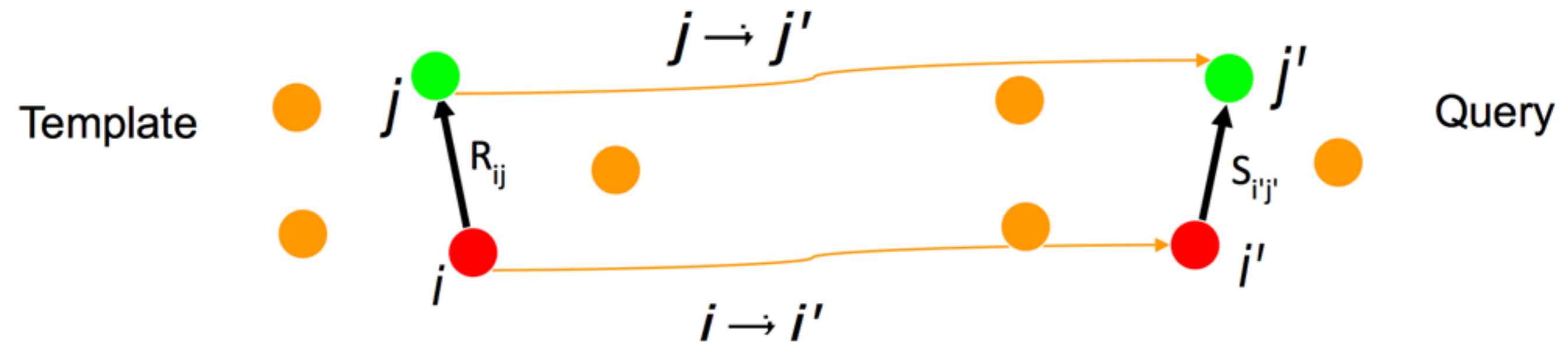
low distortion



high distortion

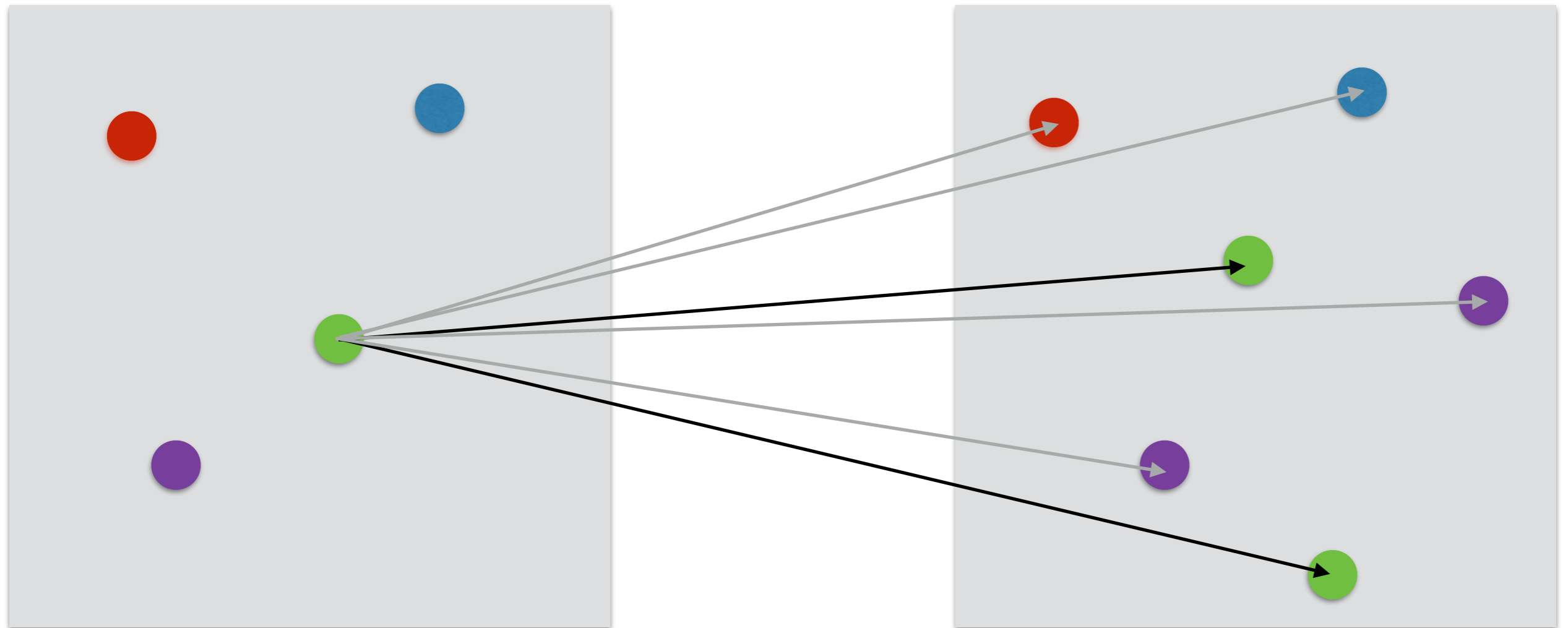


# Low distortion matching



- Measuring distortion between vectors
  - $\mathbf{R}$  and  $\mathbf{S}$  same *length* for *rotations*
  - $\mathbf{R}$  and  $\mathbf{S}$  same *angle* for *scaling*
- Distortion cost:  $\alpha f_r (|R_{ij}|, |S_{i'j'}|) + (1 - \alpha) f_s (\Theta(R_{ij}), \Theta(S_{i'j'}))$
- Matching score:  $c(i \rightarrow i') + c(j \rightarrow j')$

# Scoring matches

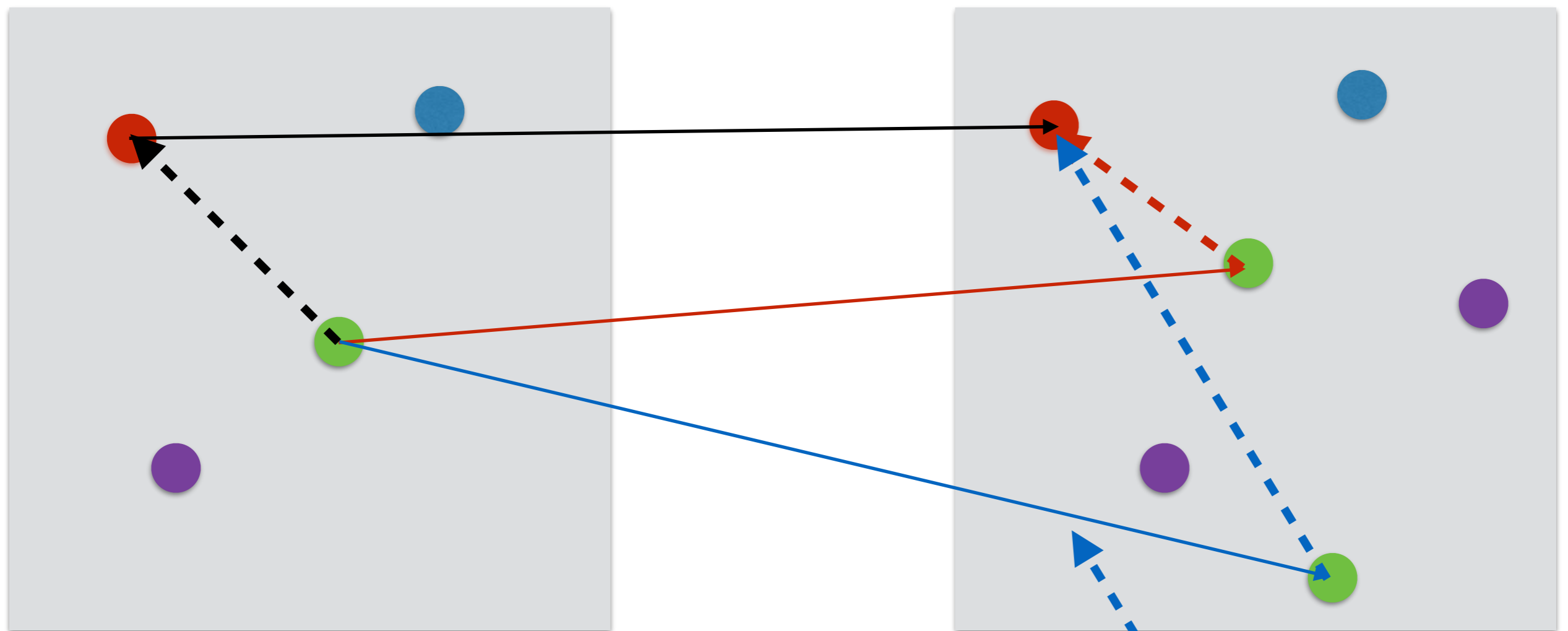


possible matches for the green point

— bad matches

— good matches

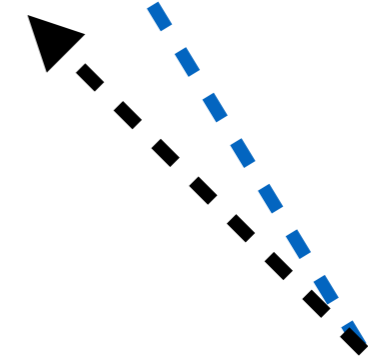
# Scoring matches



fix the red match

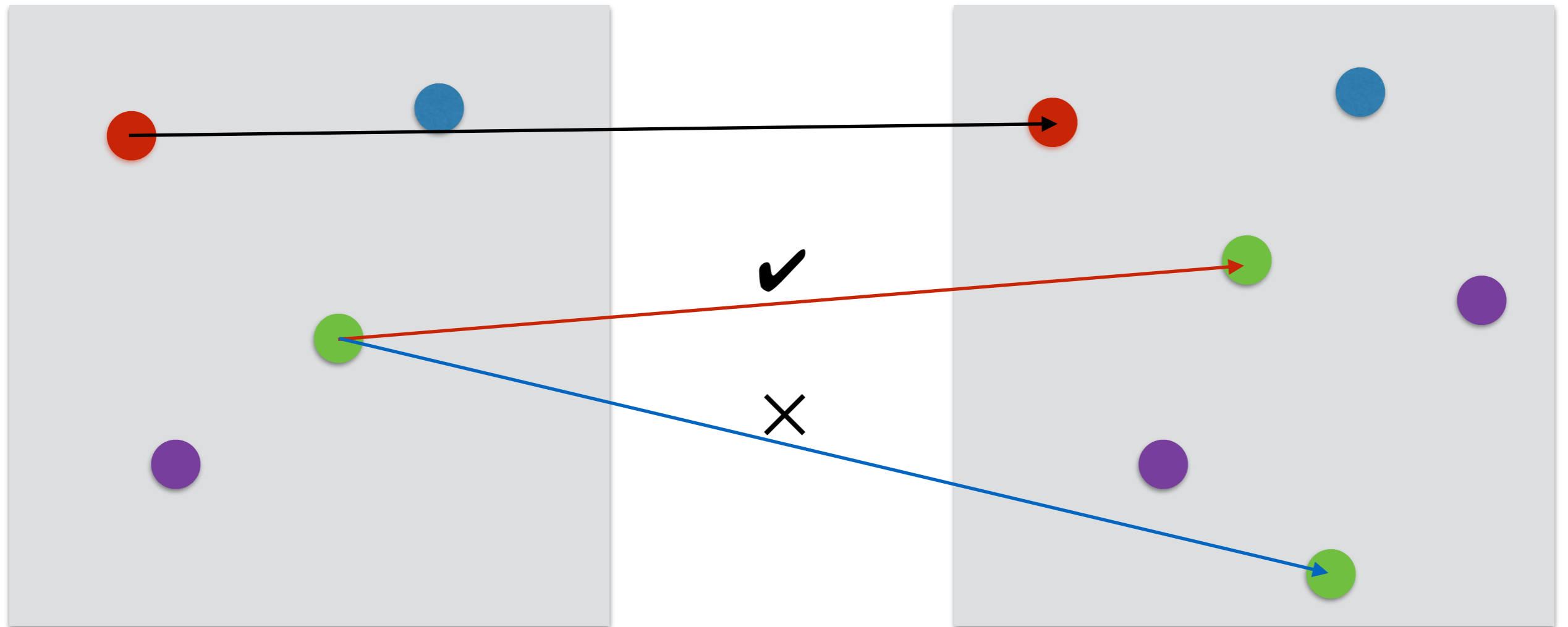


low distortion



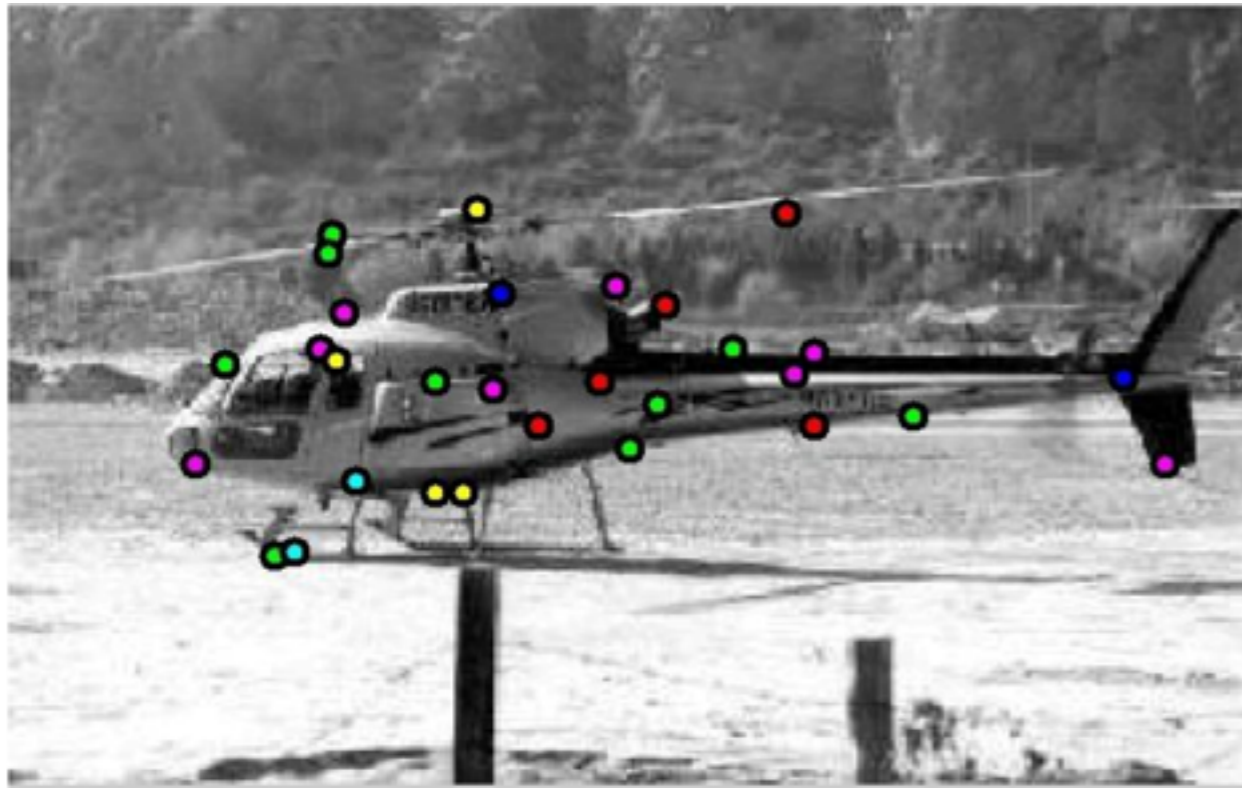
high distortion

# Scoring matches



fix the red match

# Correspondence algorithm



- Find correspondence that:
  - minimizes distortion (pairwise)
  - maximizes scores (individual) based on local descriptors matching

$$\textit{minimize} \left( - \sum_a \textit{score}(a) + \sum_{a,b} \textit{distortion}(a, b) \right)$$



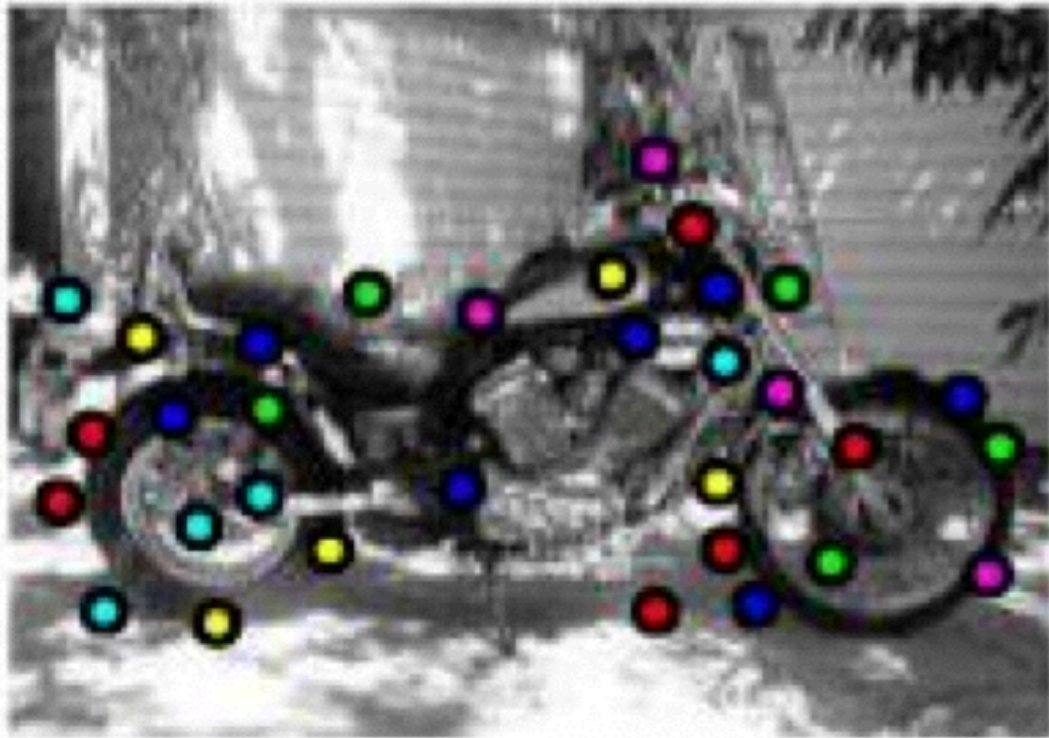
# Correspondence algorithm



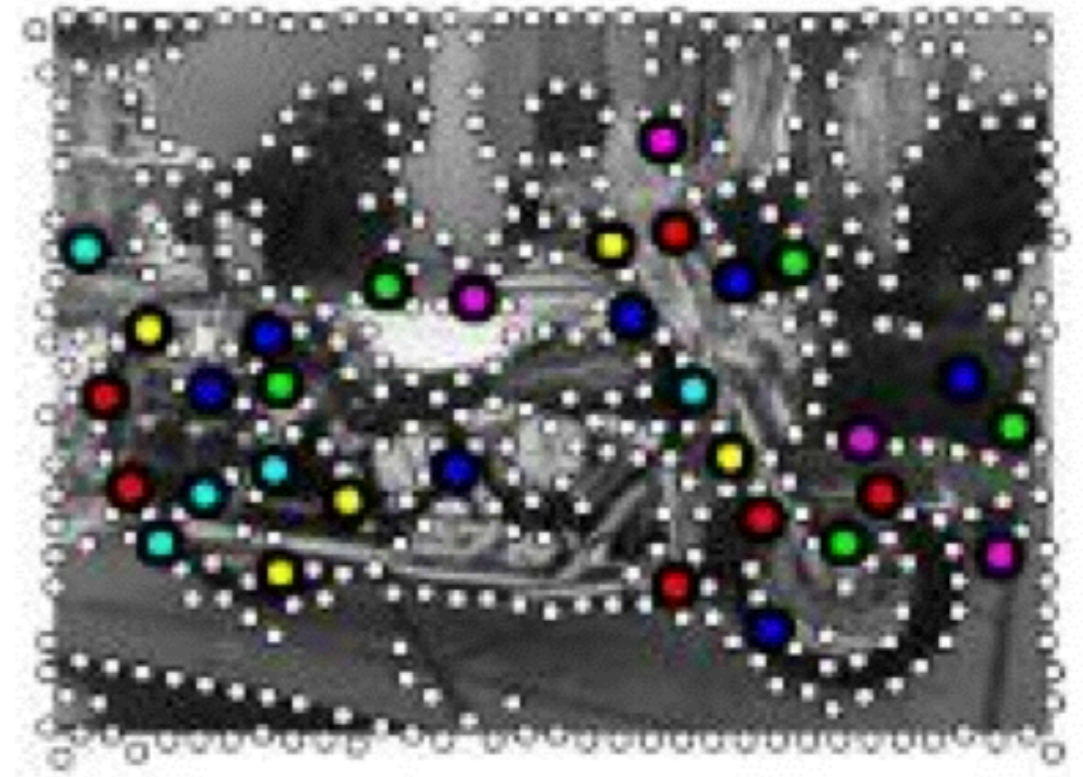
- Find correspondence that:
  - minimizes distortion (pairwise)
  - maximizes scores (individual) based on local descriptors matching
- Objective is an integer quadratic program (IQP)
  - NP-Hard
  - But approximate solutions can be found in reasonable time

# Example matches

Probe

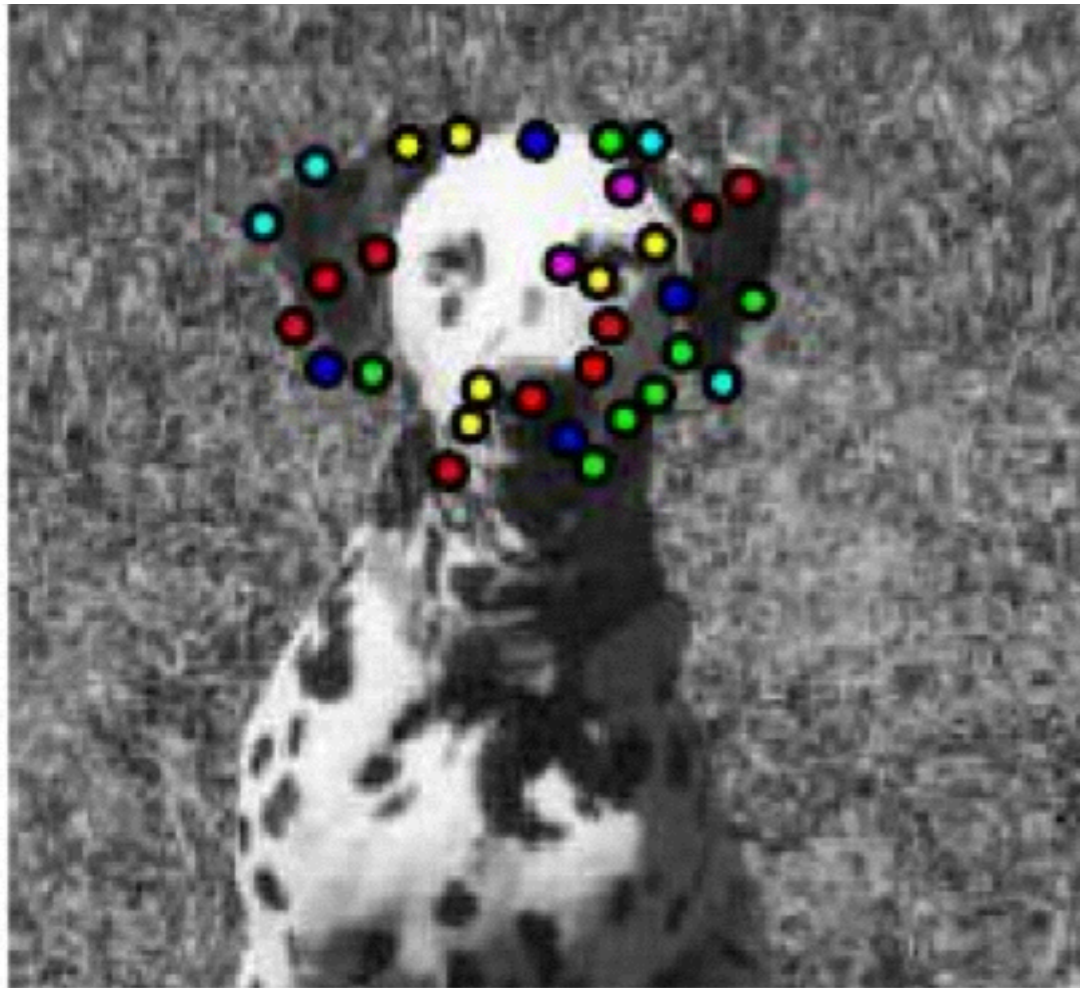


Query

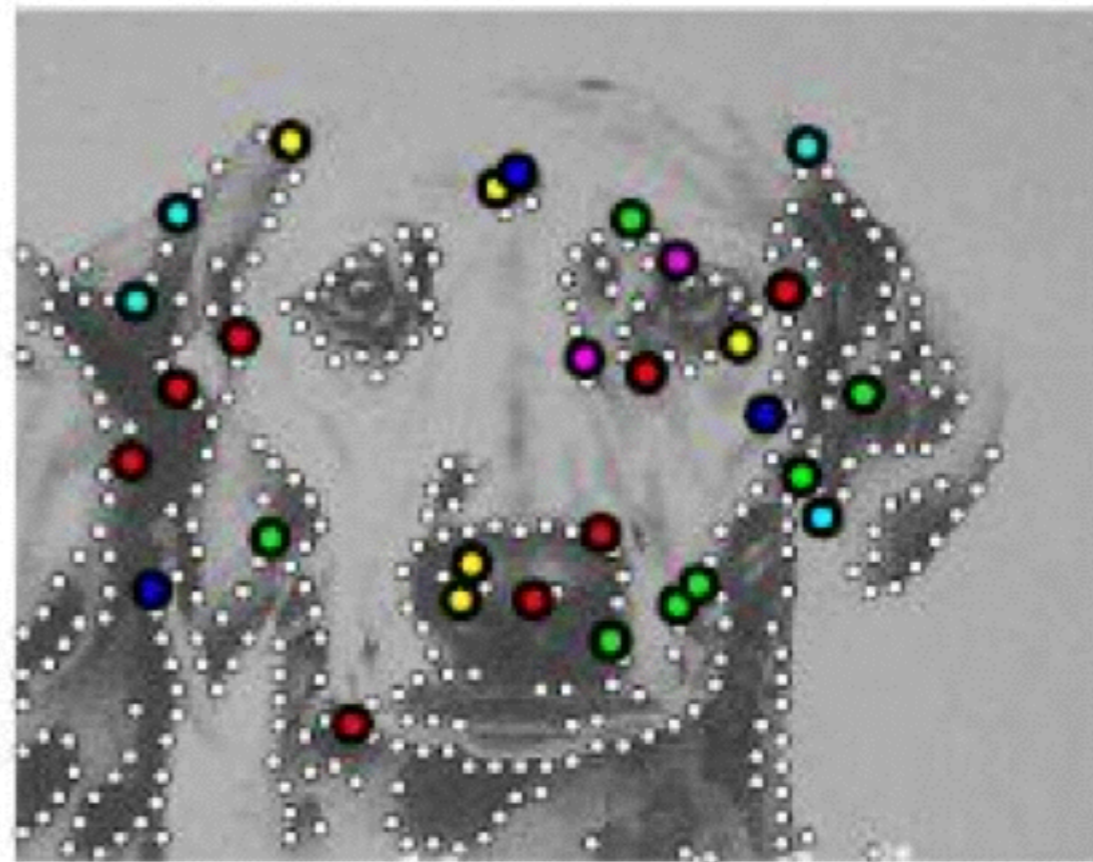


# Example matches

Probe



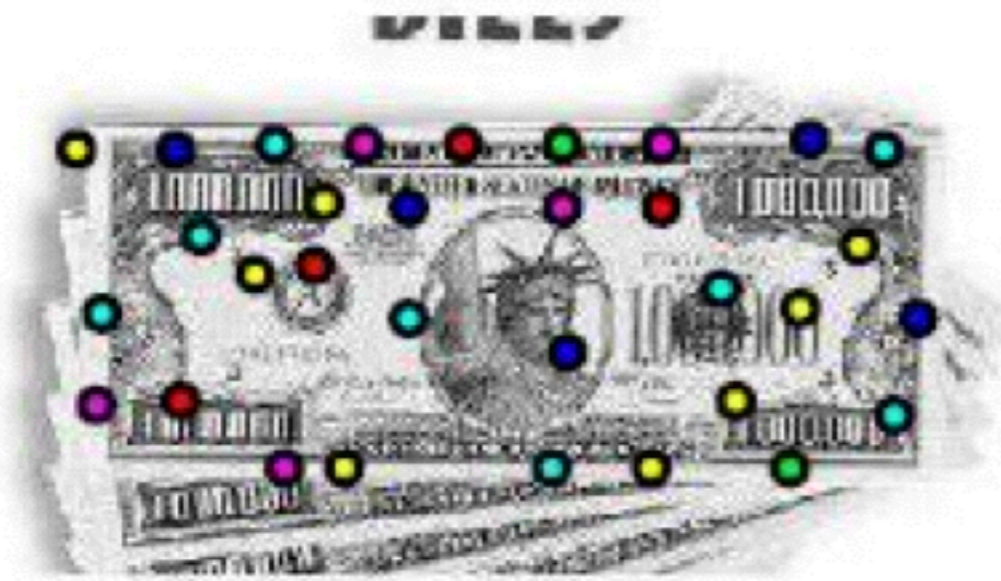
Query



# Example matches

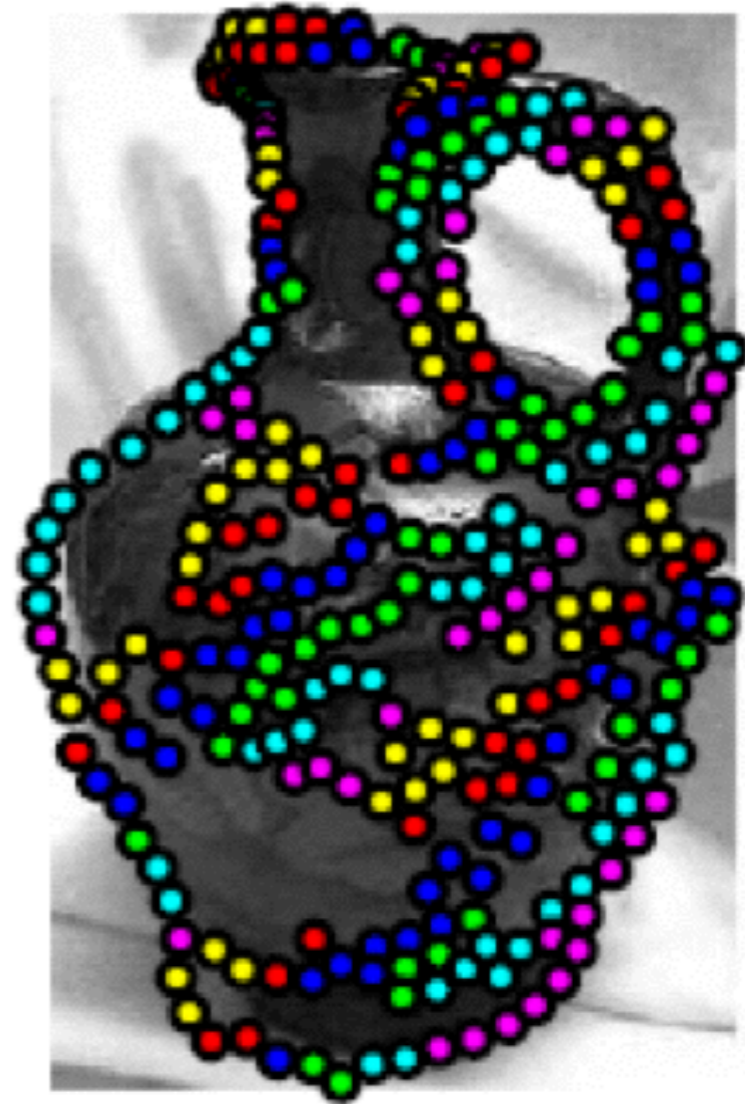
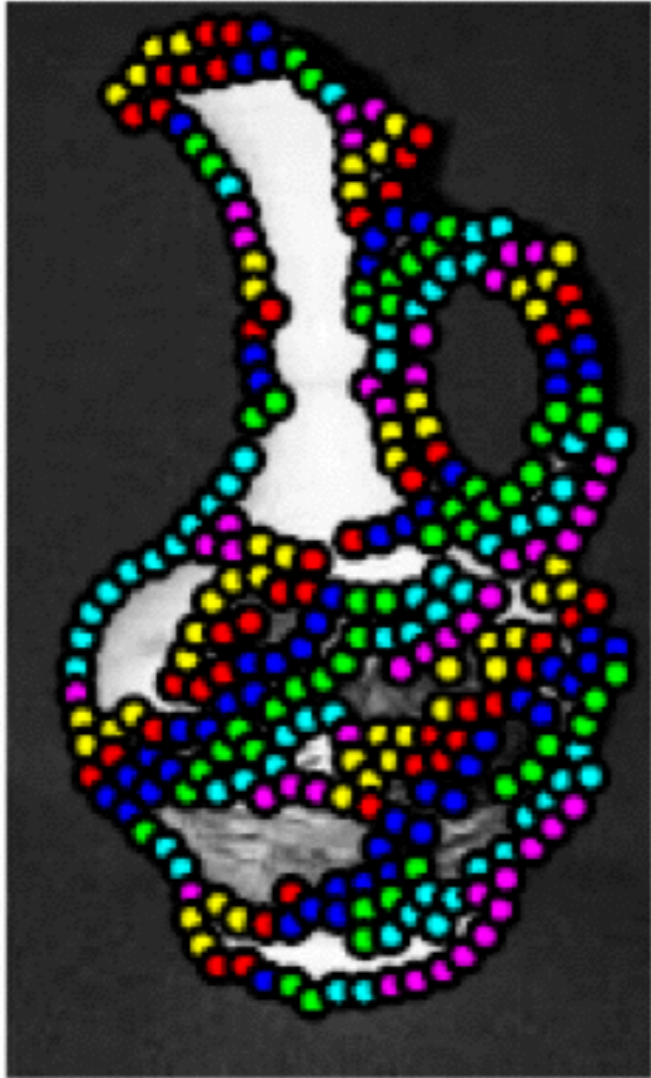
Probe

Query

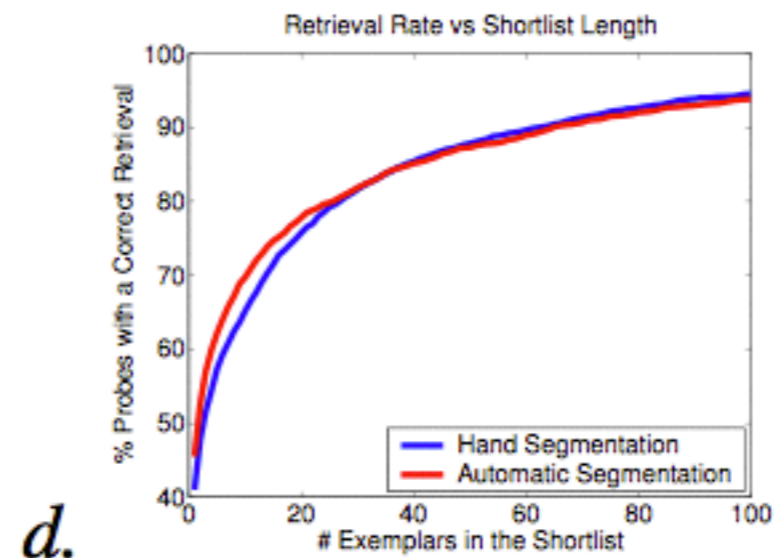
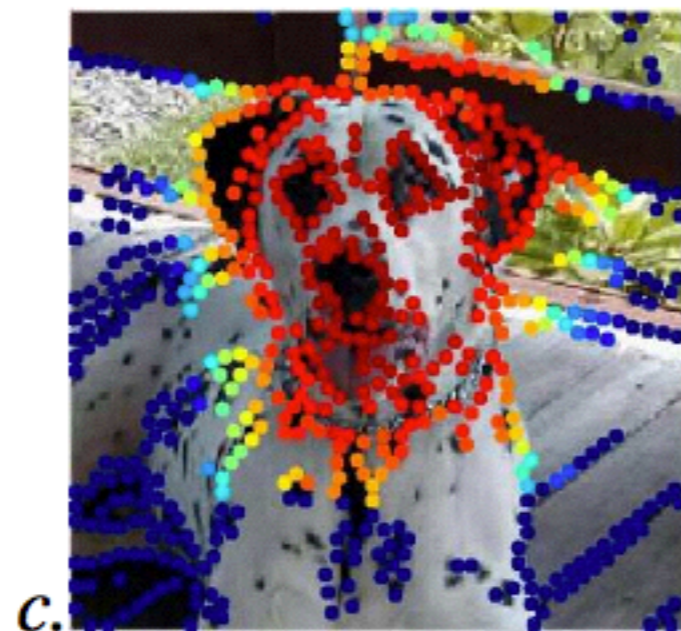
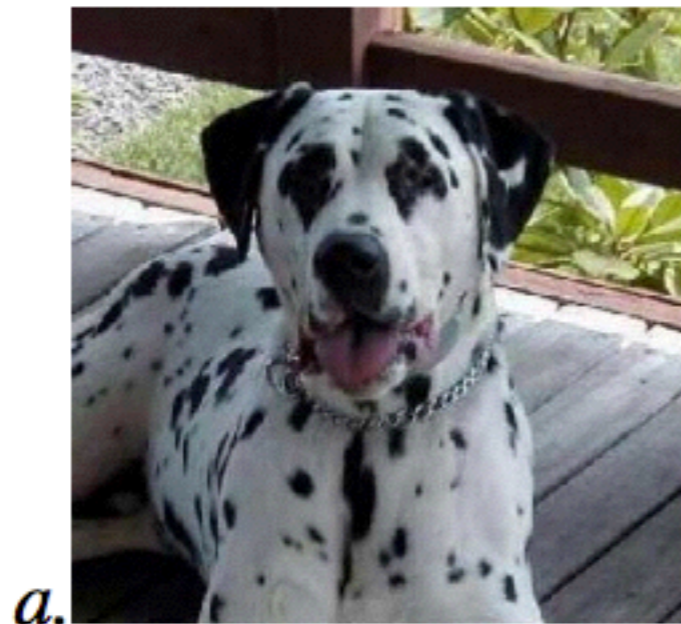


# Example matches

Interpolating the alignment



# Automatic segmentation

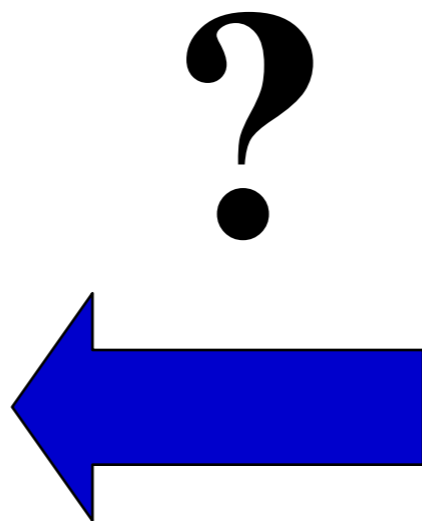
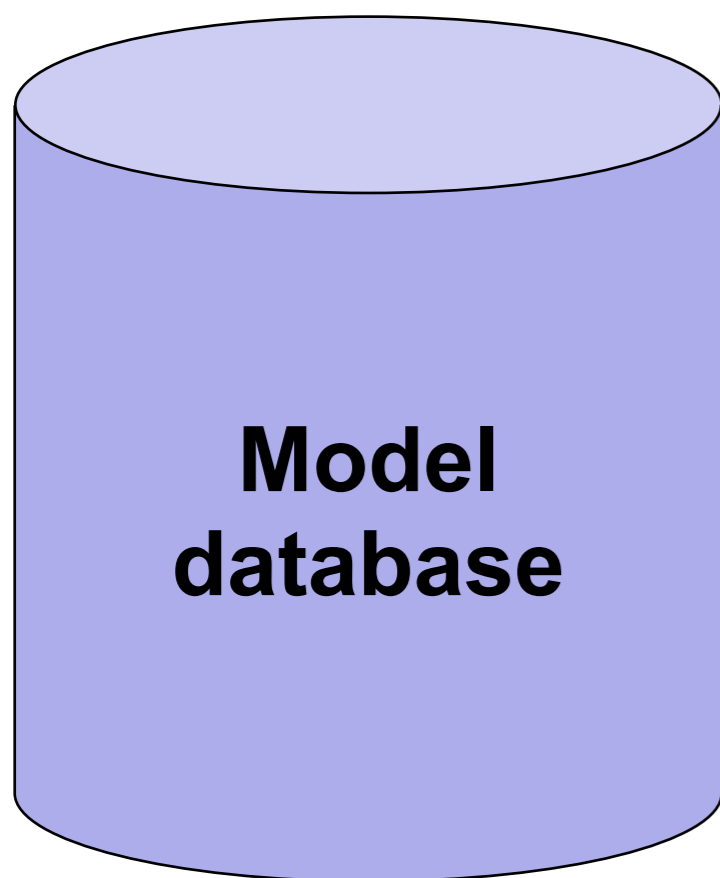


# Alignment

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# Scalability: Alignment to large databases

- What if we need to align a test image with thousands or millions of images in a model database?
  - Efficient putative match generation
    - Approximate descriptor similarity search, inverted indices

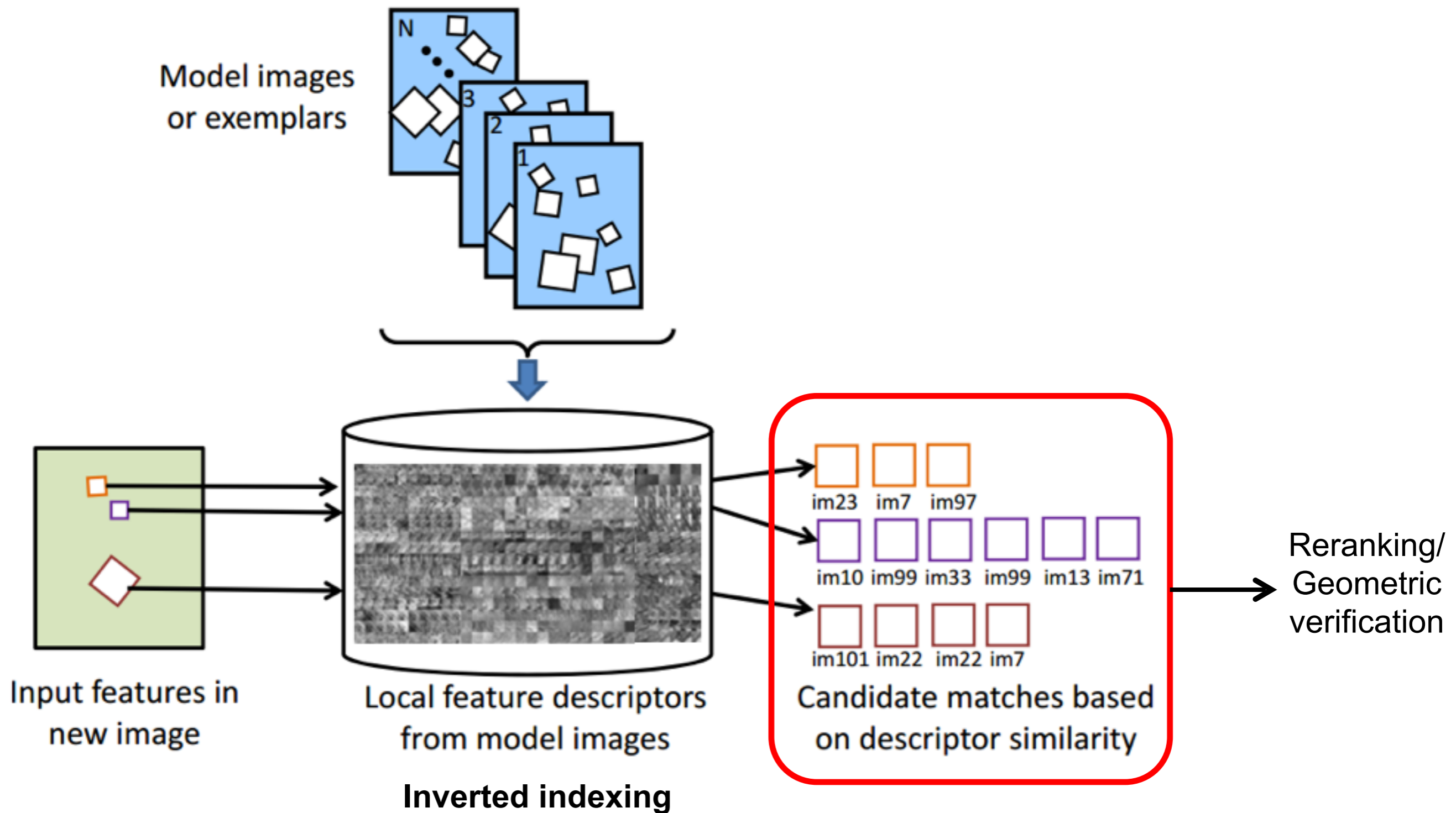


Test image

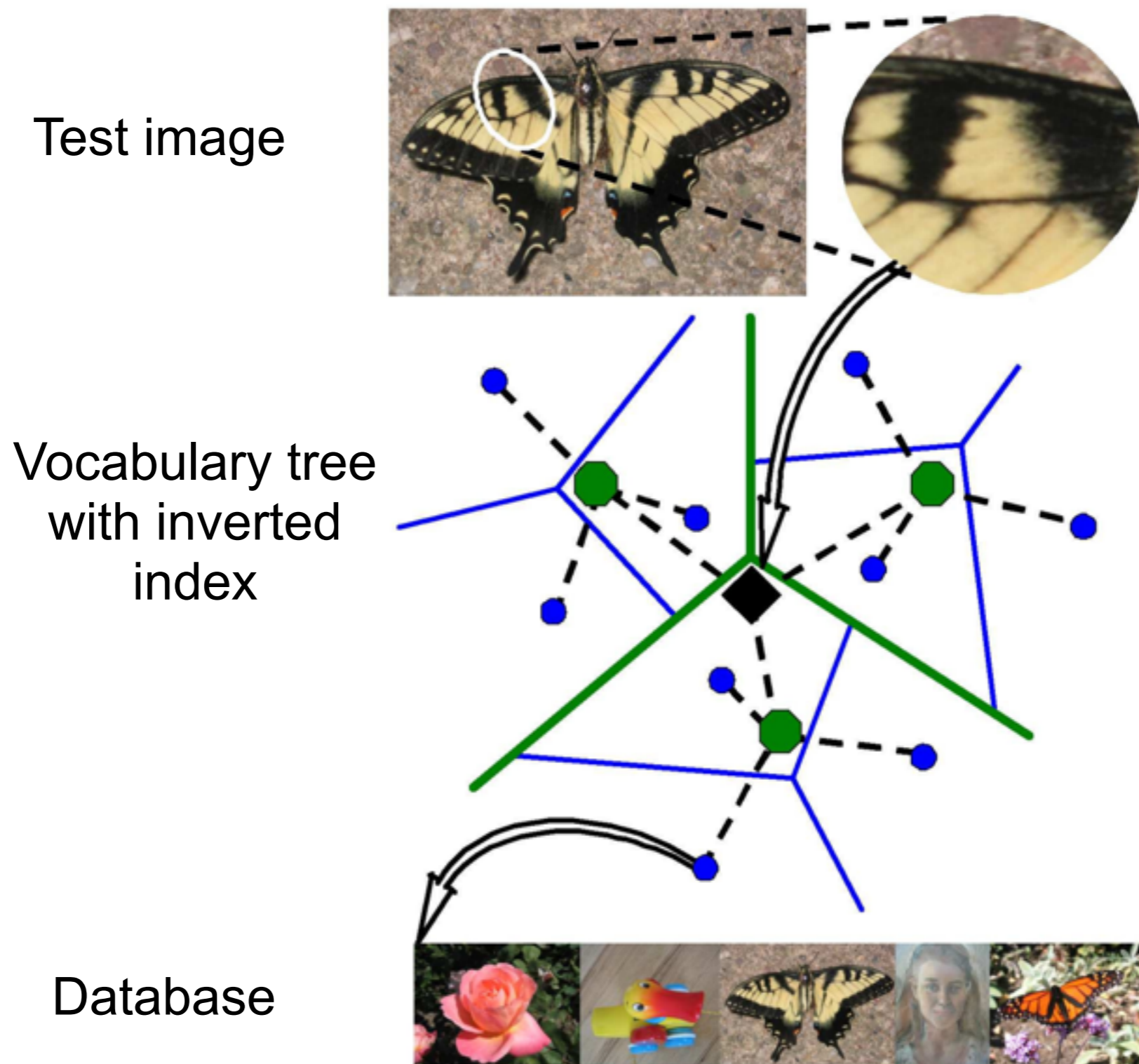




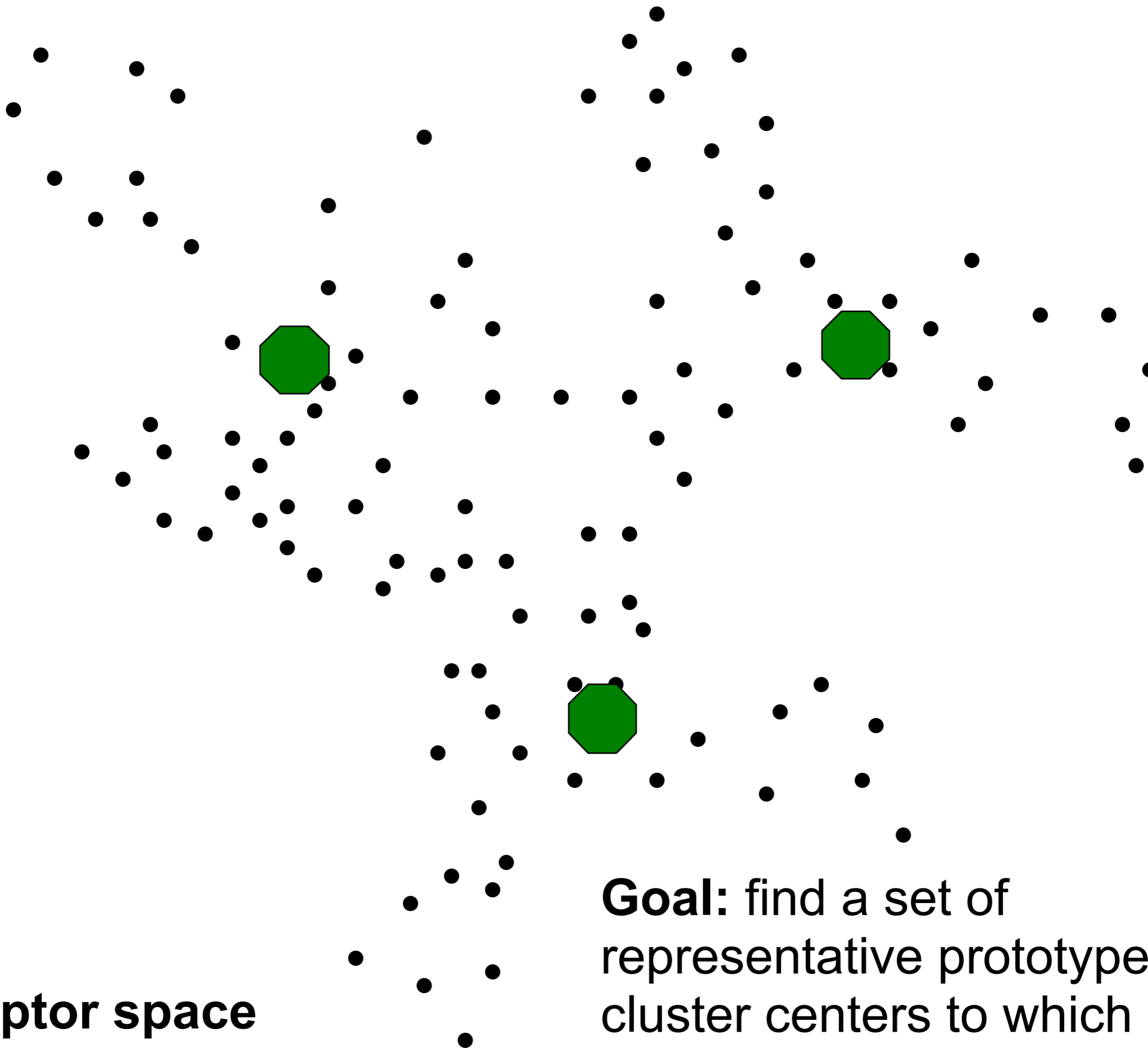
# Large-scale visual search



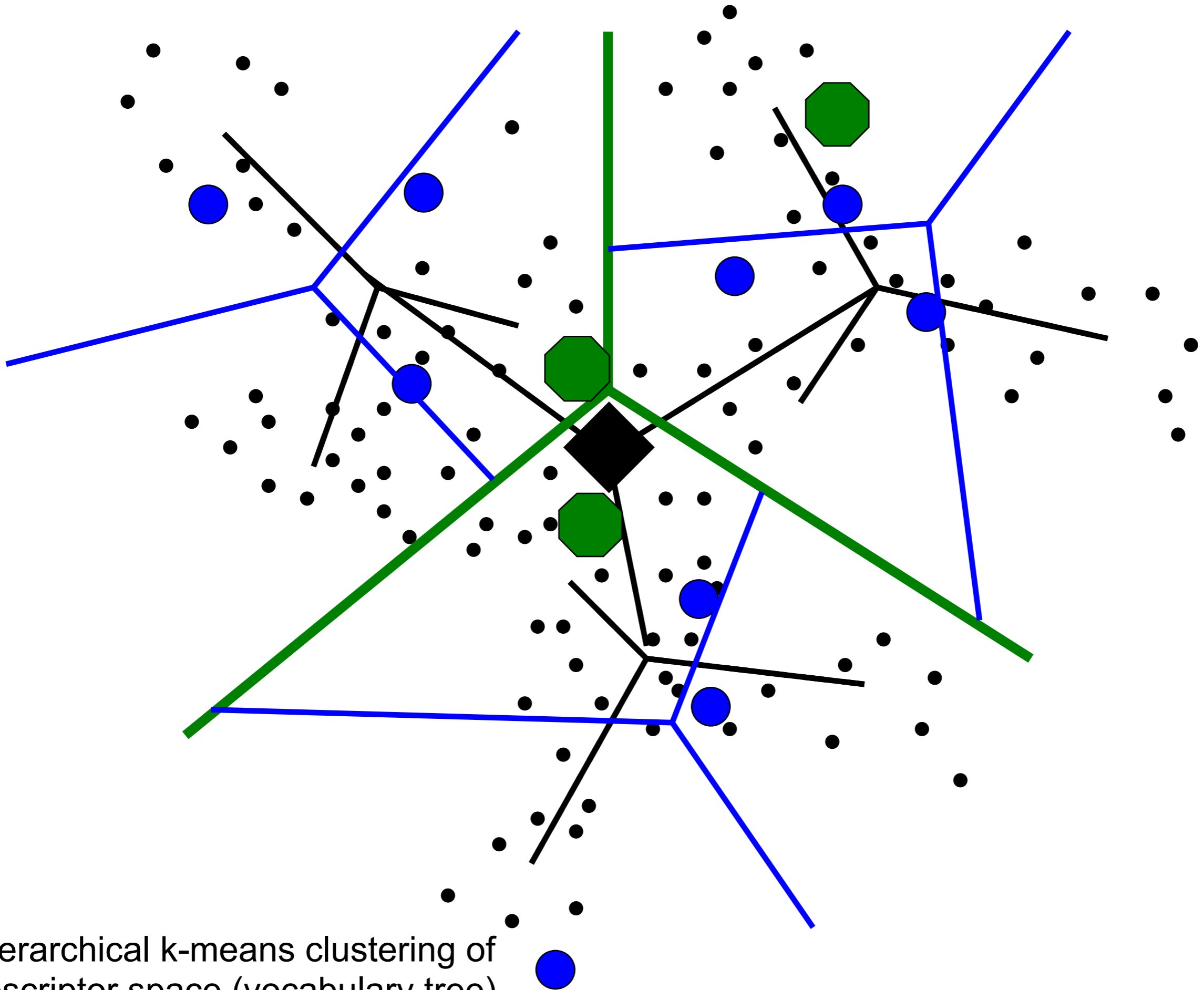
# Example indexing technique: Vocabulary trees



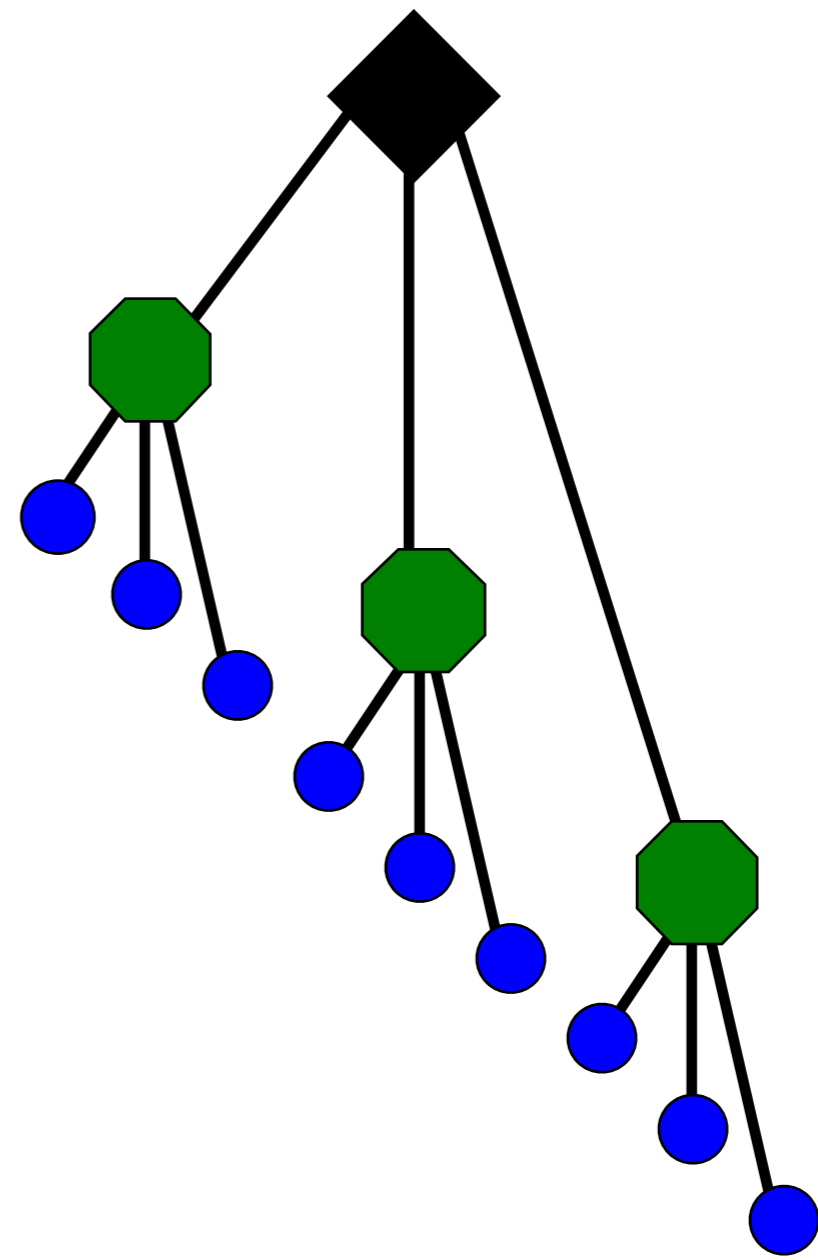
**Descriptor space**



**Goal:** find a set of representative prototypes or cluster centers to which descriptors can be quantized

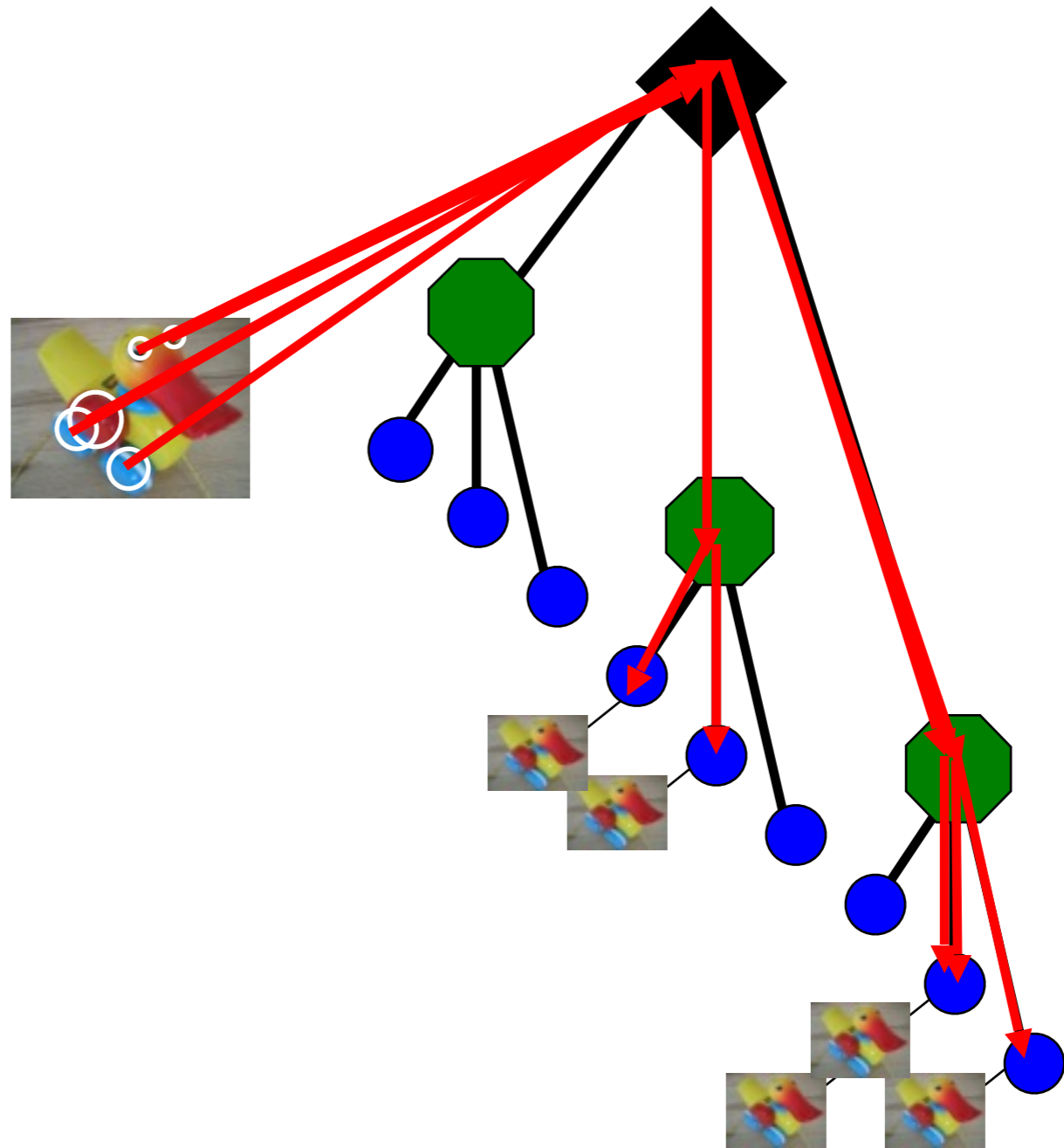


Hierarchical k-means clustering of descriptor space (vocabulary tree)



Vocabulary tree/inverted index

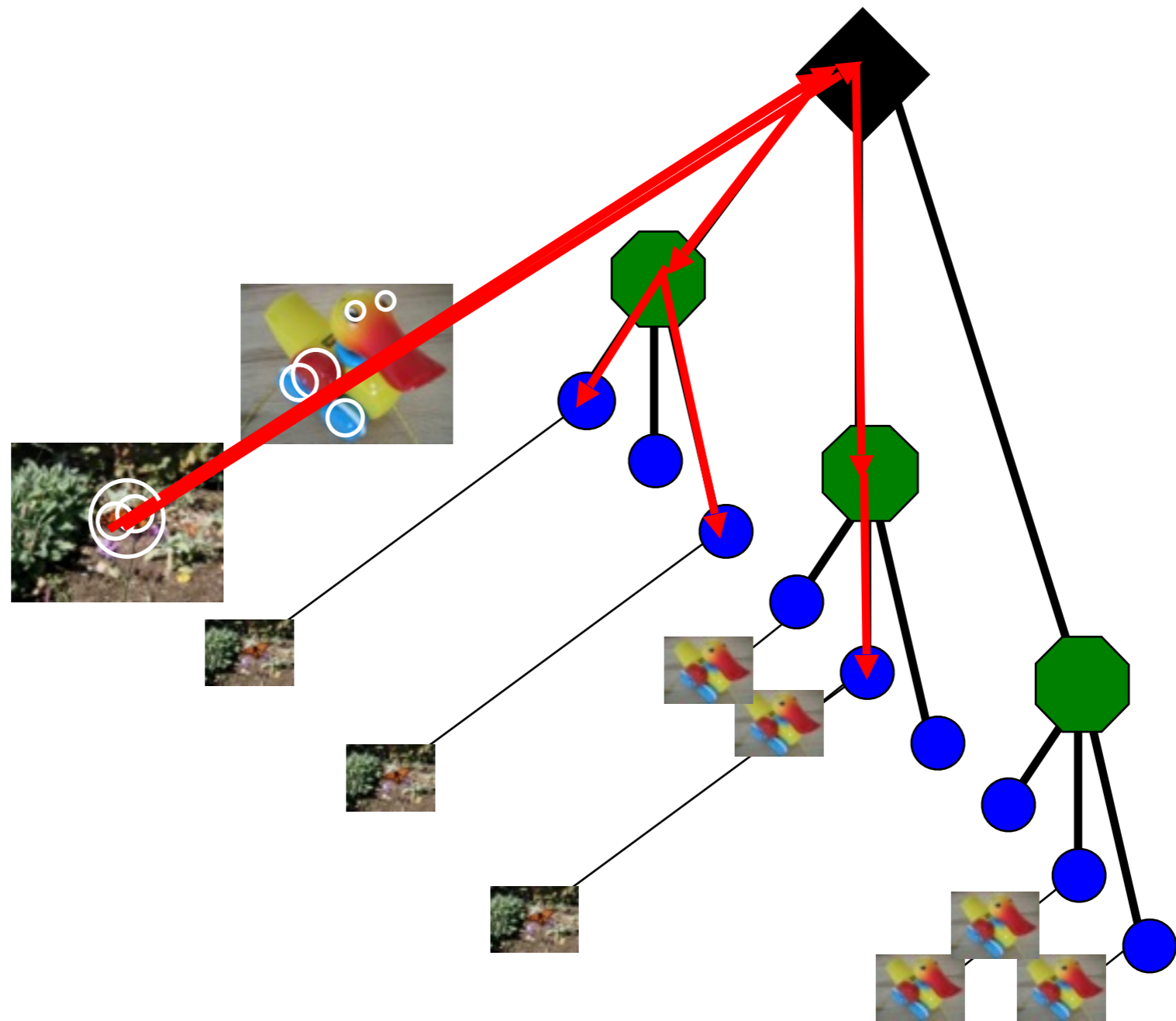
Model images



Populating the vocabulary tree/inverted index

Slide credit: D. Nister

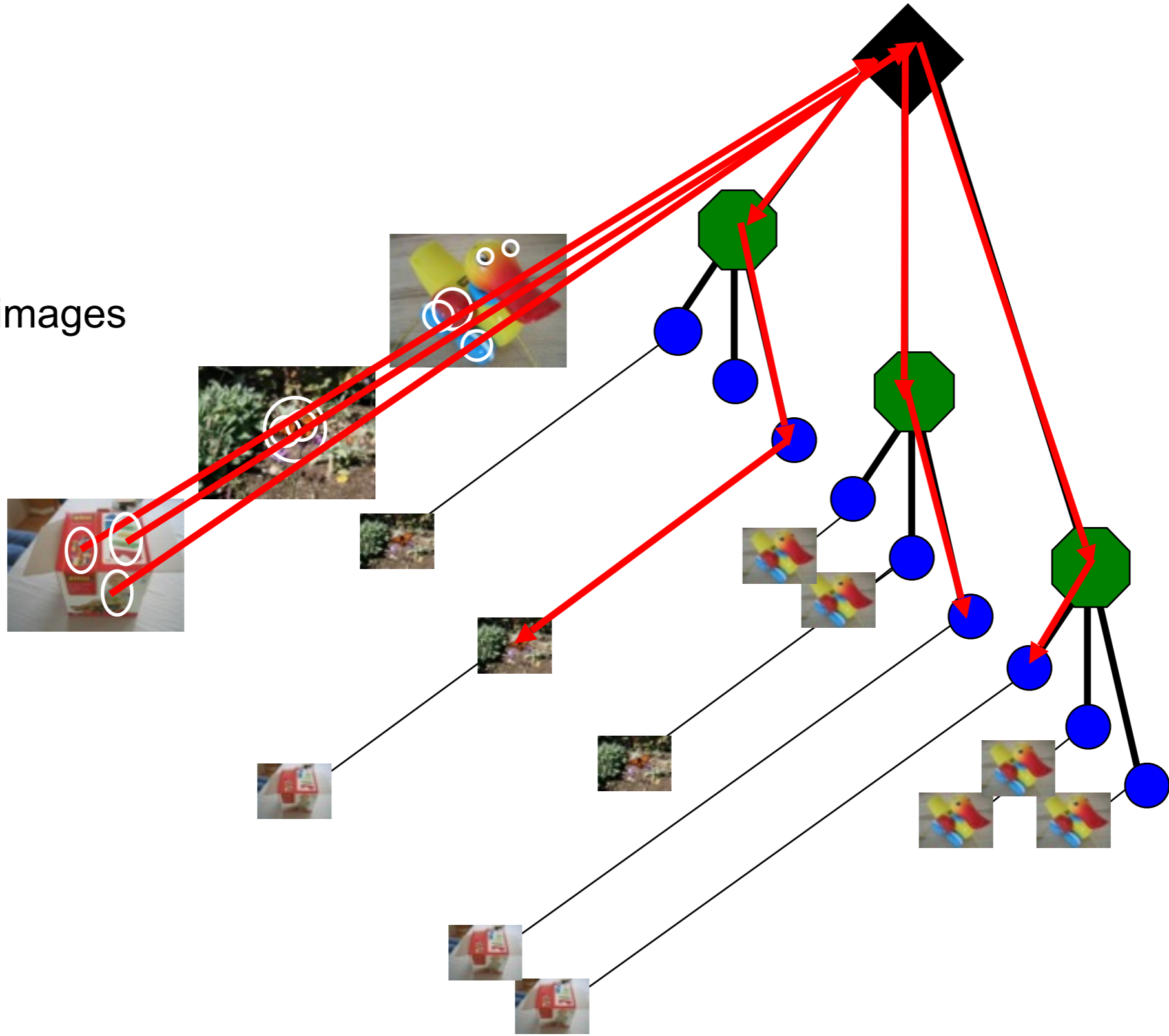
Model images



Populating the vocabulary tree/inverted index

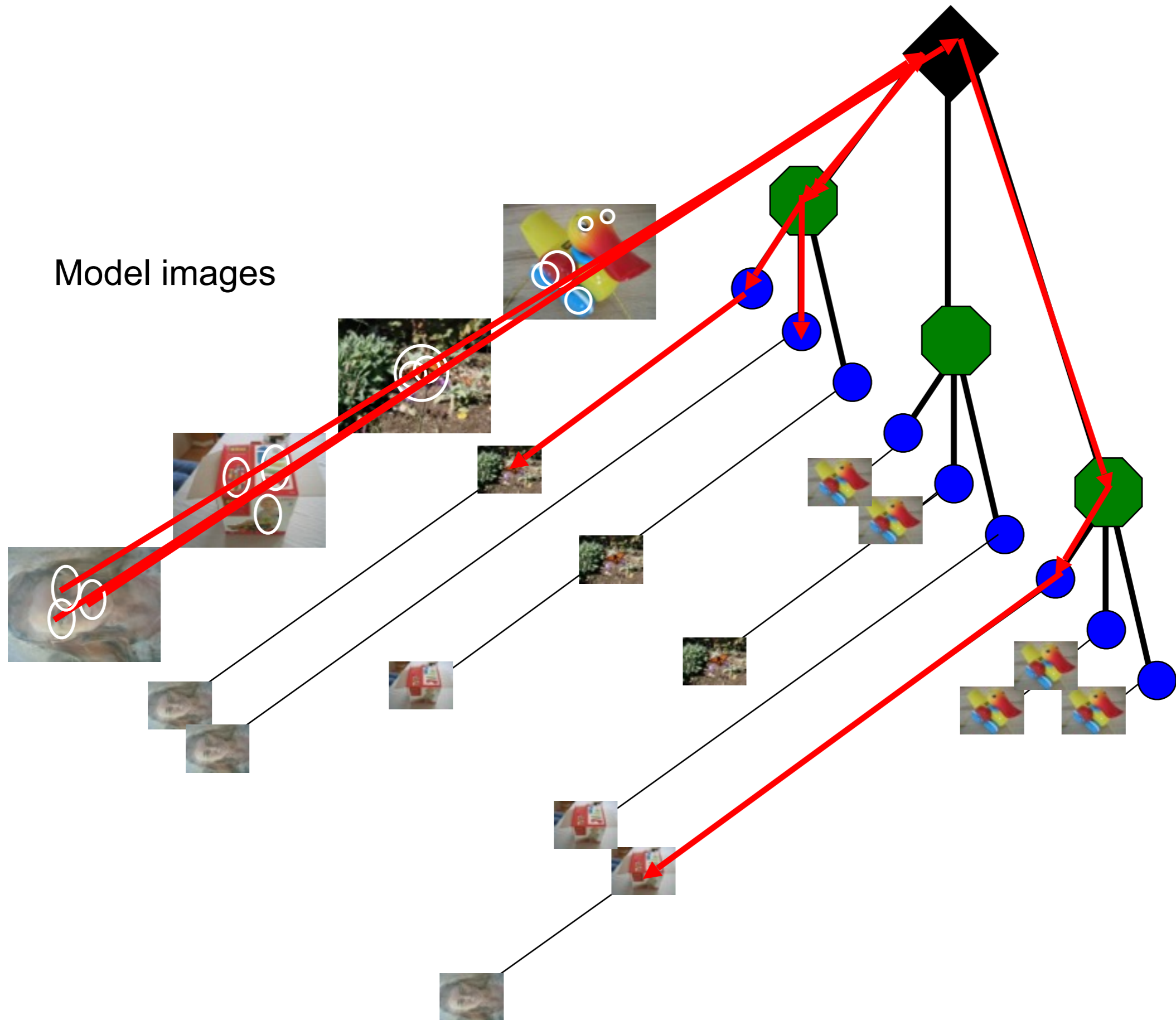
Slide credit: D. Nister

Model images



Populating the vocabulary tree/inverted index

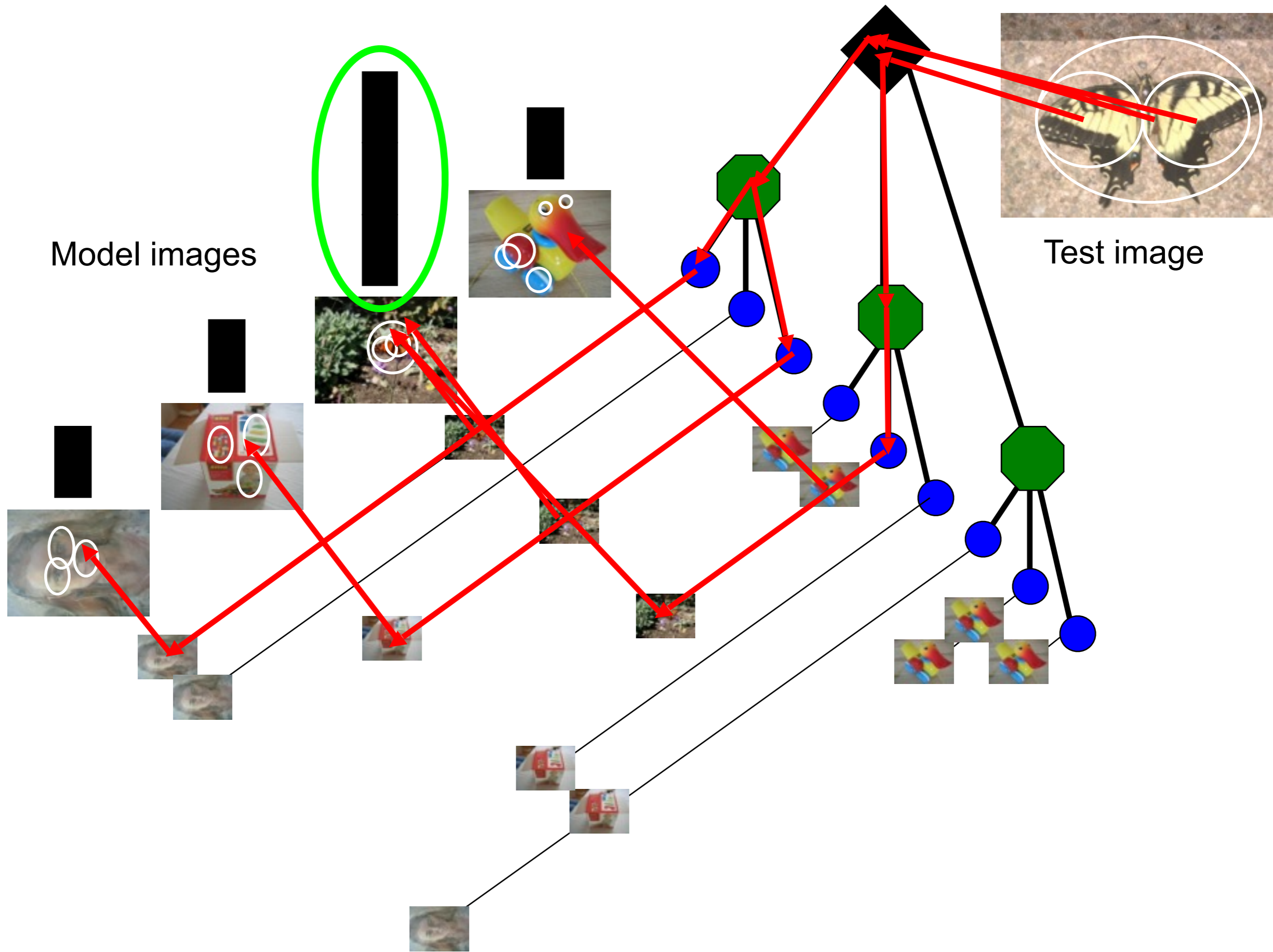




Model images

Populating the vocabulary tree/inverted index

Slide credit: D. Nister



Model images

Test image

Looking up a test image

Slide credit: D. Nister

# Further thoughts and readings ...

- Chapters 6, 7 and 9 from Richard Szeliski's book.
- Shape matching references
  - Shape matching and object recognition using shape contexts, Belongie, Malik and Puzicha, PAMI 2002 ([paper](#))
  - Shape matching and object recognition using low distortion correspondences, Berg, Berg and Malik, CVPR 2005 ([paper](#))
  - Hierarchical matching of deformable shapes, Felzenszwalb and Schwartz, CVPR 2007 ([paper](#))
  - David Nister's vocabulary tree [paper](#)
- Web demos from Oxford VGG group
  - [Video google](#), [Oxford building search](#), [Sculpture retrieval](#)

# Cool application of large-scale alignment: searching the night sky

<http://www.astrometry.net/>

