CMPSCI 670: Computer Vision

Corner detection

University of Massachusetts, Amherst

September 29, 2014

Instructor: Subhransu Maji

Administrivia

- Homework 2 code.zip had a bug (or two)
  - Download the latest code.zip from the homework 2 page
  - For those of who who already started:
    - The bugs were in the **evalCode.m**, function calls had the wrong syntax and a variable (**integrationMethod**) was not defined
- Get started with the homework!
- Office hours on Wednesday after class in case you have questions

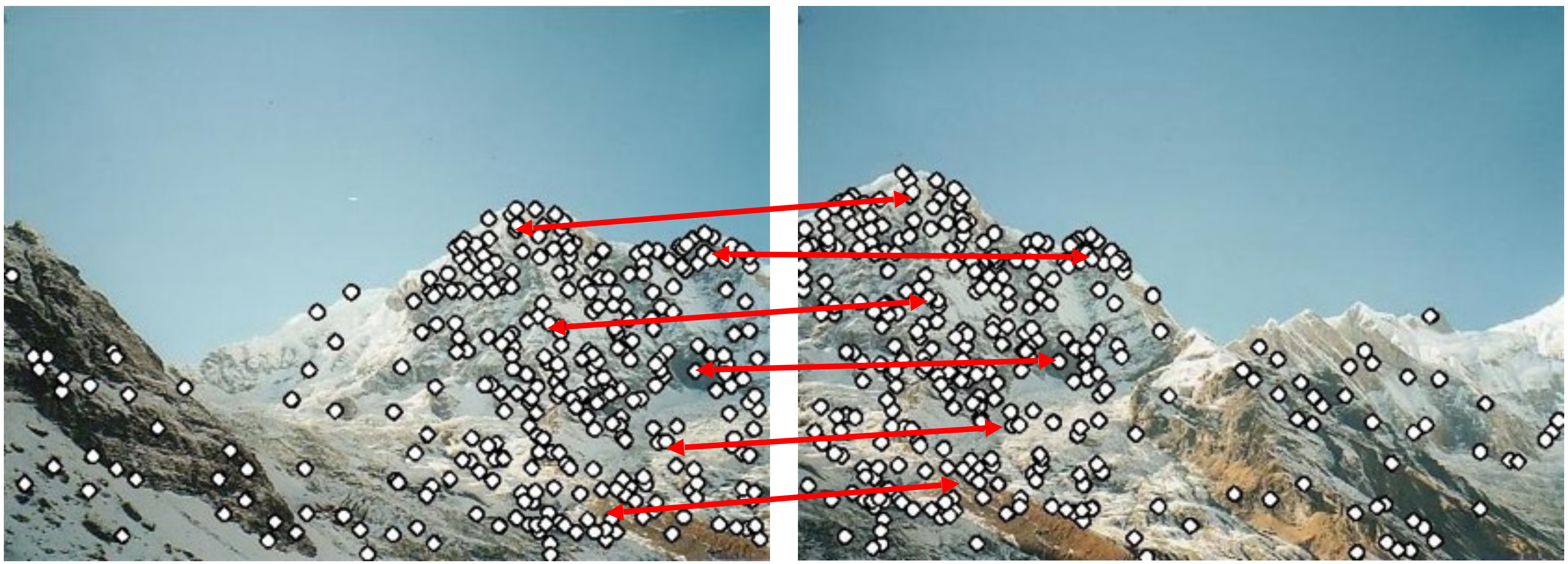
Feature extraction: Corners

Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract features

Step 2: match features

Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

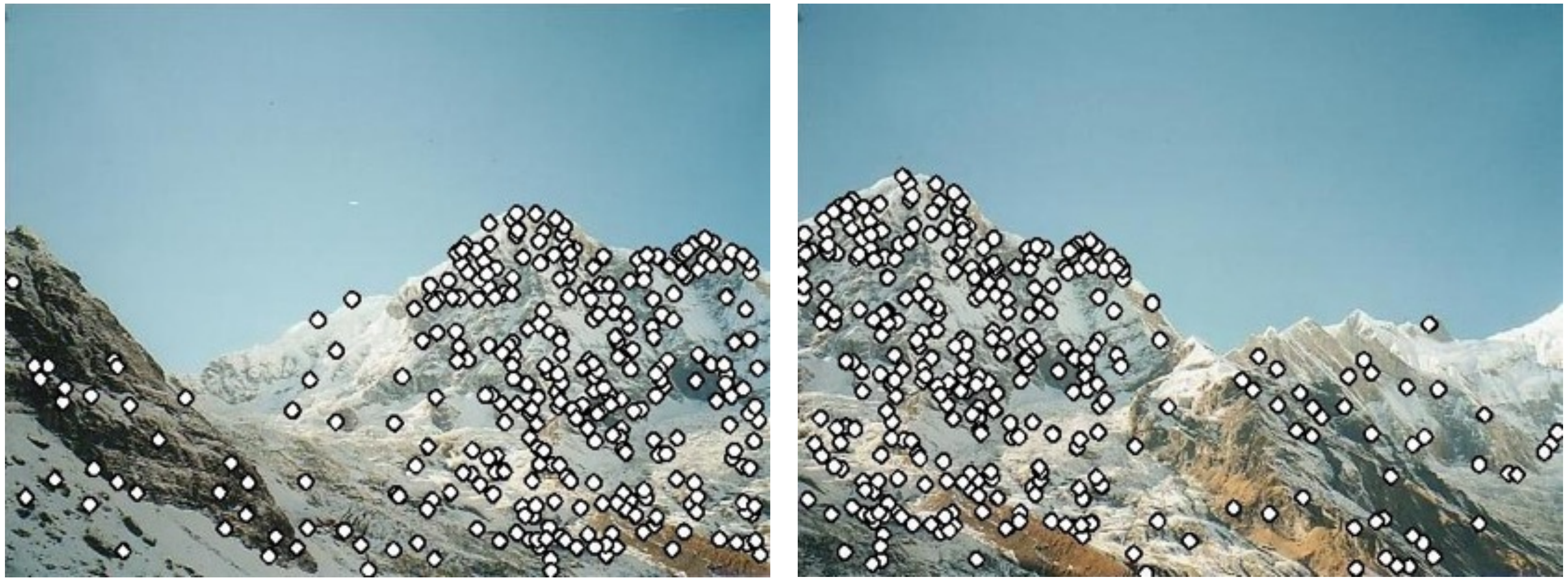
Step 1: extract features

Step 2: match features

Step 3: align images

Slide credit: L. Lazebnik

Characteristics of good features



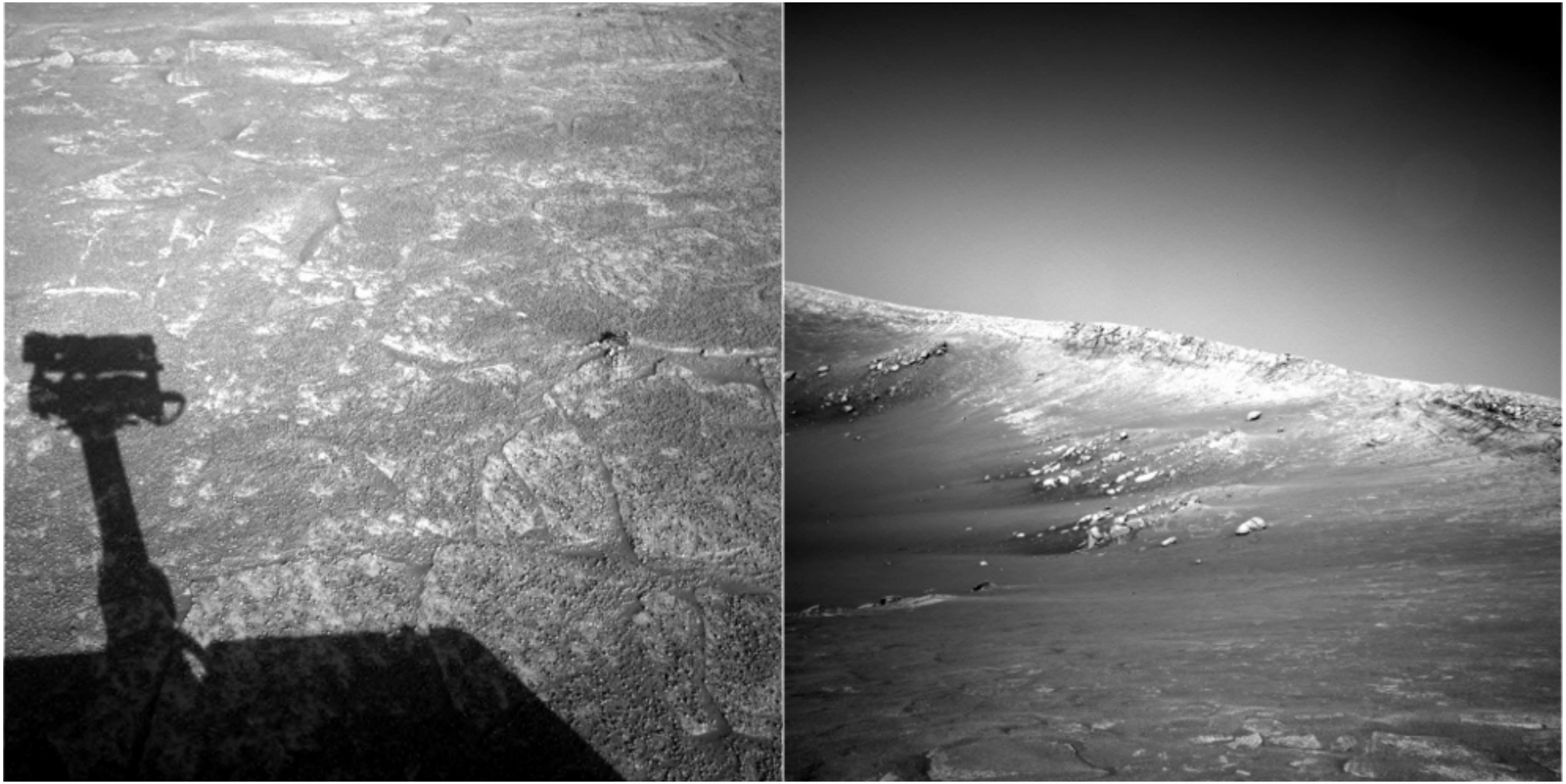
- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Applications

Feature points are used for:

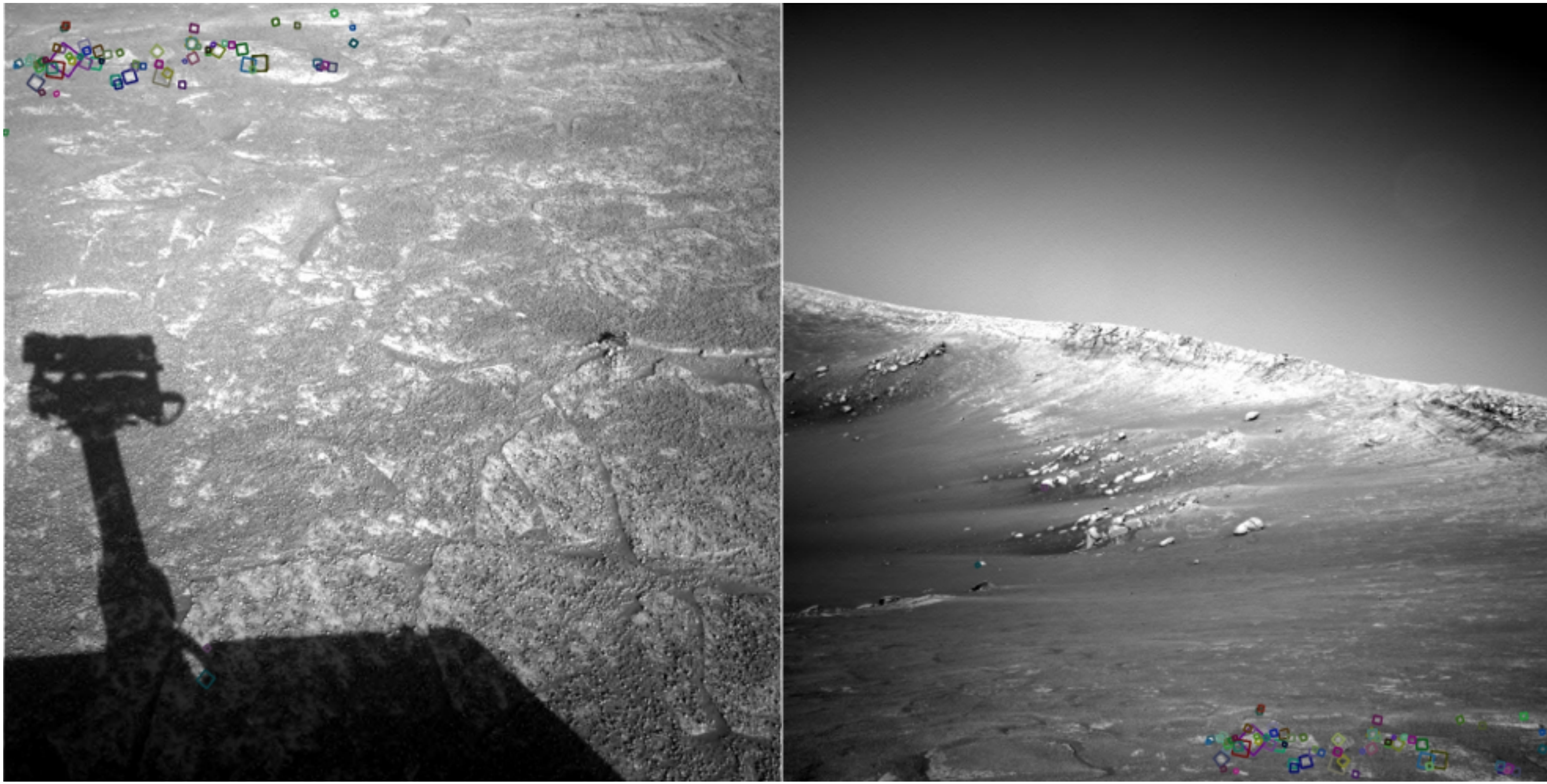
- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Indexing and database retrieval
- Object recognition

A hard feature matching problem



NASA Mars Rover images

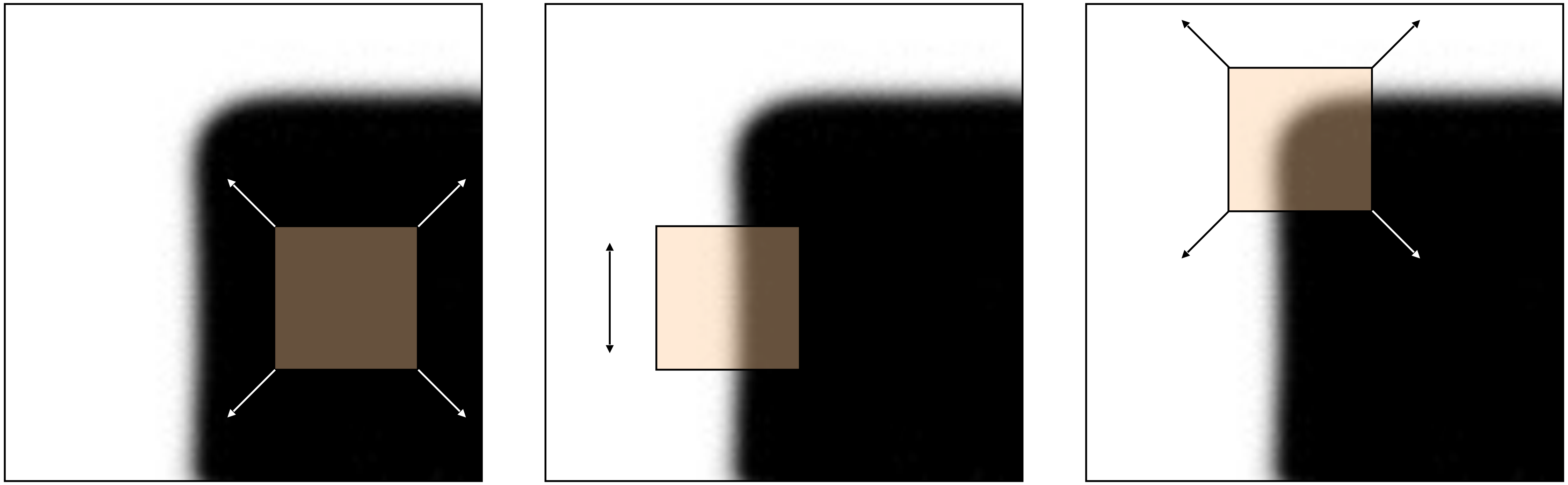
Answer below (look for tiny colored squares…)



NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Corner Detection: Basic Idea

- We should easily recognize the corners by looking through a small window
- Shifting a window in ***any direction*** should give ***a large change*** in intensity at a corner



"flat" region:
no change in
all directions

"edge":
no change along
the edge
direction

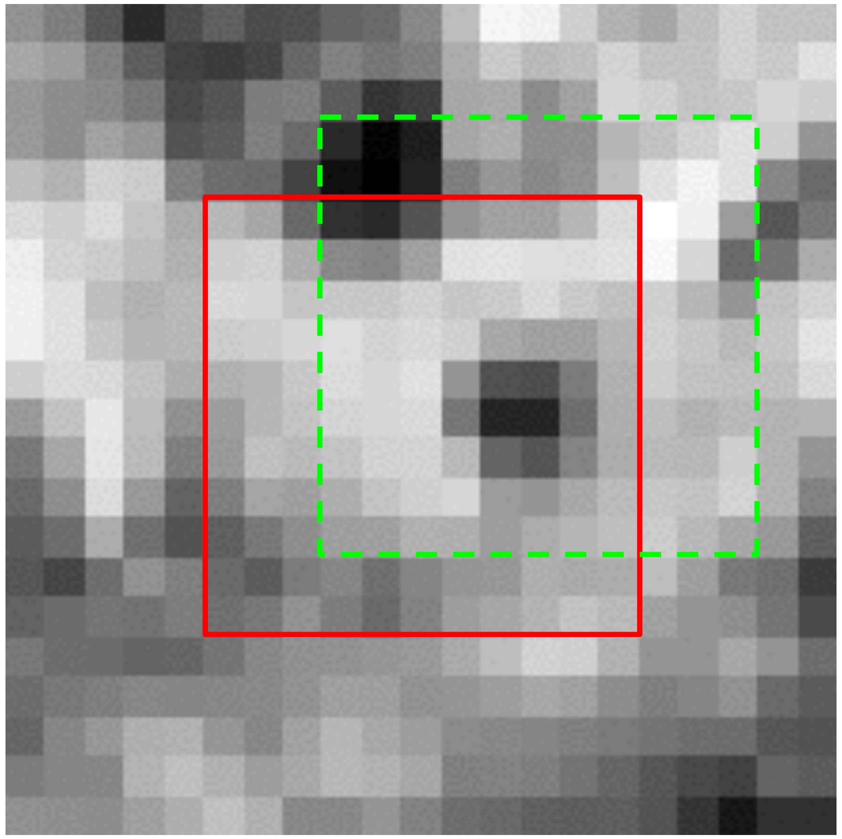
"corner":
significant
change in all
directions

Corner Detection: Mathematics

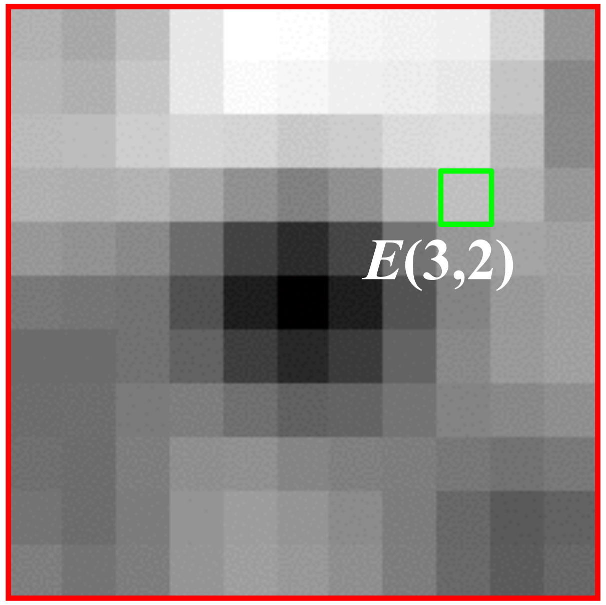
Change in appearance of window *W* for the shift [*u*,*v*]:

$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y)\right]^2$$

$I(x,y)$



$E(u,v)$

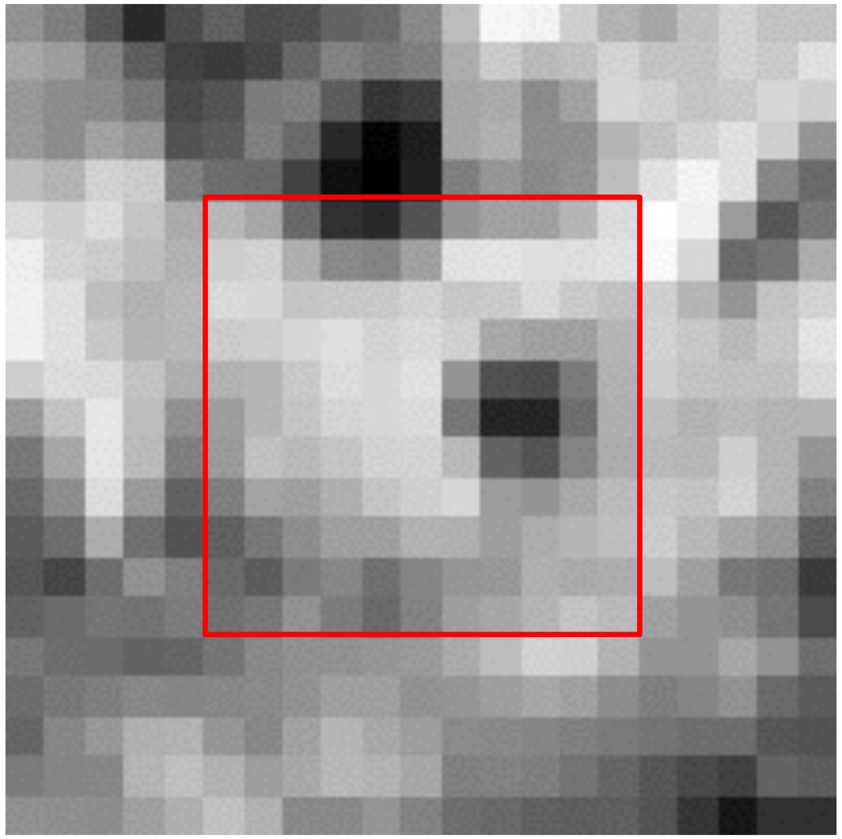


Corner Detection: Mathematics

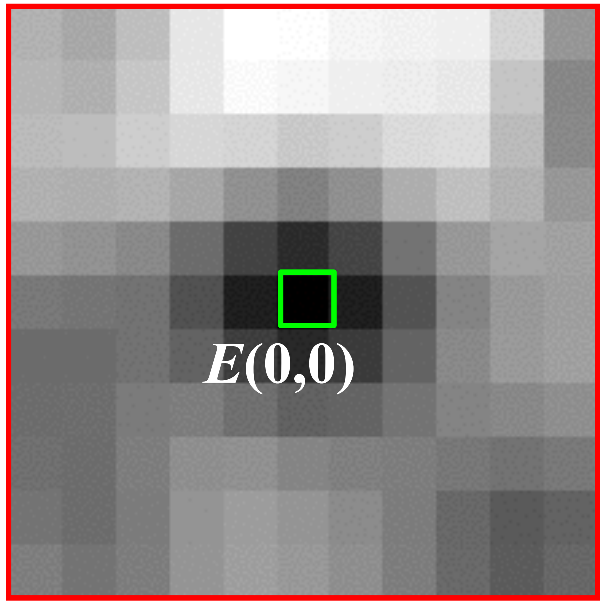
Change in appearance of window *W* for the shift [*u*,*v*]:

$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y)\right]^2$$

$I(x,y)$



$E(u,v)$



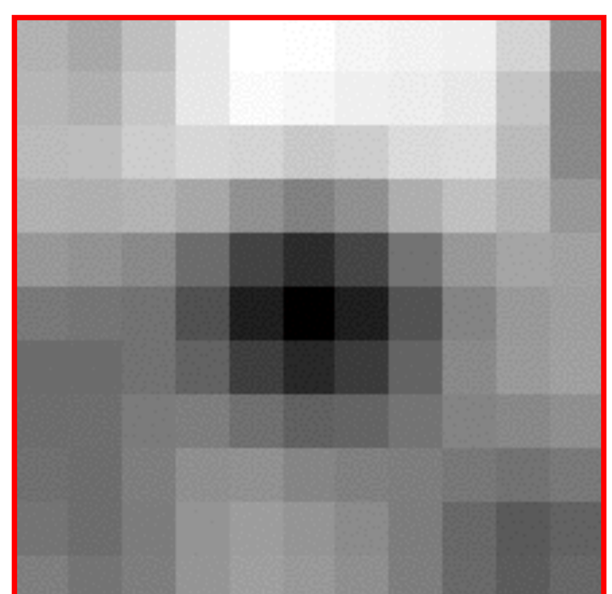
Corner Detection: Mathematics

Change in appearance of window *W* for the shift [*u*,*v*]:

$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

$E(u,v)$



Corner Detection: Mathematics

- First-order Taylor approximation for small motions [*u*, *v*]:

$$I(x+u, y+v) = I(x, y) + I_x u + I_y v$$

- Let's plug this into *E*(*u*,*v*)

$$\begin{aligned} E(u,v) &= \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y)\right]^2 \\ &\simeq \sum_{(x,y)\in W} \left[I(x,y) + I_x u + I_y v - I(x,y)\right]^2 \\ &= \sum_{(x,y)\in W} \left[I_x u + I_y v\right]^2 \\ &= \sum_{(x,y)\in W} \left[I_x^2 u^2 + I_x I_y uv + I_y I_x uv + I_y^2 v^2\right] \end{aligned}$$

Corner Detection: Mathematics

The quadratic approximation can be written as

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a *second moment matrix* computed from image derivatives:

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

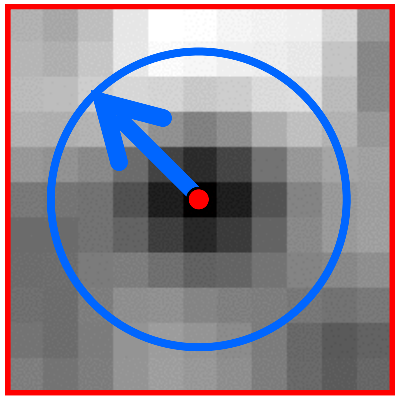
(the sums are over all the pixels in the window *W*)

Interpreting the second moment matrix

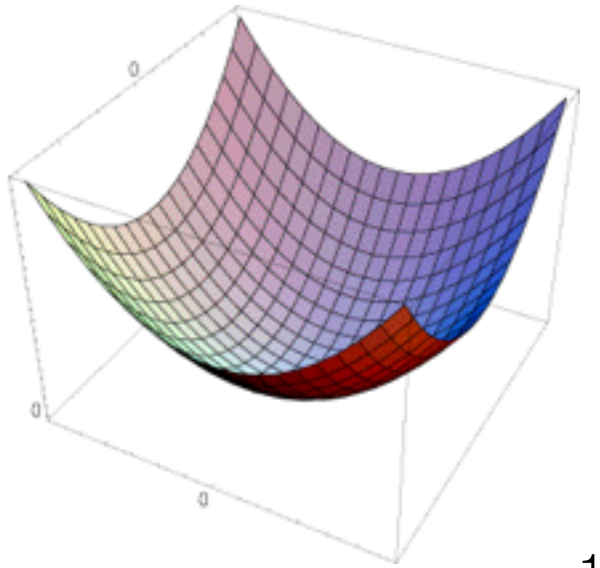
- The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.
  - Specifically, in which directions does it have the smallest/greatest change?

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$E(u, v)$



$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

First, consider the axis-aligned case
(gradients are either horizontal or vertical)

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

If either *a* or *b* is close to 0, then this is **not** a corner, so look for locations where both are large.

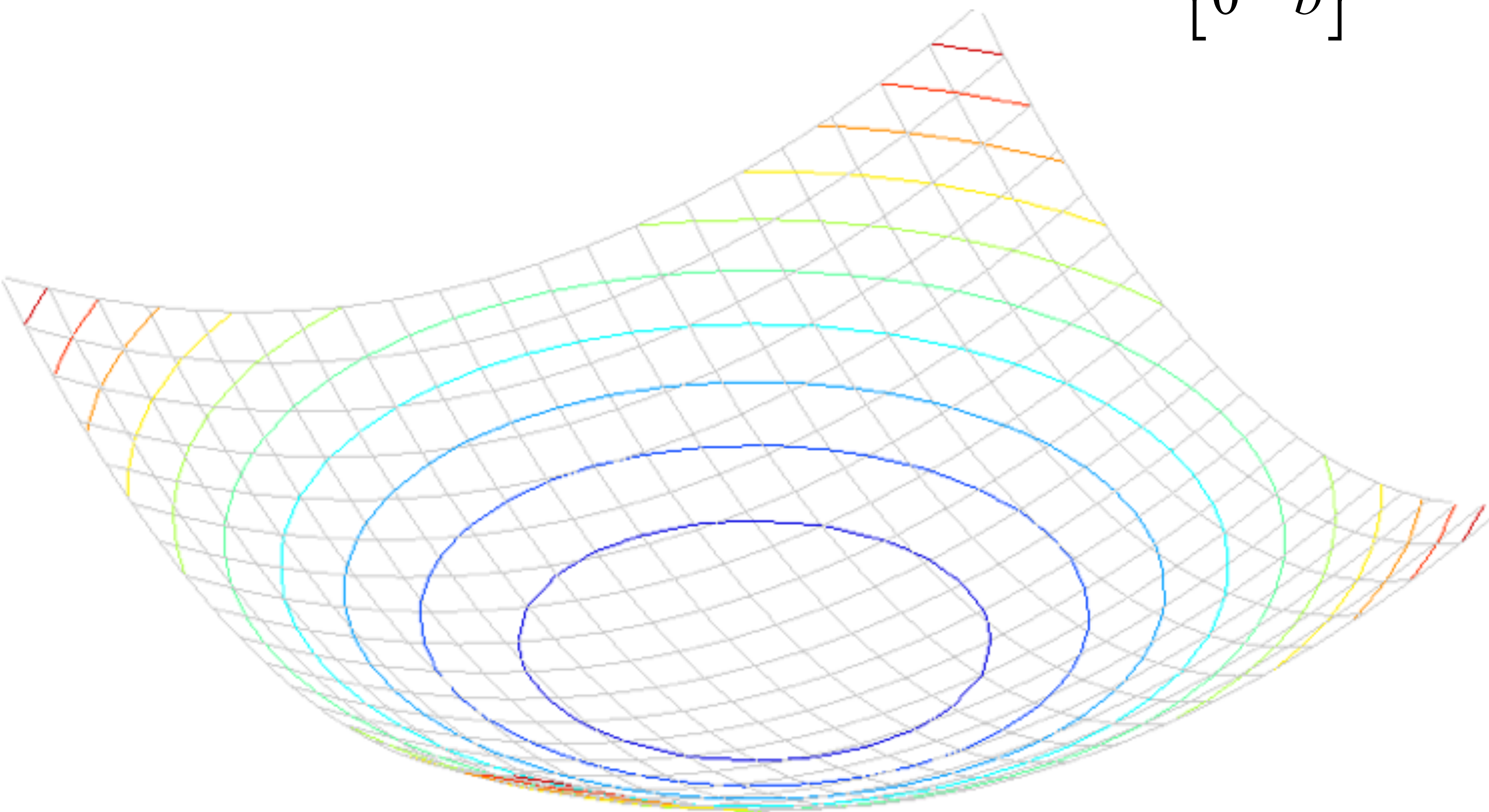
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$:

$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

$$M \rightarrow \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

This is the equation of an ellipse.



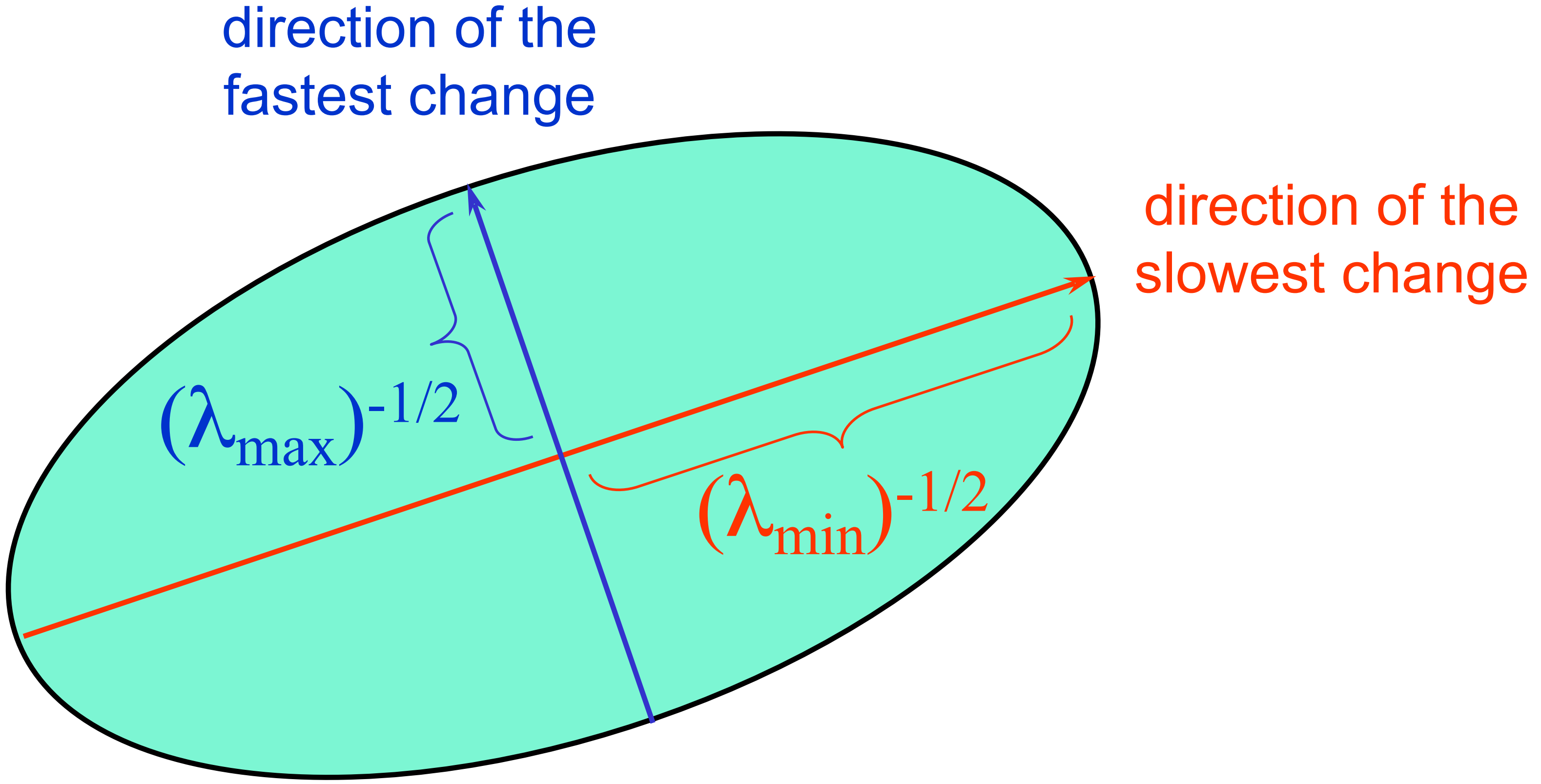
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

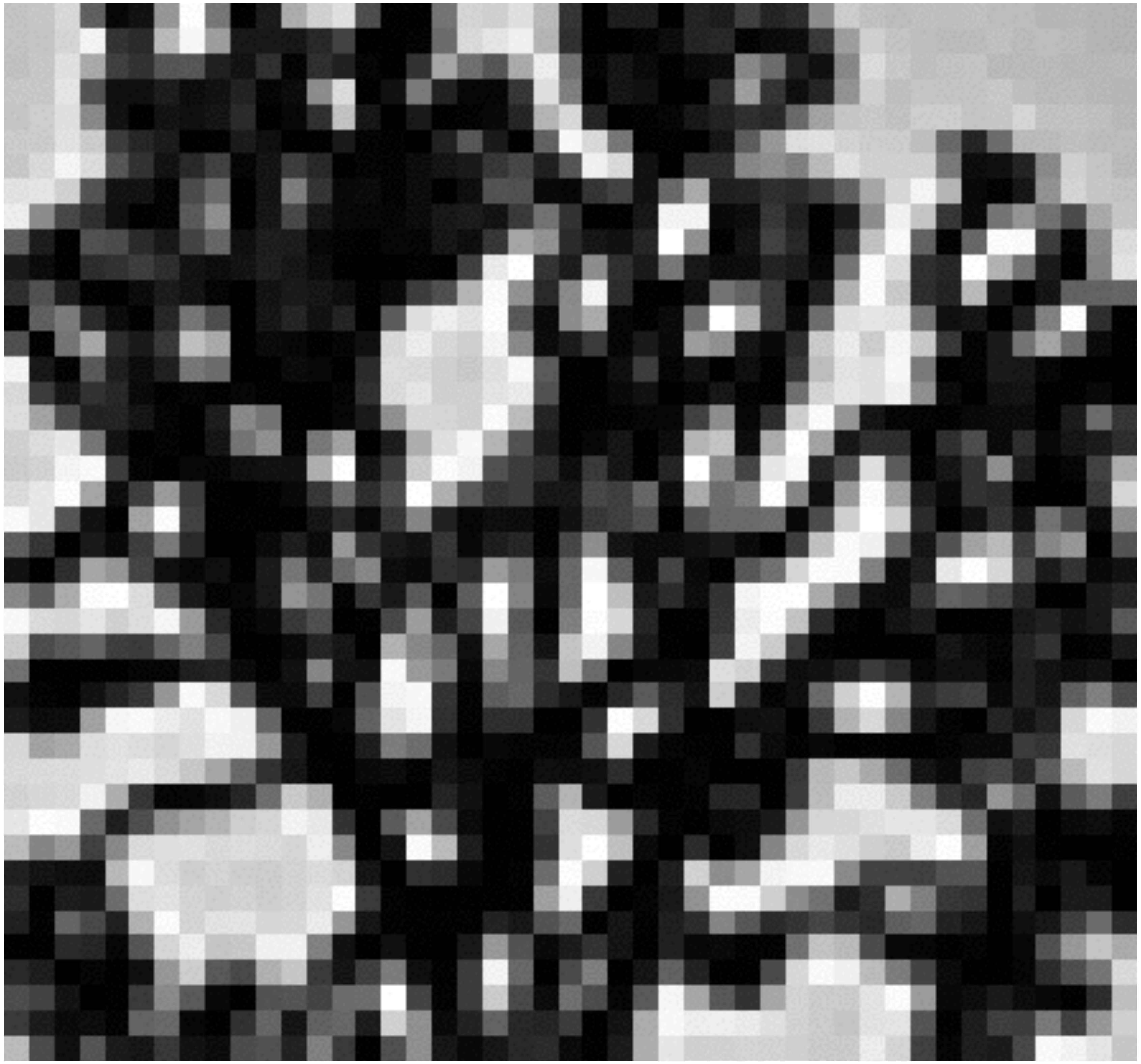
This is the equation of an ellipse.

Diagonalization of M: $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

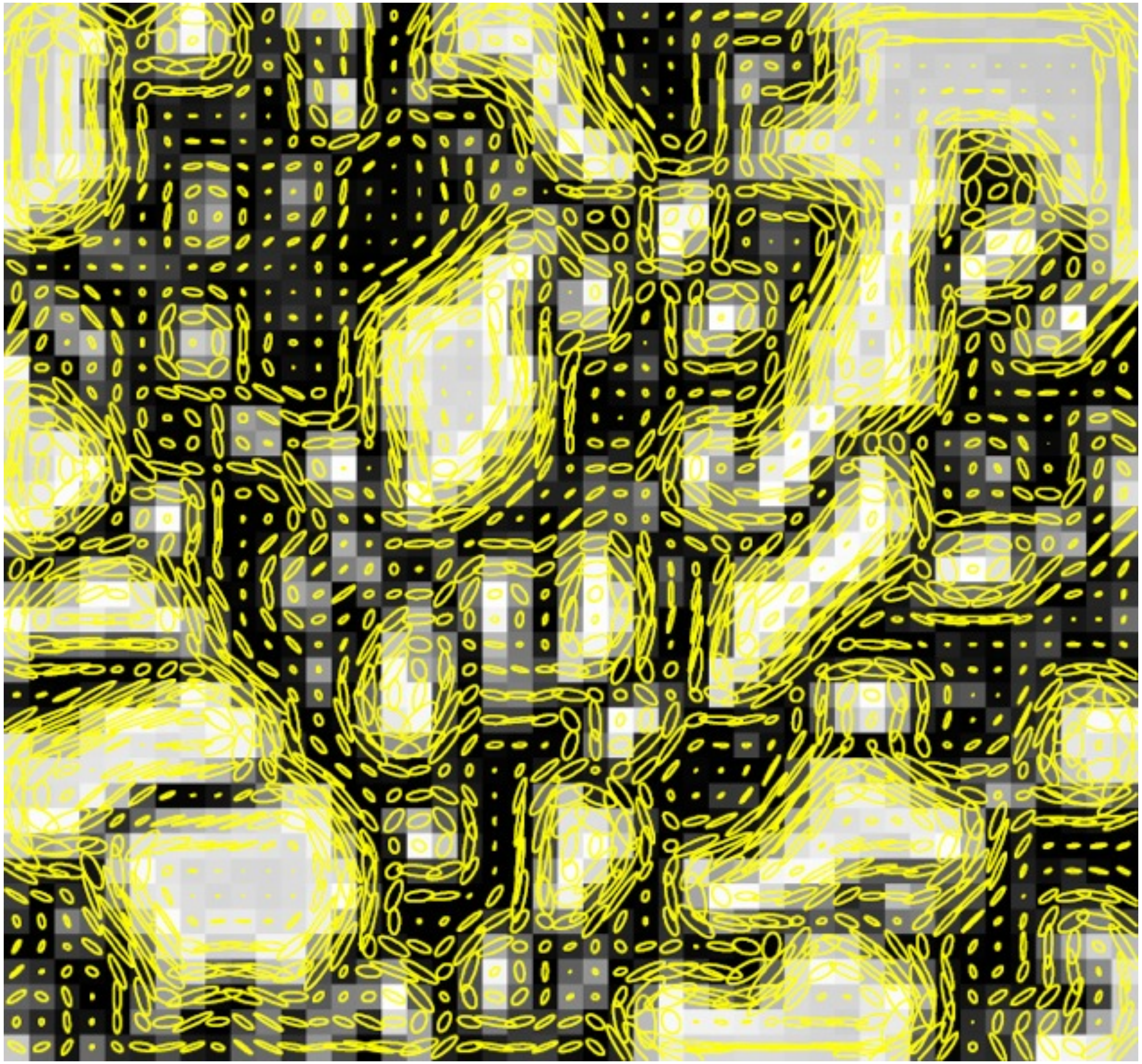
The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*



Visualization of second moment matrices

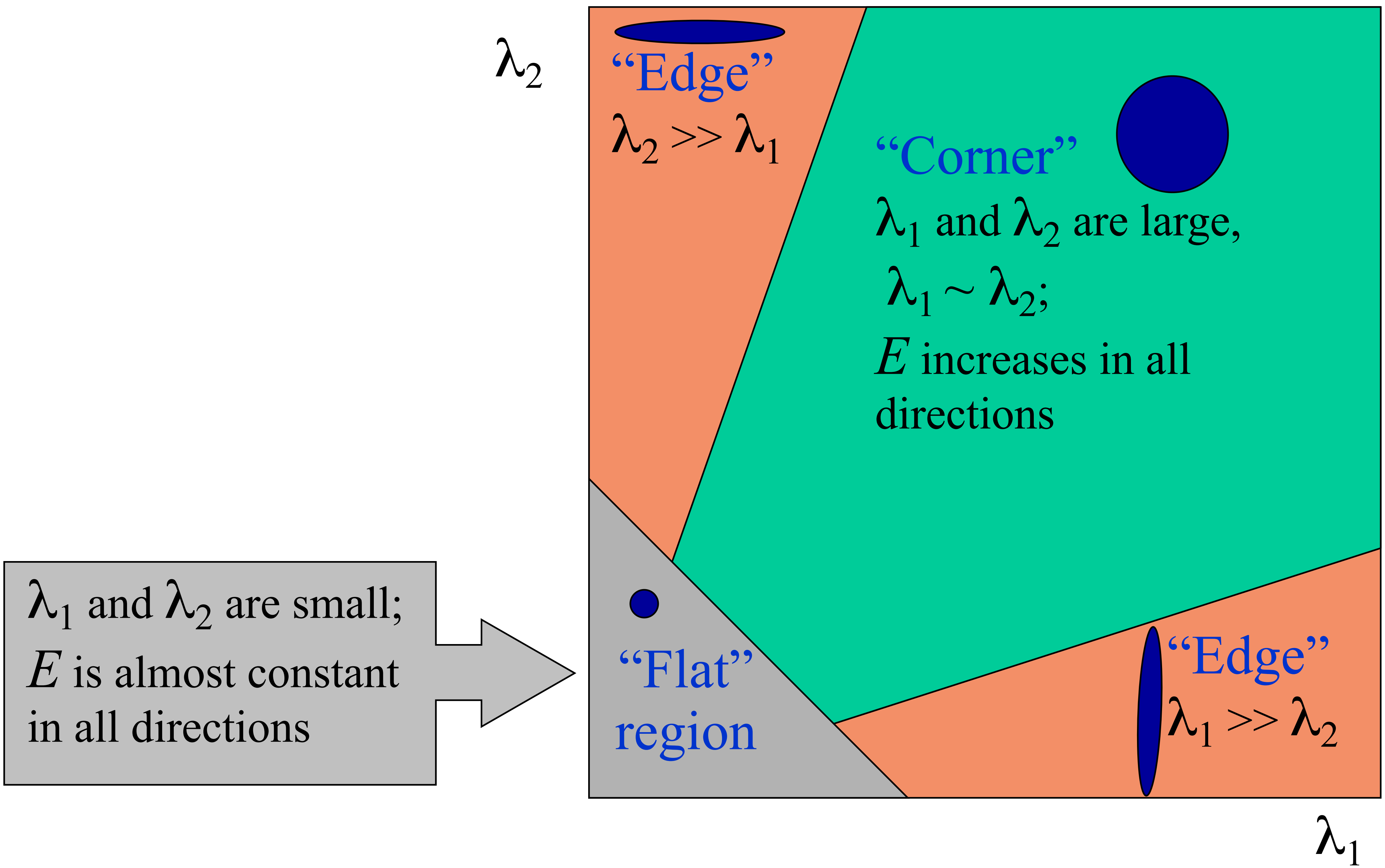


Visualization of second moment matrices



Interpreting the eigenvalues

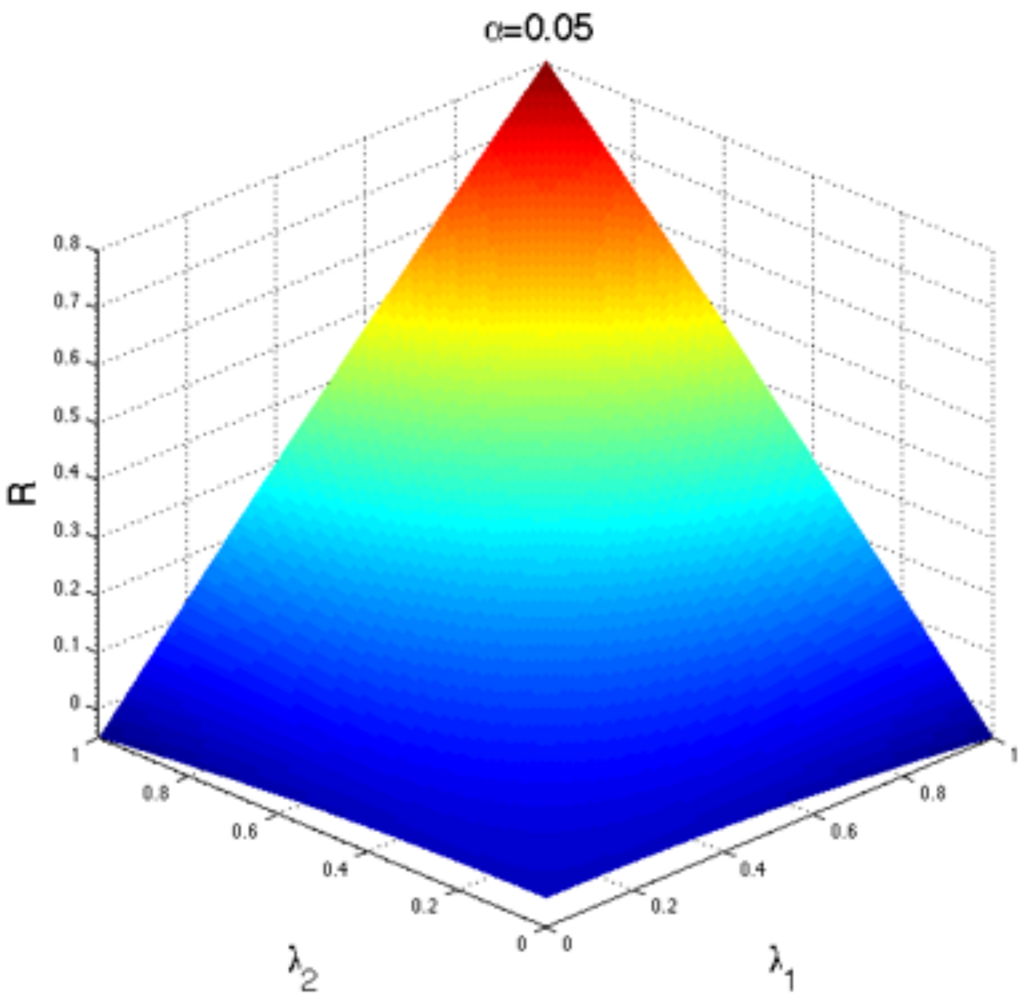
Classification of image points using eigenvalues of *M*:

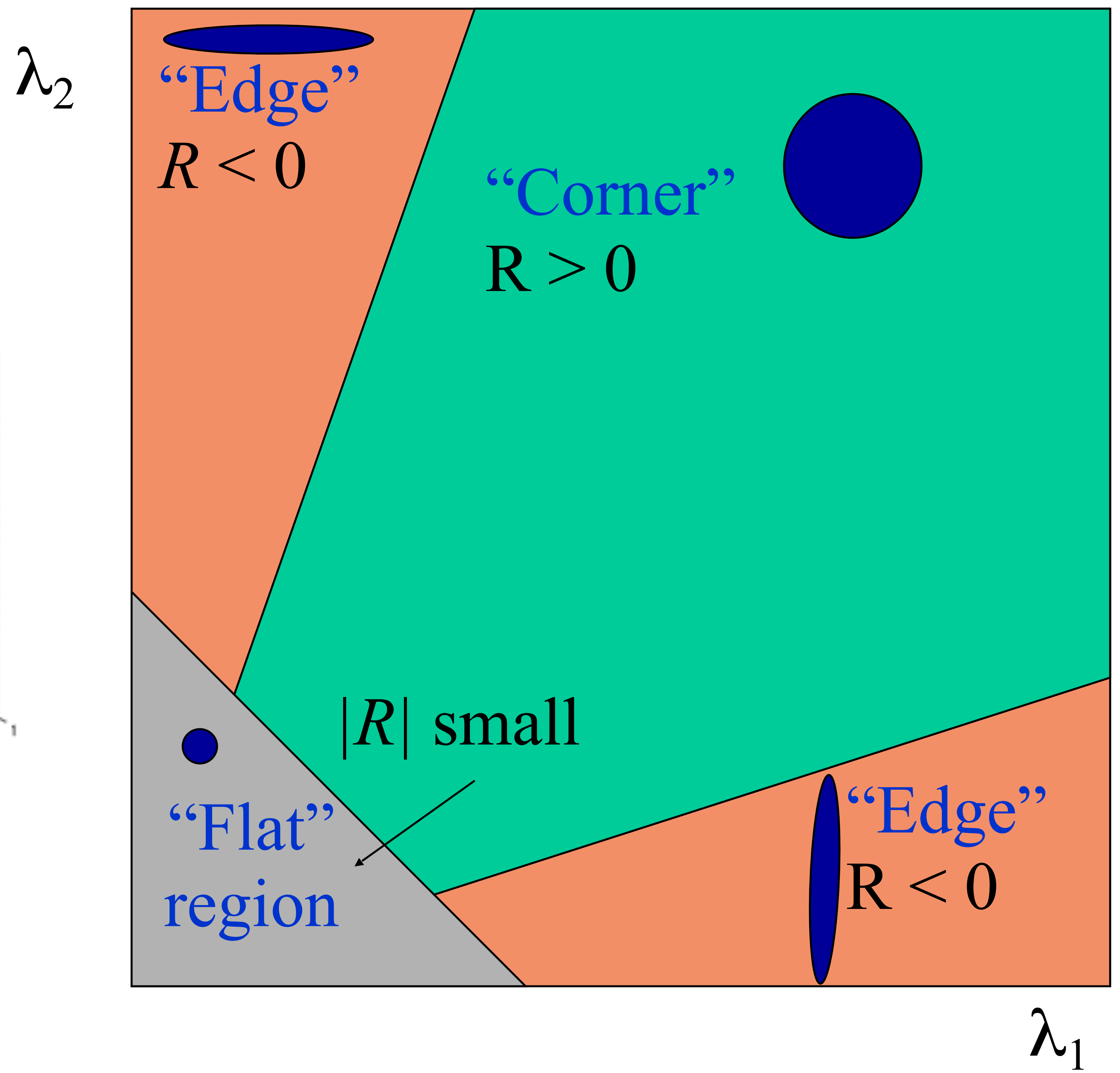


Corner response function

$$R = \det(M) - \alpha \,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)





The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. **"A Combined Corner and Edge Detector."** *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

The Harris corner detector

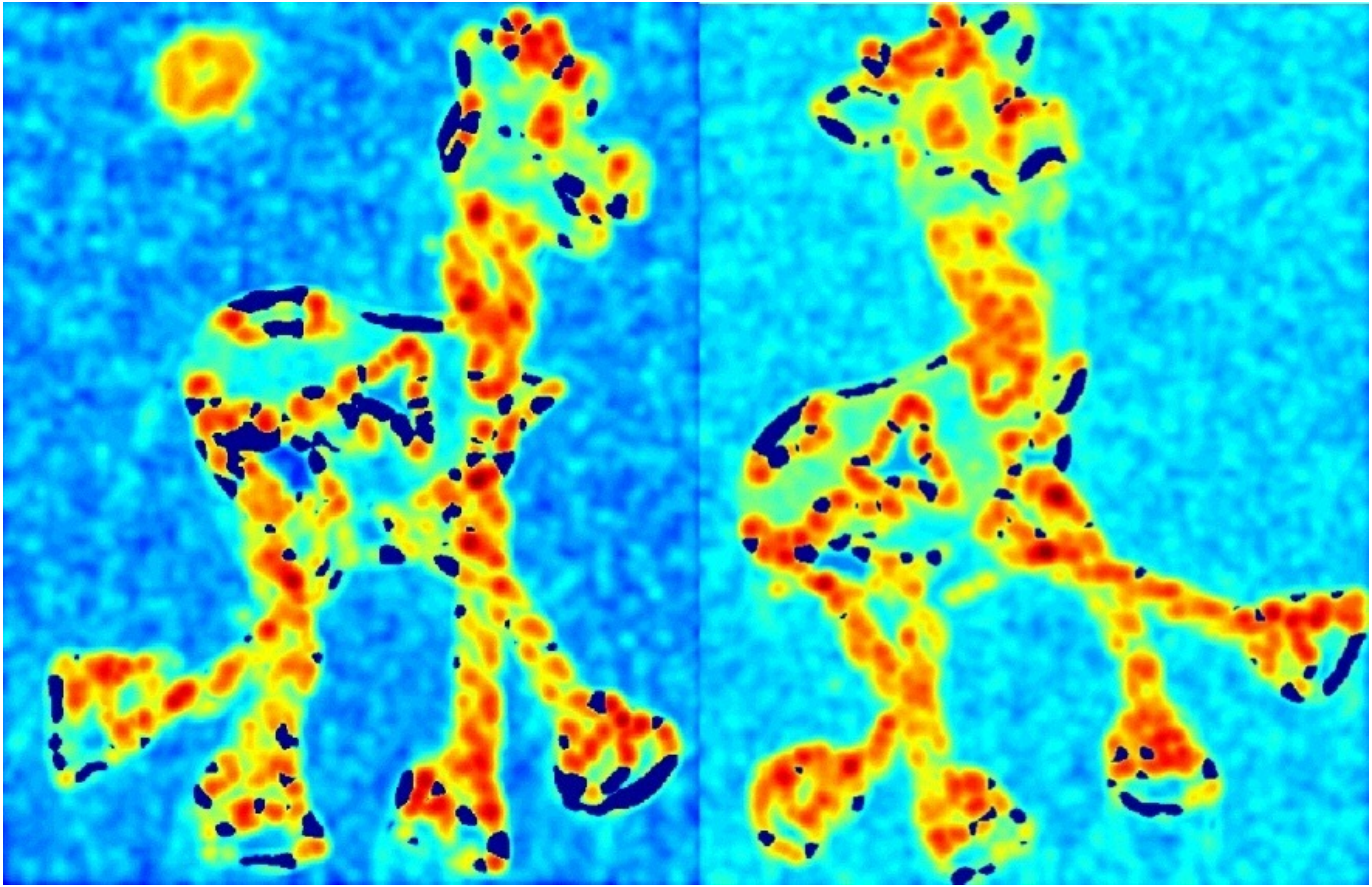
1. Compute partial derivatives at each pixel
2. Compute second moment matrix M in a Gaussian window around each pixel
3. Compute corner response function R

C.Harris and M.Stephens. **"A Combined Corner and Edge Detector."** *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Harris Detector: Steps

Harris Detector: Steps

Compute corner response *R*



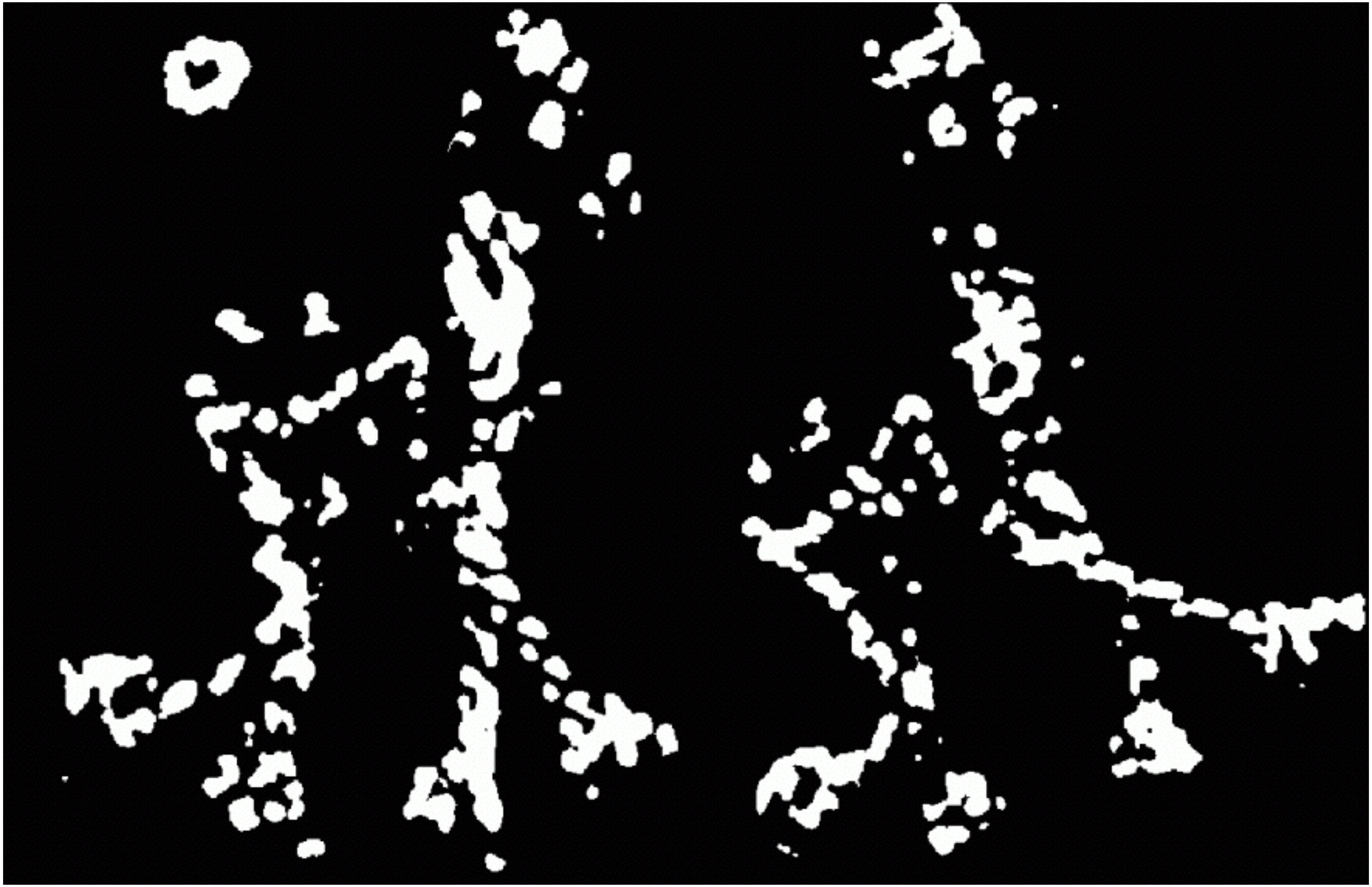
The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix M in a Gaussian window around each pixel
3. Compute corner response function R
4. Threshold R
5. Find local maxima of response function (non-maximum suppression)

C.Harris and M.Stephens. **"A Combined Corner and Edge Detector."** *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

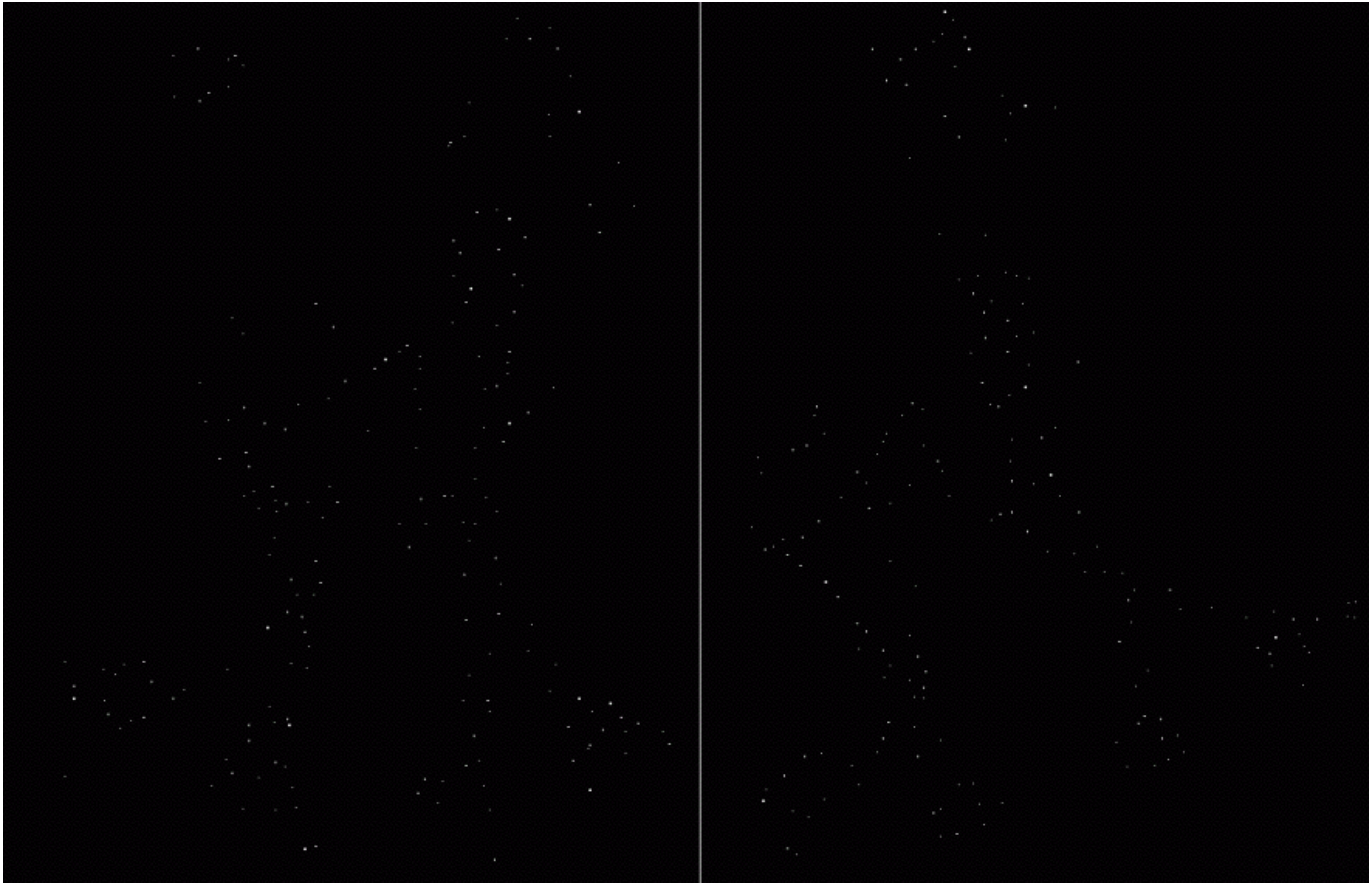
Harris Detector: Steps

Find points with large corner response: R > threshold



Harris Detector: Steps

Take only the points of local maxima of *R*

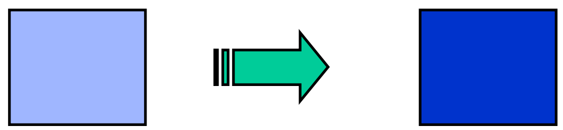


Harris Detector: Steps

Invariance and covariance

- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
  - **Invariance:** image is transformed and corner locations do not change
  - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

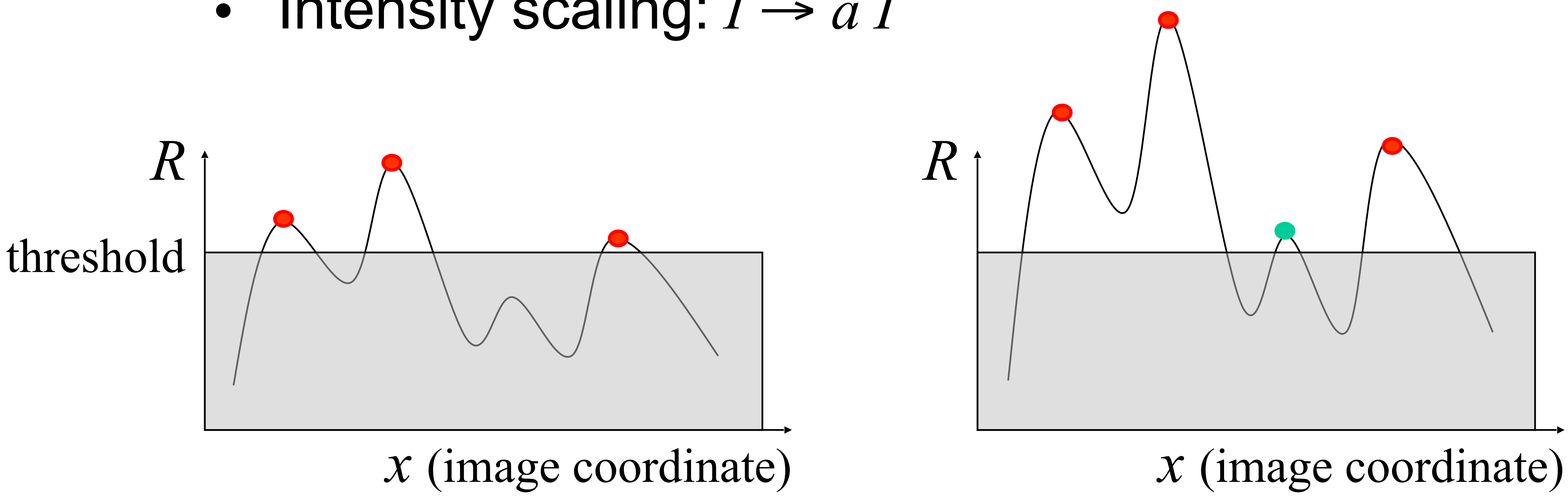
Affine intensity change



$$I \rightarrow a\,I + b$$

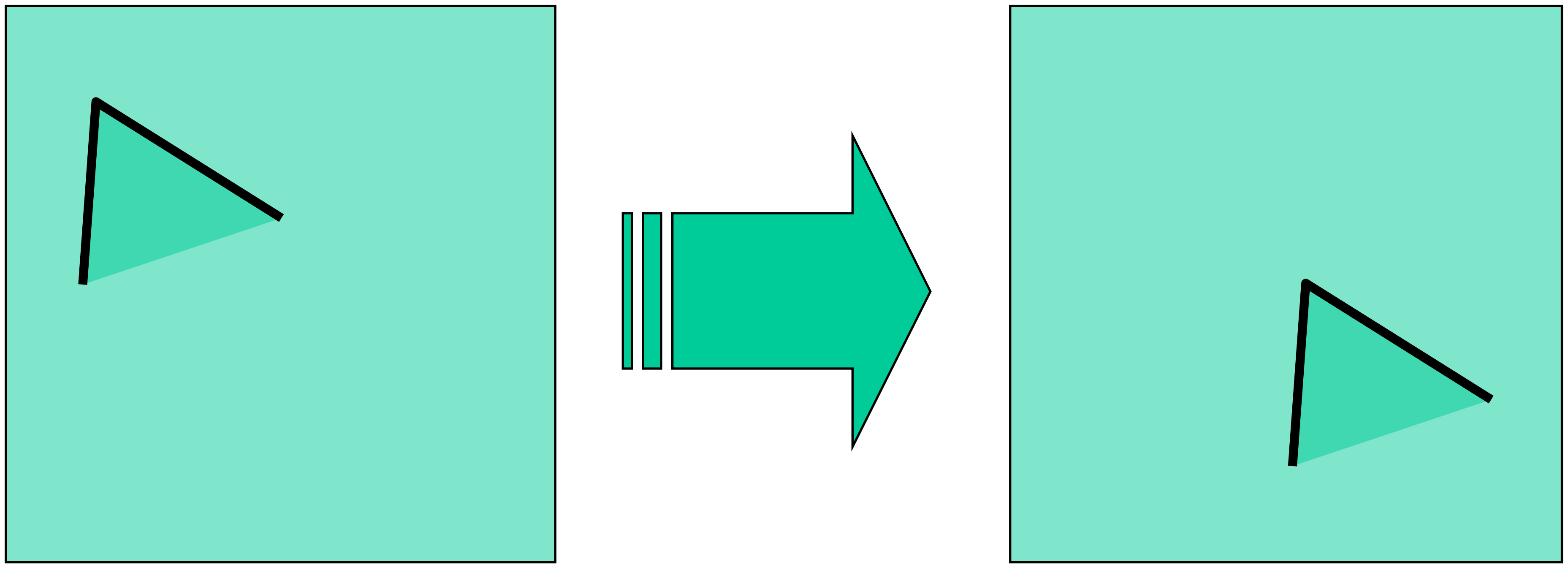
- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

- Intensity scaling: $I \rightarrow a\,I$



Partially invariant to affine intensity change

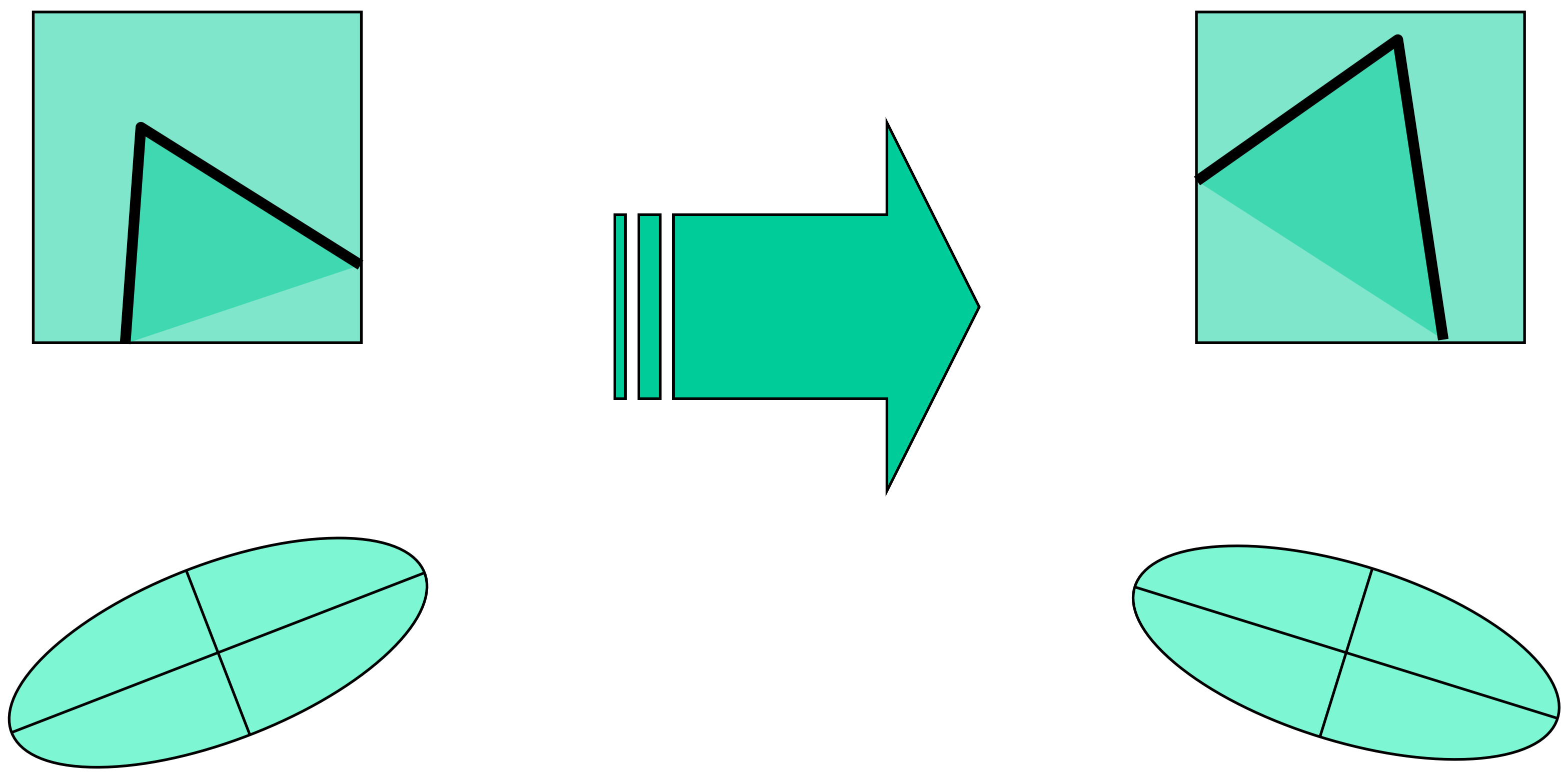
Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

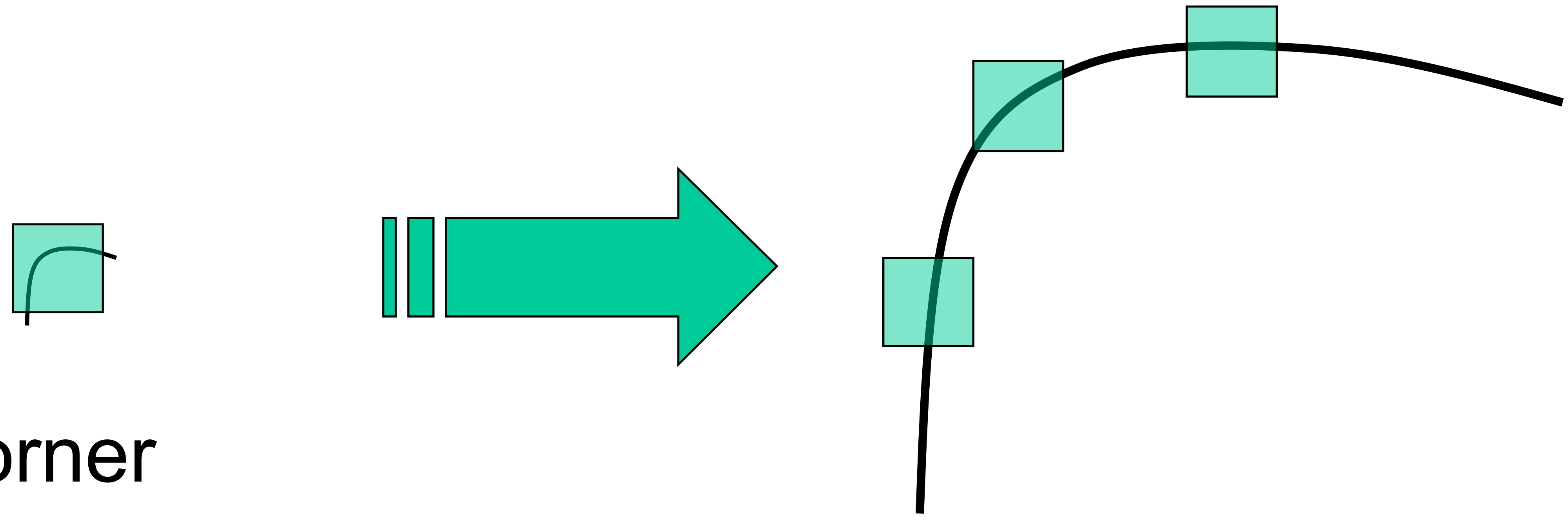
Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



Corner

All points will be classified as edges

Corner location is not covariant to scaling!

Further thoughts and readings…

- Original corner detector paper
  - C.Harris and M.Stephens, **"A Combined Corner and Edge Detector."** Proceedings of the 4th Alvey Vision Conference,1988
- Other corner functions
  - Can you think of other $f(\lambda_1, \lambda_2)$ that work for finding corners?
- How can we make the Harris corner detector scale covariant?