CMPSCI 670: Computer Vision

Linear filtering cont., Edge detection

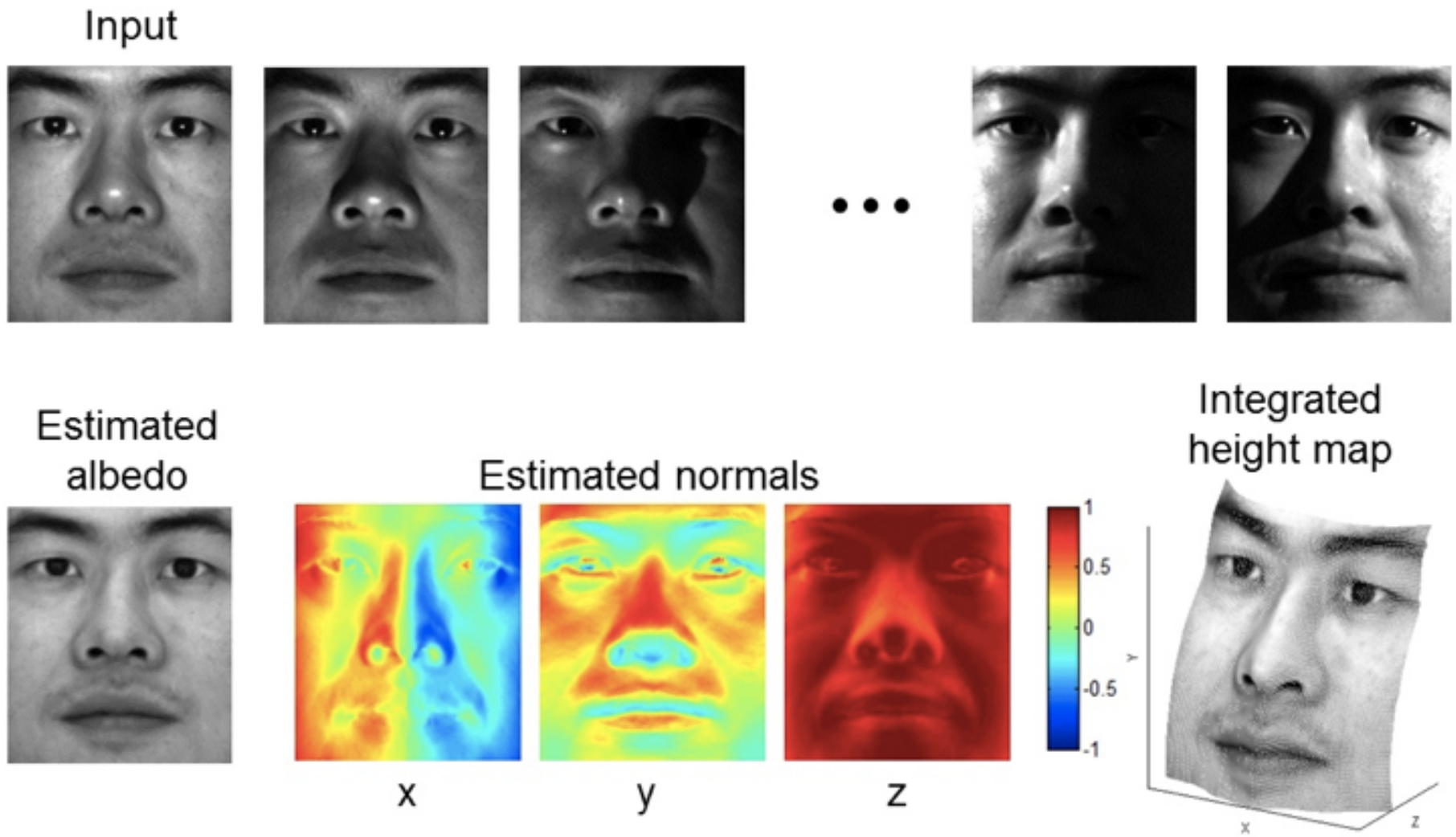
University of Massachusetts, Amherst

September 24, 2014

Instructor: Subhransu Maji

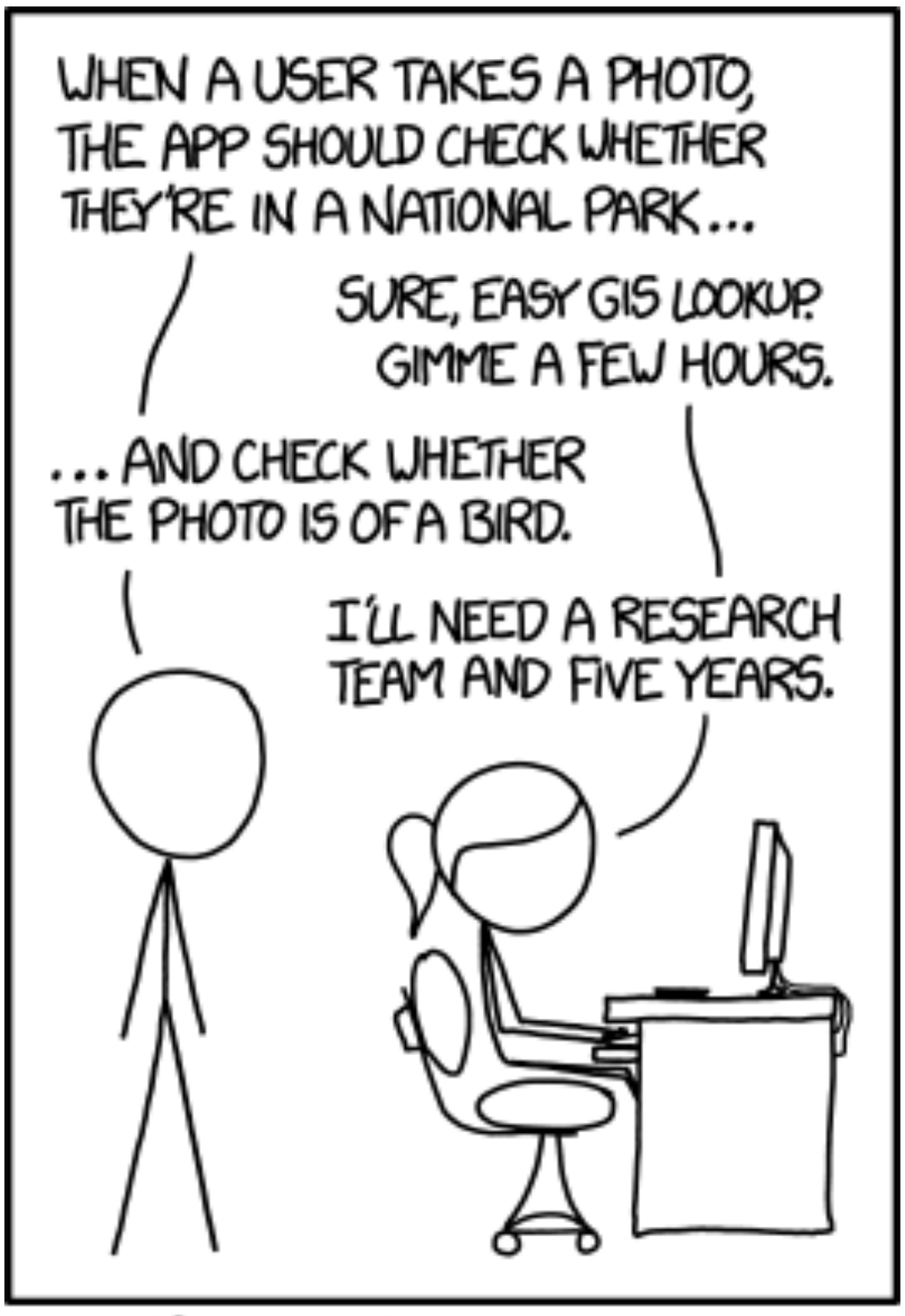
Administrivia

- Homework 2 is up (due October 6 before class starts)
  - Photometric stereo



- Office hours this week (~~today~~ tomorrow) Th 3:45-4:45, CS274

Joys of computer vision research



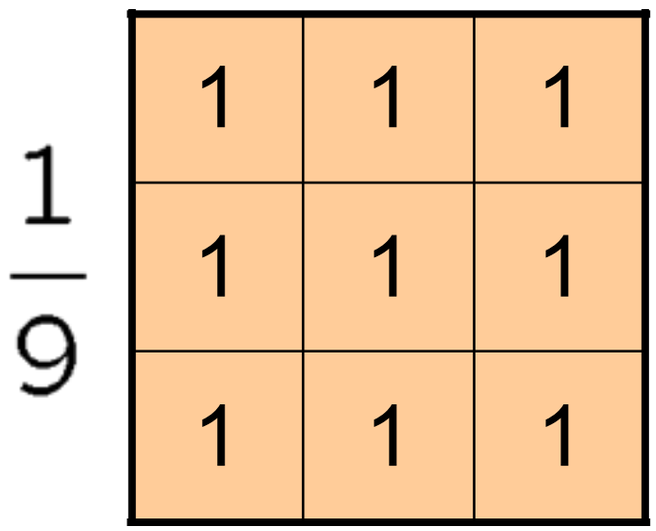
http://xkcd.com/1425/

Motivation: Image de-noising

- How can we reduce noise in a photograph?

Moving average

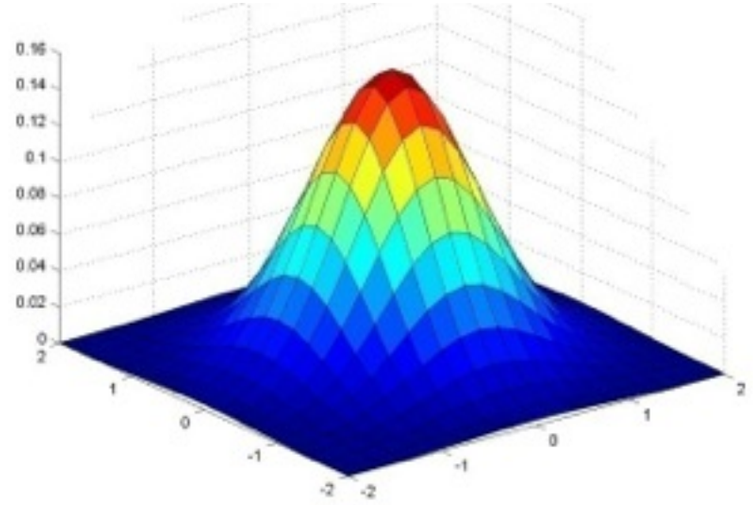
- Let's replace each pixel with a ***weighted*** average of its neighborhood
- The weights are called the *filter kernel*
- What are the weights for the average of a 3x3 neighborhood?



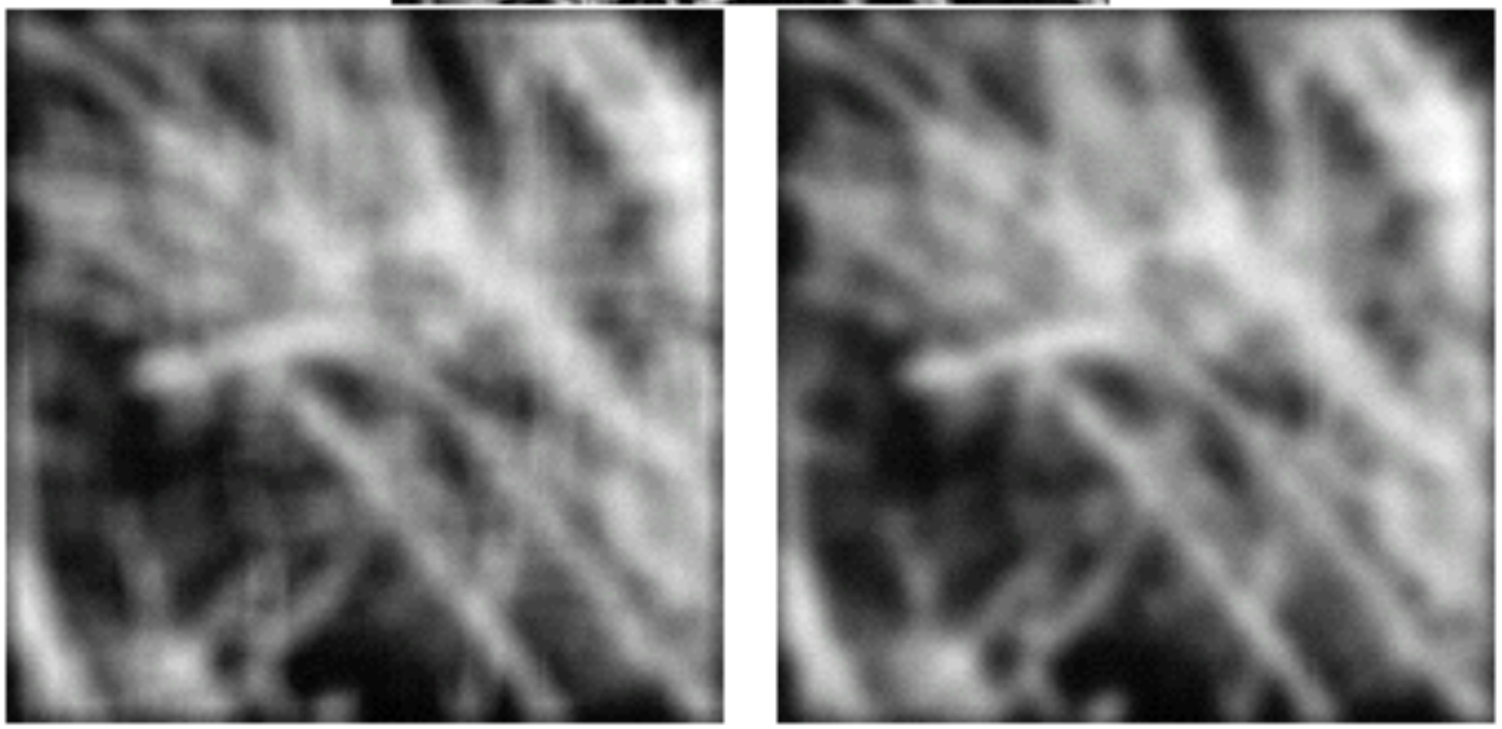
"box filter"

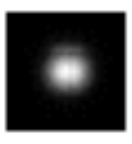
Source: D. Lowe

Gaussian vs. box filtering



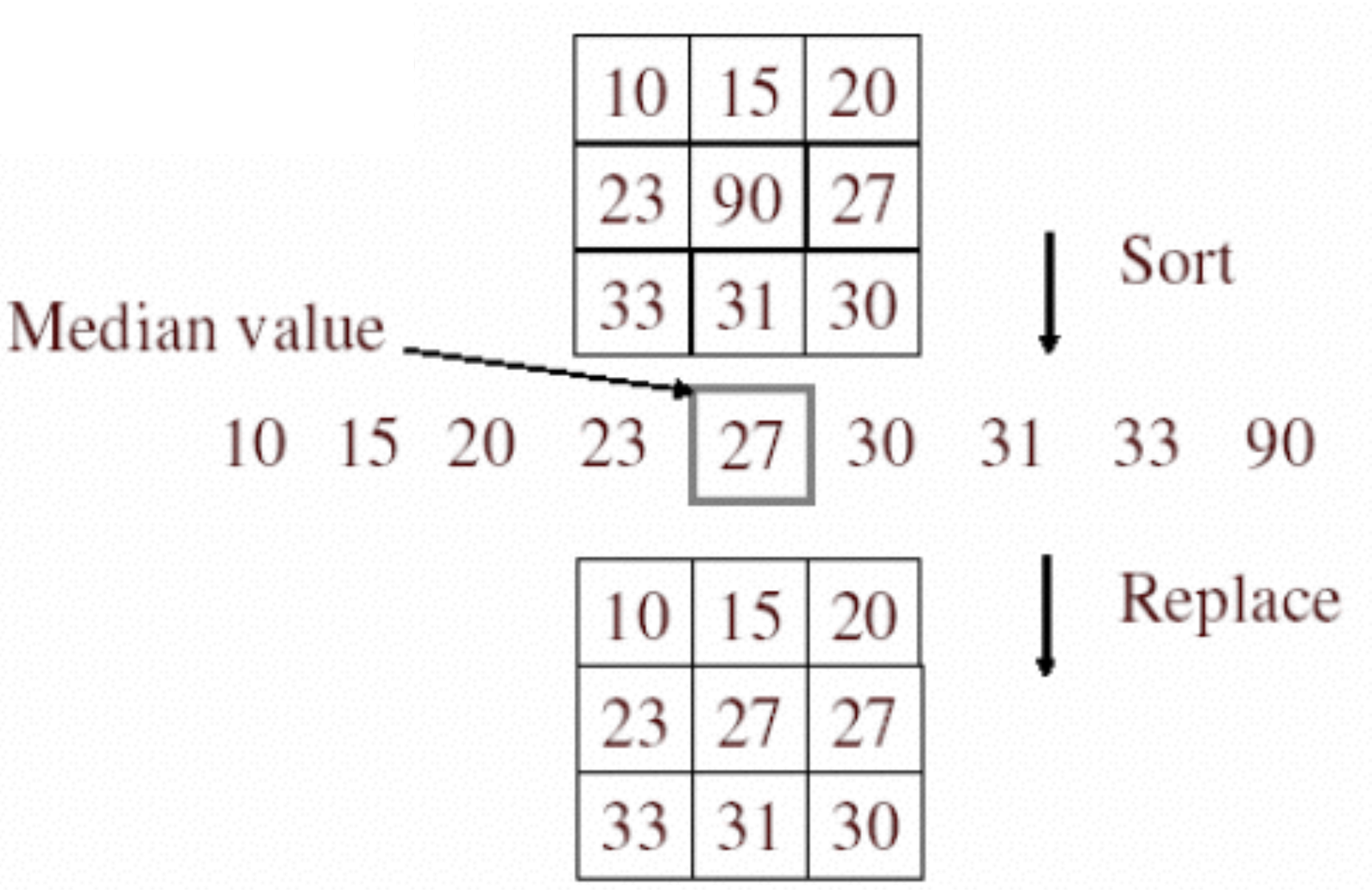
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$





Alternative idea: Median filtering

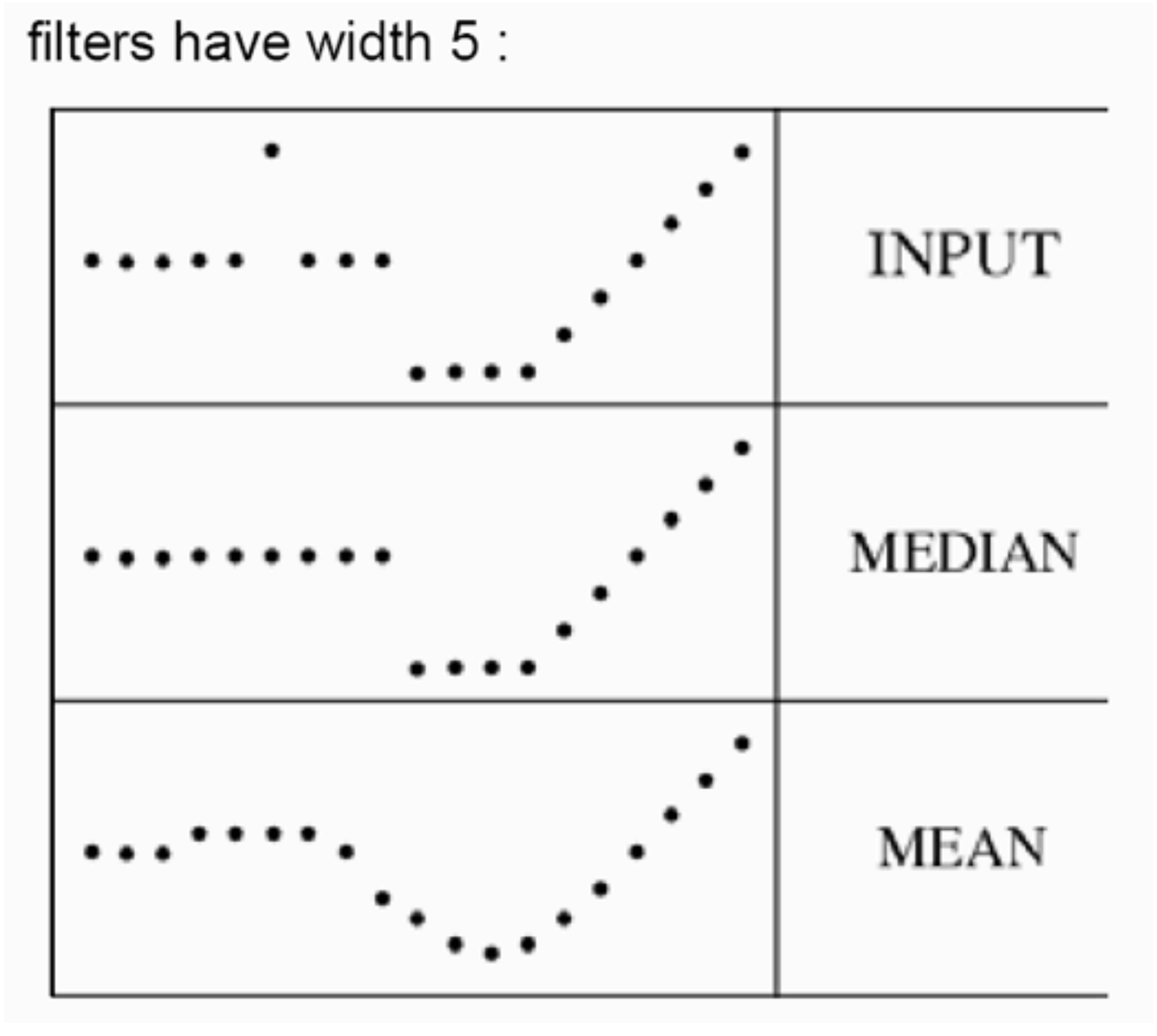
- A **median filter** operates over a window by selecting the median intensity in the window



- Is median filtering linear?

Median filter

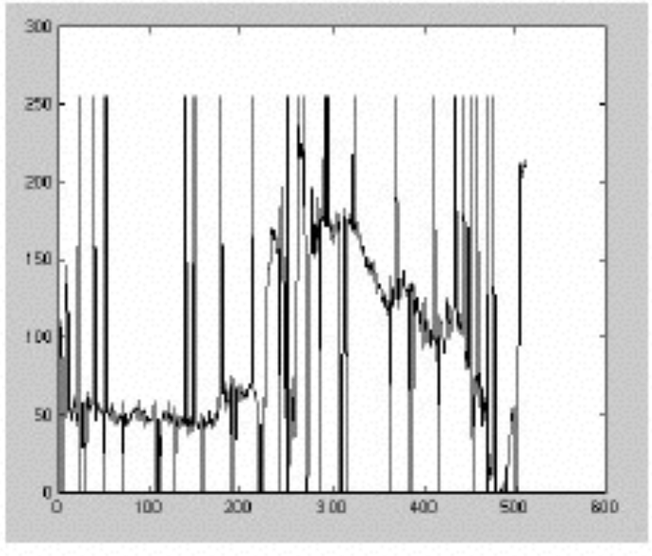
- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers



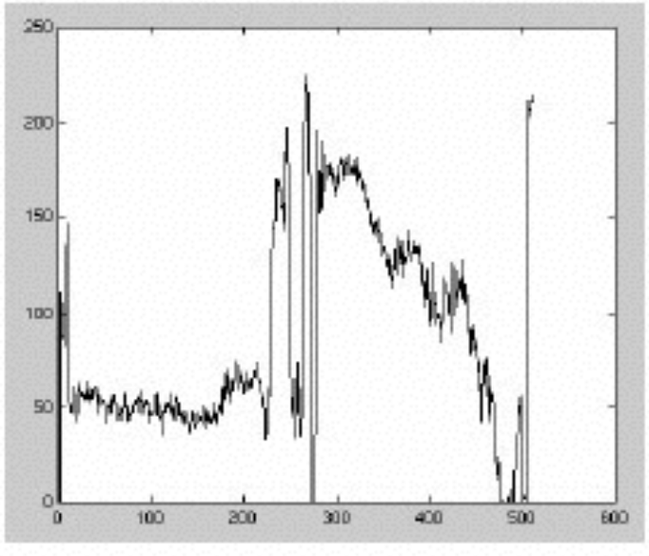
Source: K. Grauman

Median filter

Salt-and-pepper noise



Median filtered

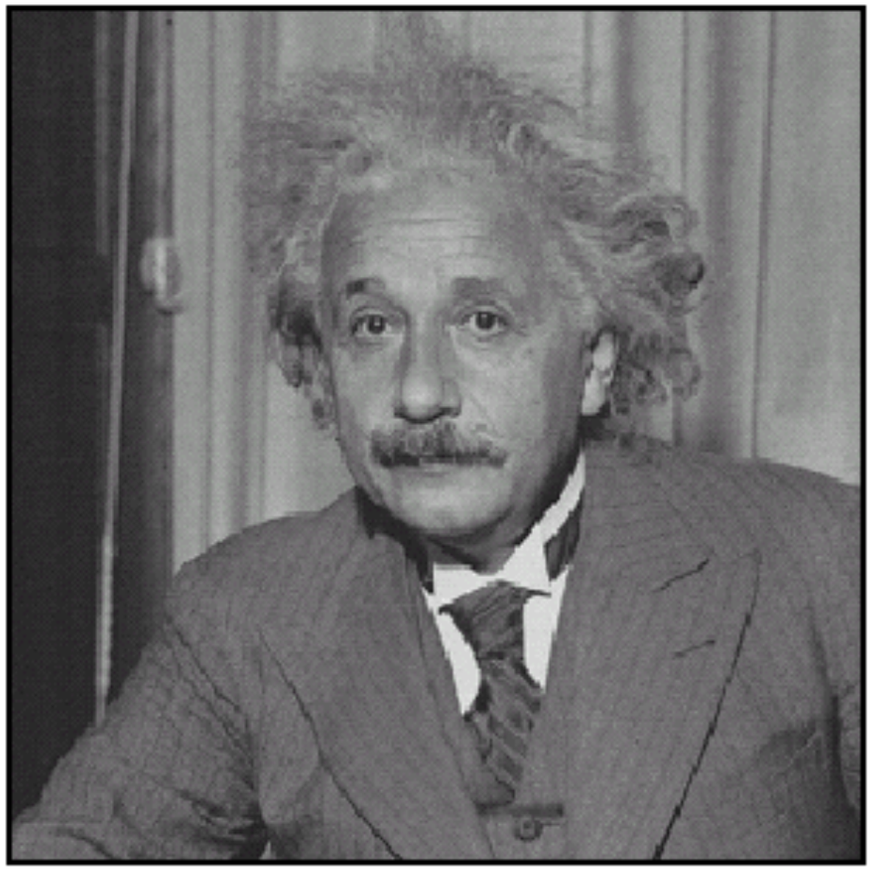


MATLAB: medfilt2(image, [h w])

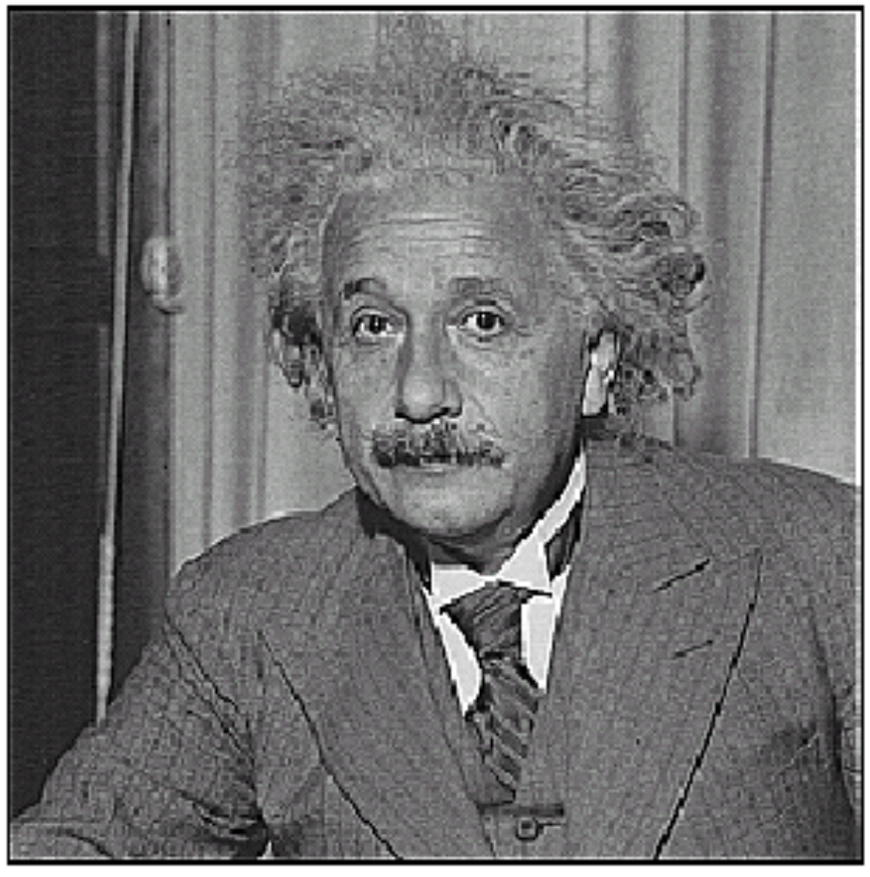
Gaussian vs. median filtering

	3x3	5x5	7x7
Gaussian			
Median			

Sharpening revisited



before



after

Source: D. Lowe

Sharpening revisited

What does blurring take away?

–

=

Let’s add it back:

+ α

=

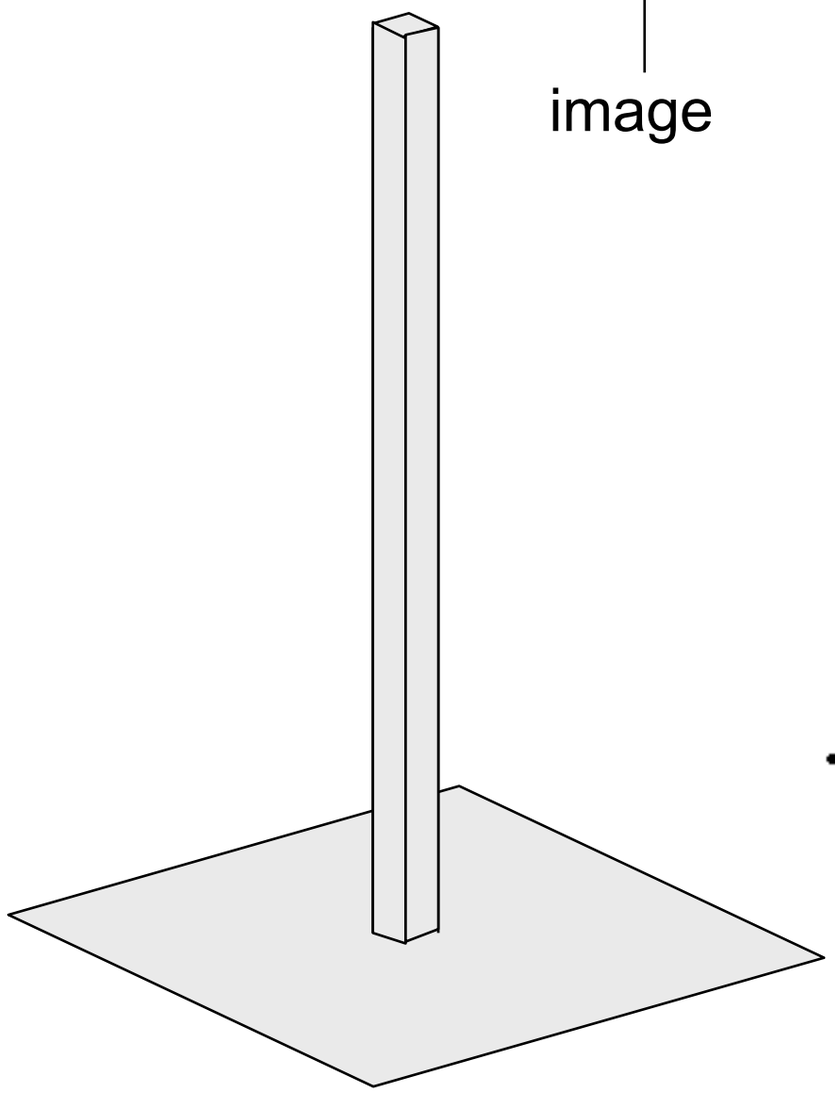
Unsharp mask filter

$$f + \alpha(f - f * g) = (1+\alpha)f - \alpha\, f * g = f * ((1+\alpha)e - g)$$

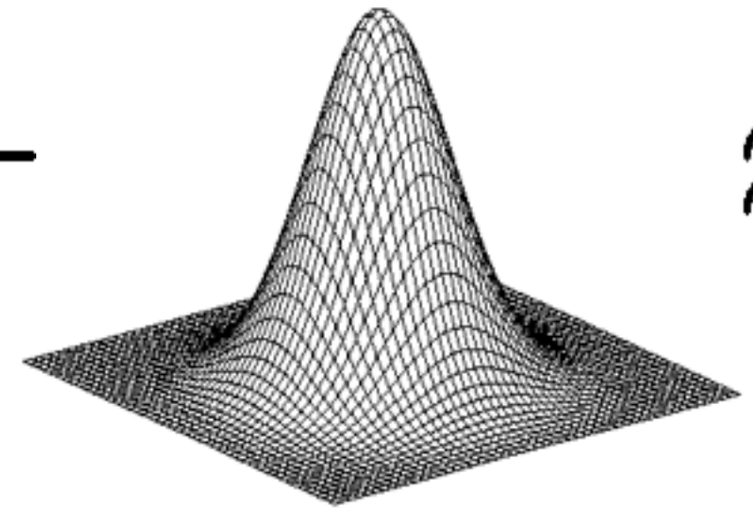
image

blurred image

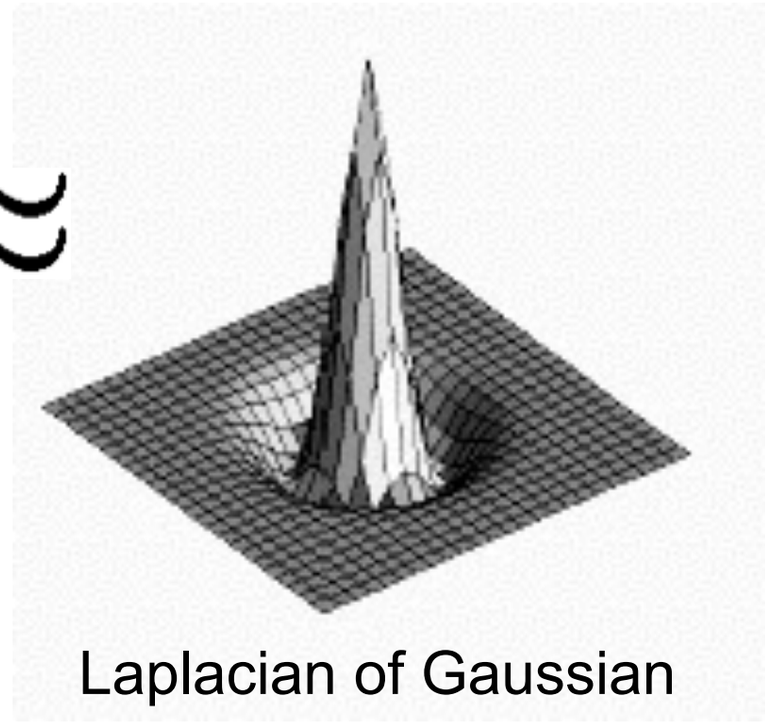
unit impulse (identity)



unit impulse

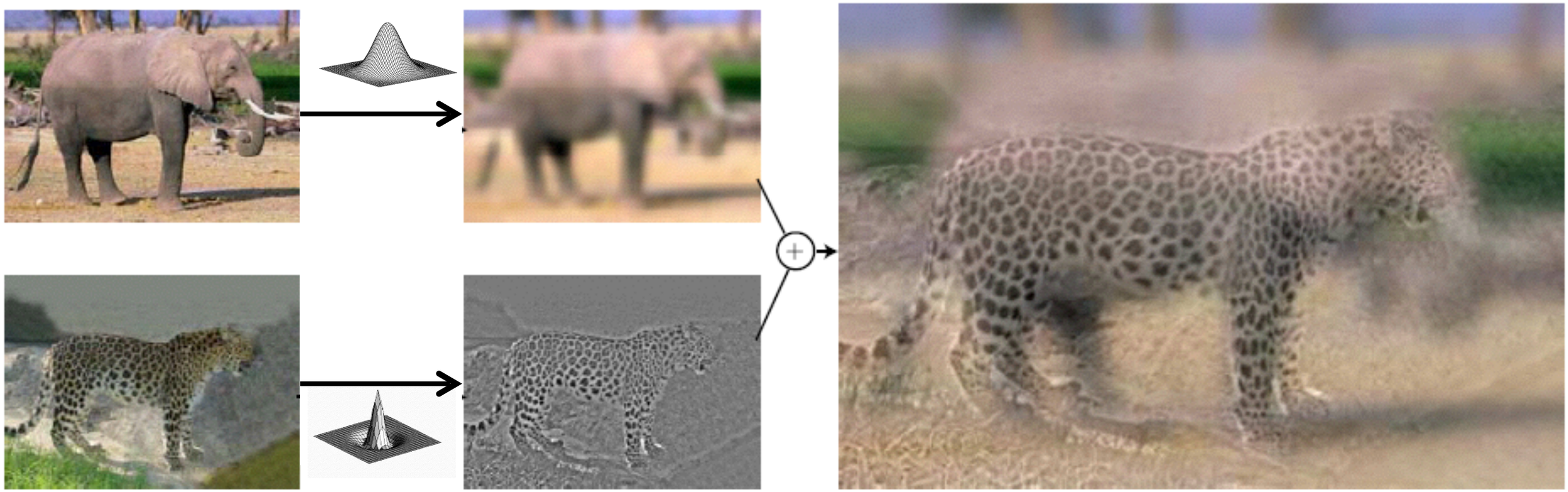


Gaussian



Laplacian of Gaussian

Application: Hybrid Images

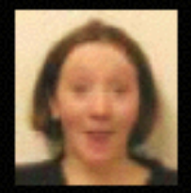


A. Oliva, A. Torralba, P.G. Schyns,
“Hybrid Images,” SIGGRAPH 2006

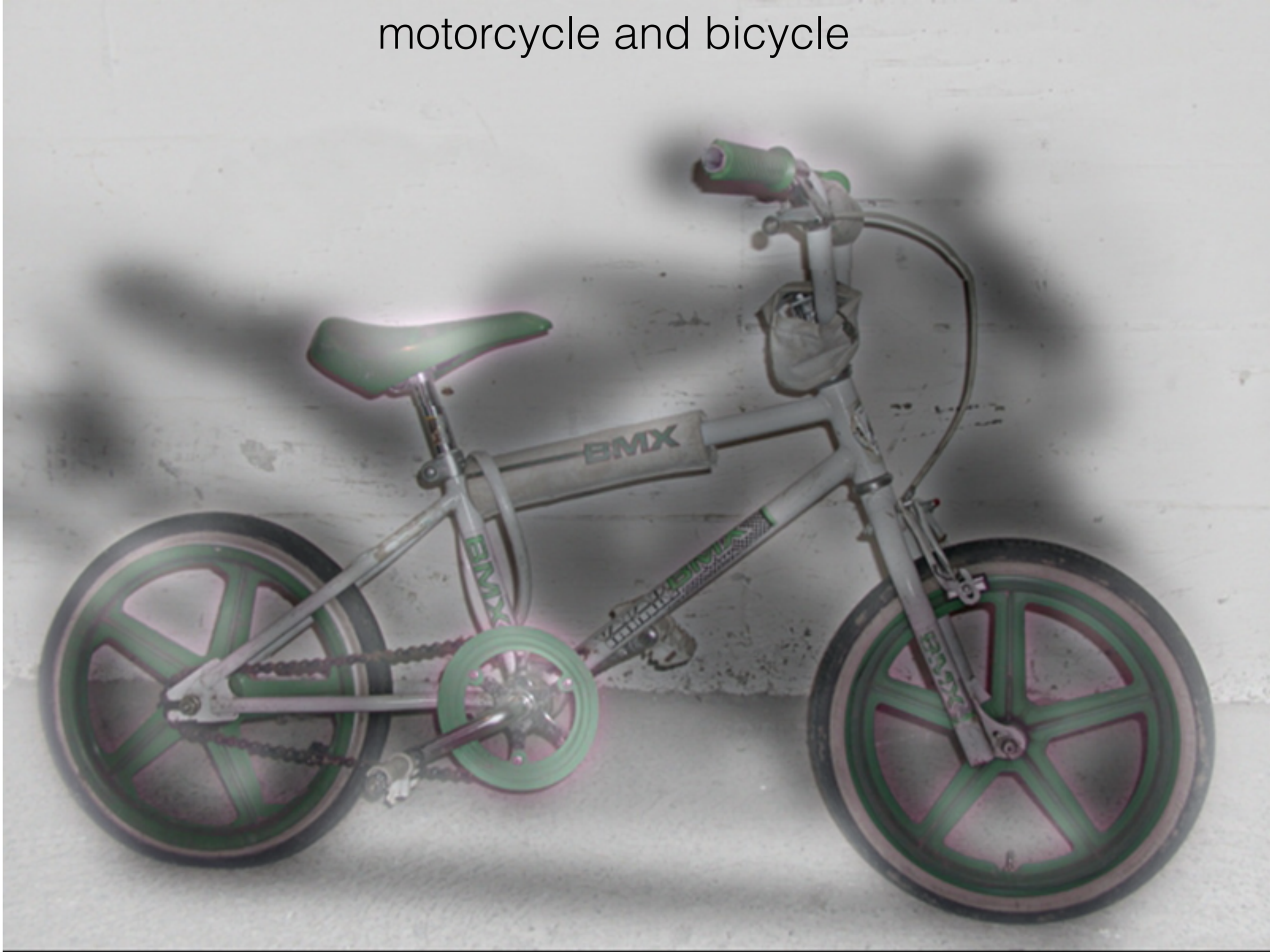
Changing expression

SIGGRAPH2006

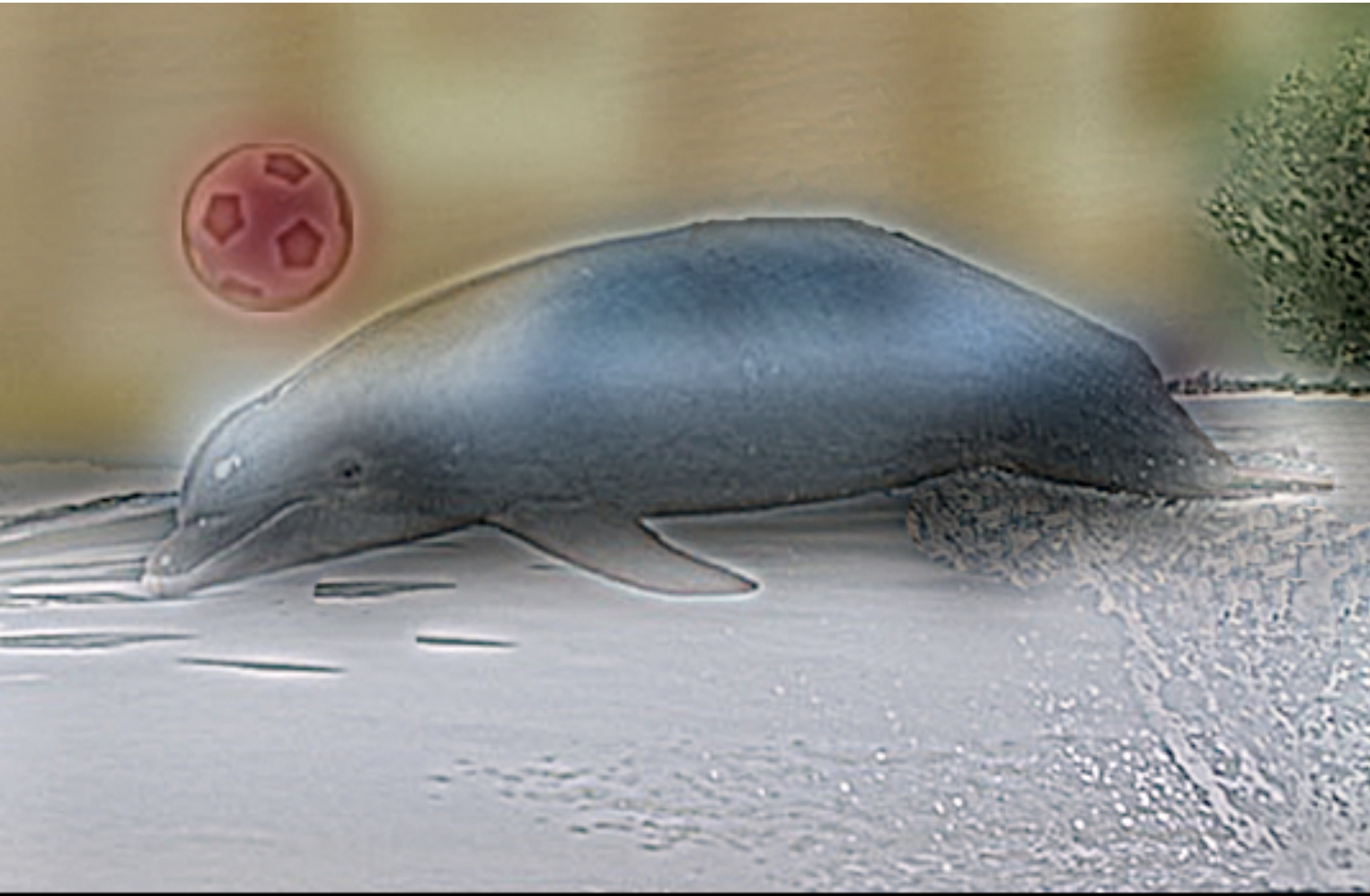
Sad ⟷ Surprised



motorcycle and bicycle



dolphin and car



Edge detection

[Winter in Kraków photographed by Marcin Ryczek]

Edge detection

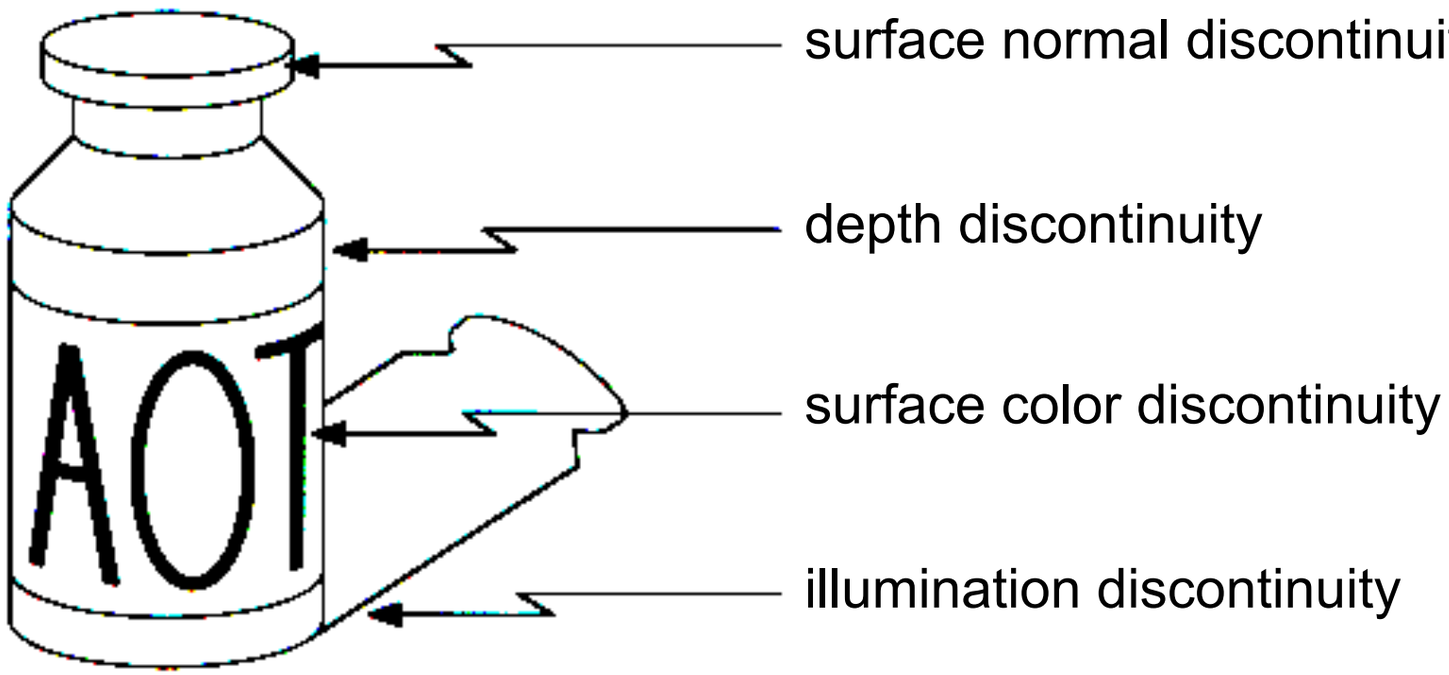
- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels

- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Source: D. Lowe

Origin of edges

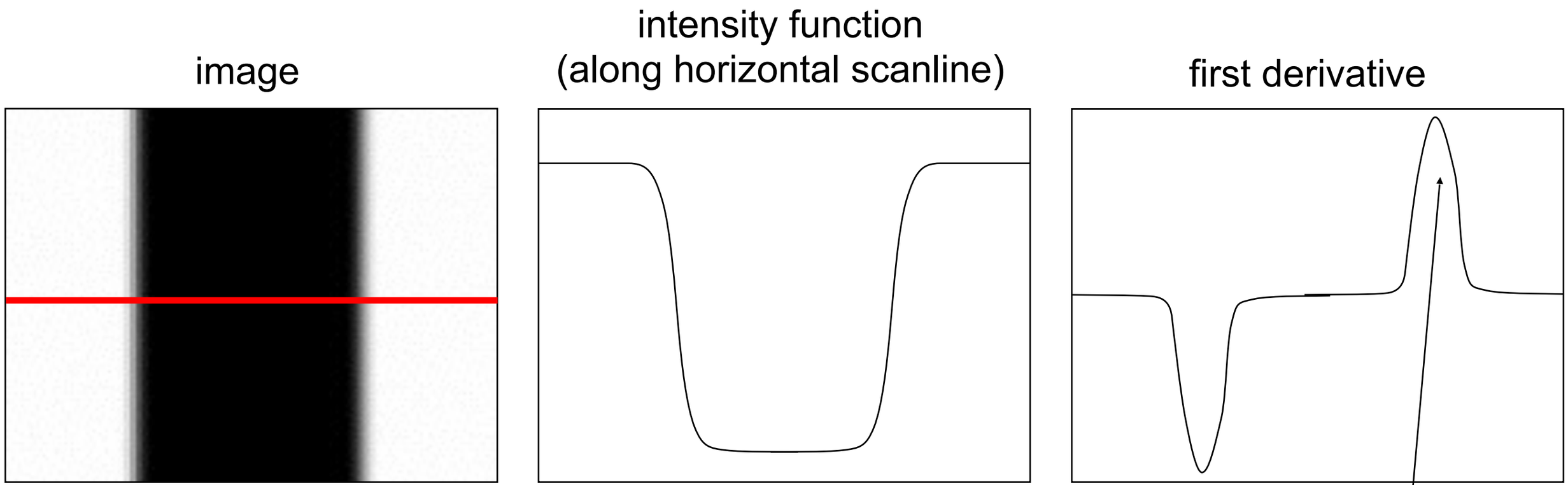
Edges are caused by a variety of factors:



Source: Steve Seitz

Edge detection

- An edge is a place of rapid change in the image intensity function



Derivatives with convolution

For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1,y) - f(x,y)}{1}$$

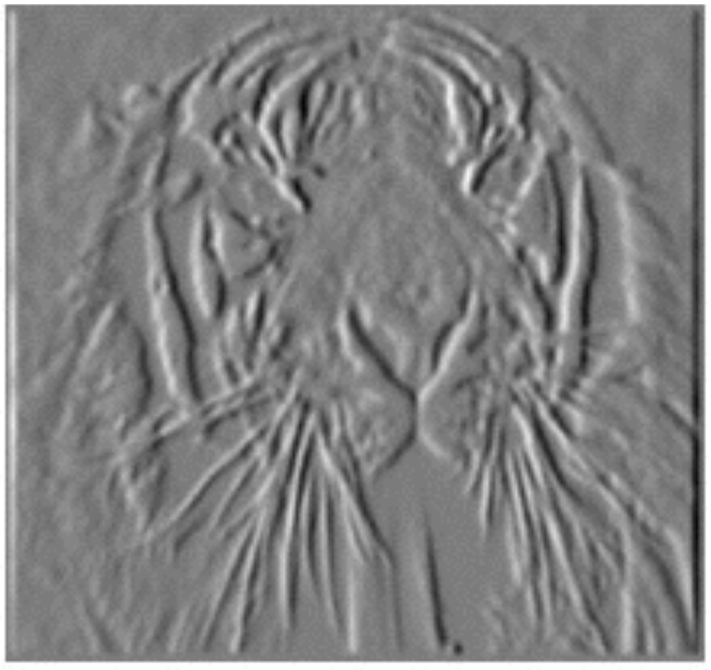
To implement the above as convolution, what would be the associated filter?

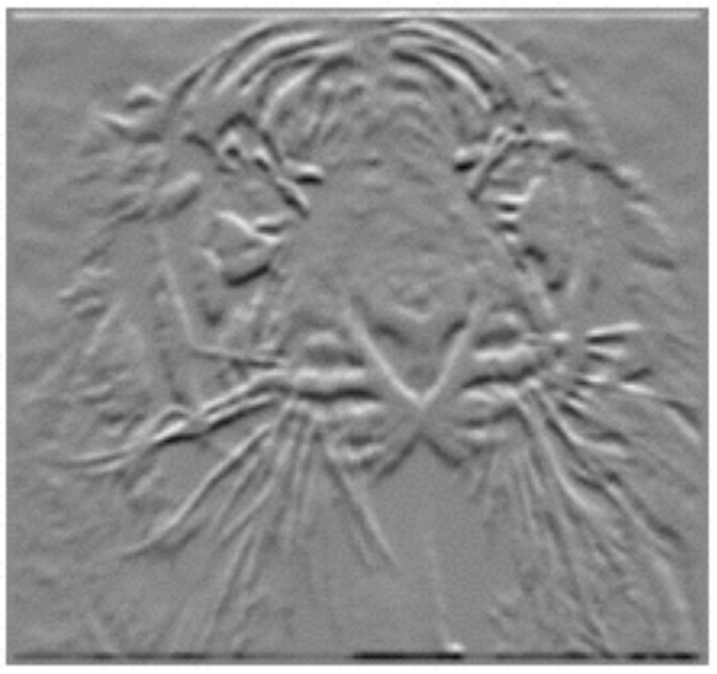
Source: K. Grauman

Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$

-1	1





$$\frac{\partial f(x,y)}{\partial y}$$

-1
1

or

1
-1

Which one shows changes with respect to x?

Finite difference filters

Other approximations of derivative filters exist:

Prewitt: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Sobel: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

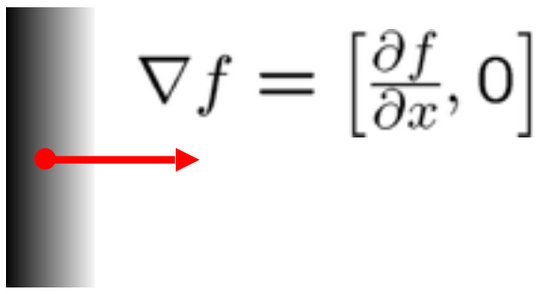
Roberts: $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

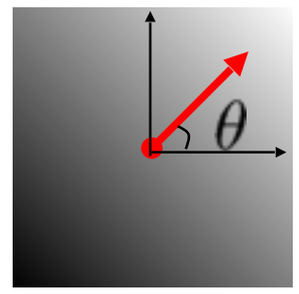
Source: K. Grauman

Image gradient

The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$





The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

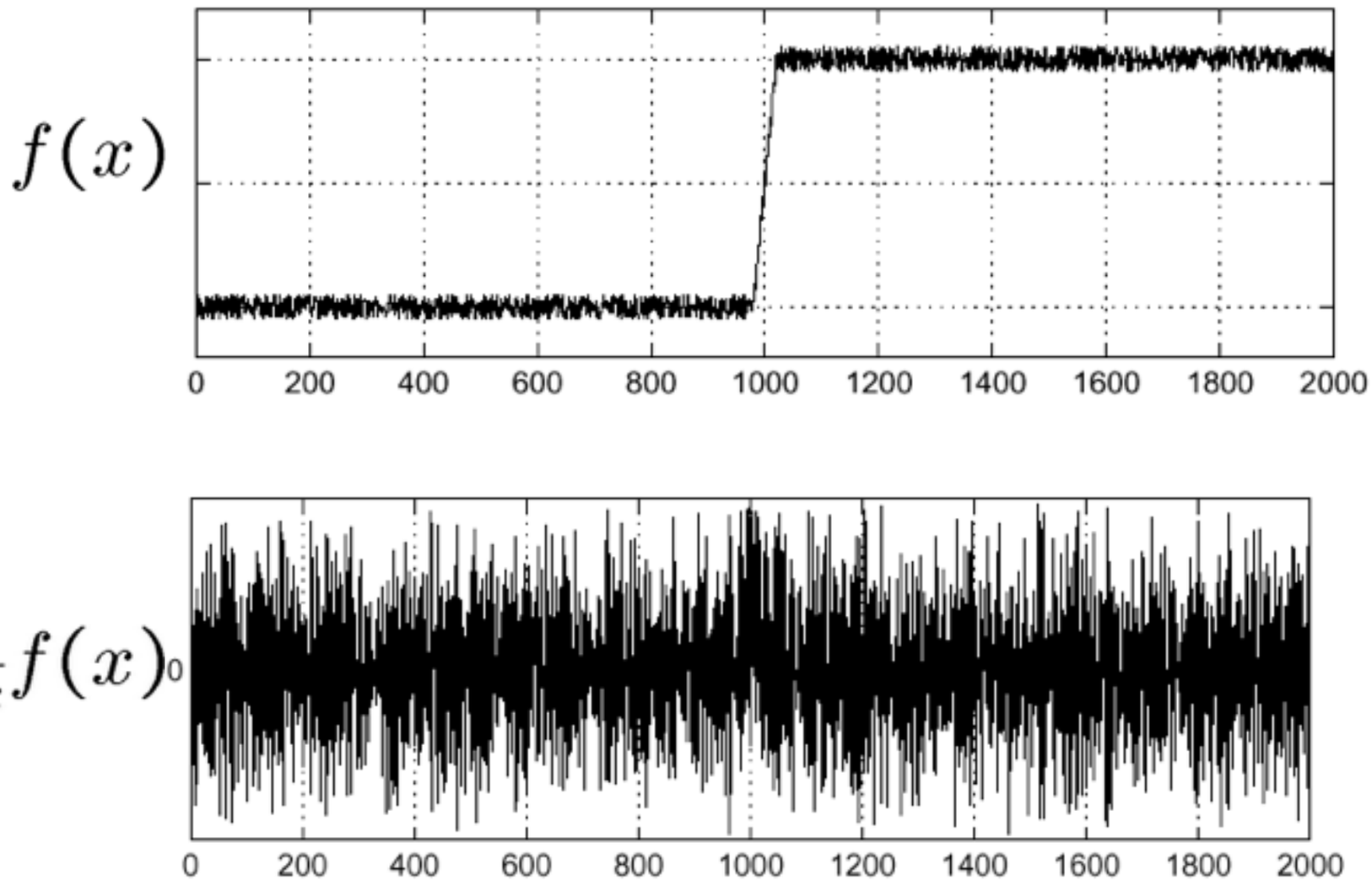
The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

Effects of noise

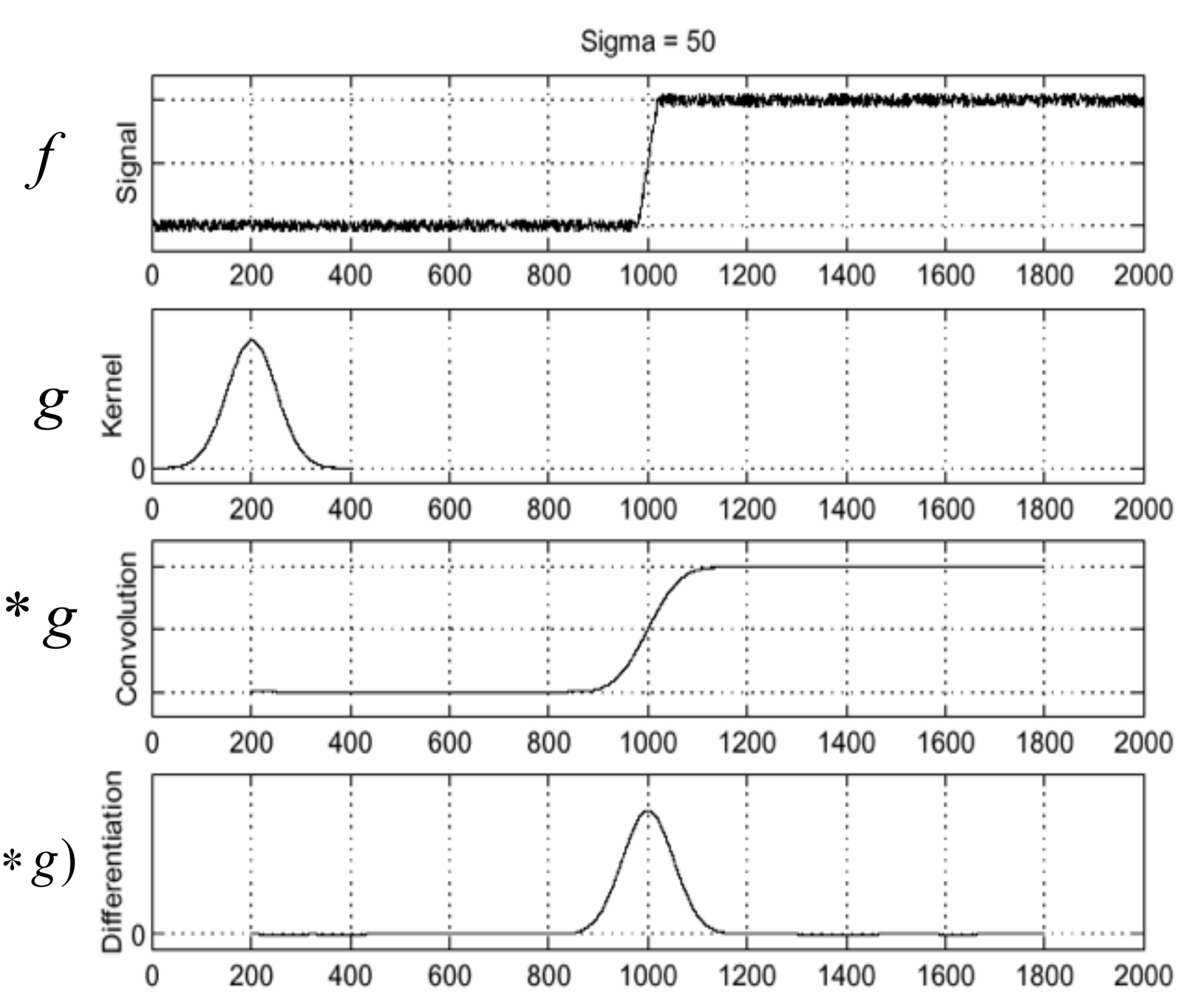
Consider a single row or column of the image



Where is the edge?

Source: S. Seitz

Solution: smooth first



f

g

$f * g$

$\frac{d}{dx}(f * g)$

- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative:

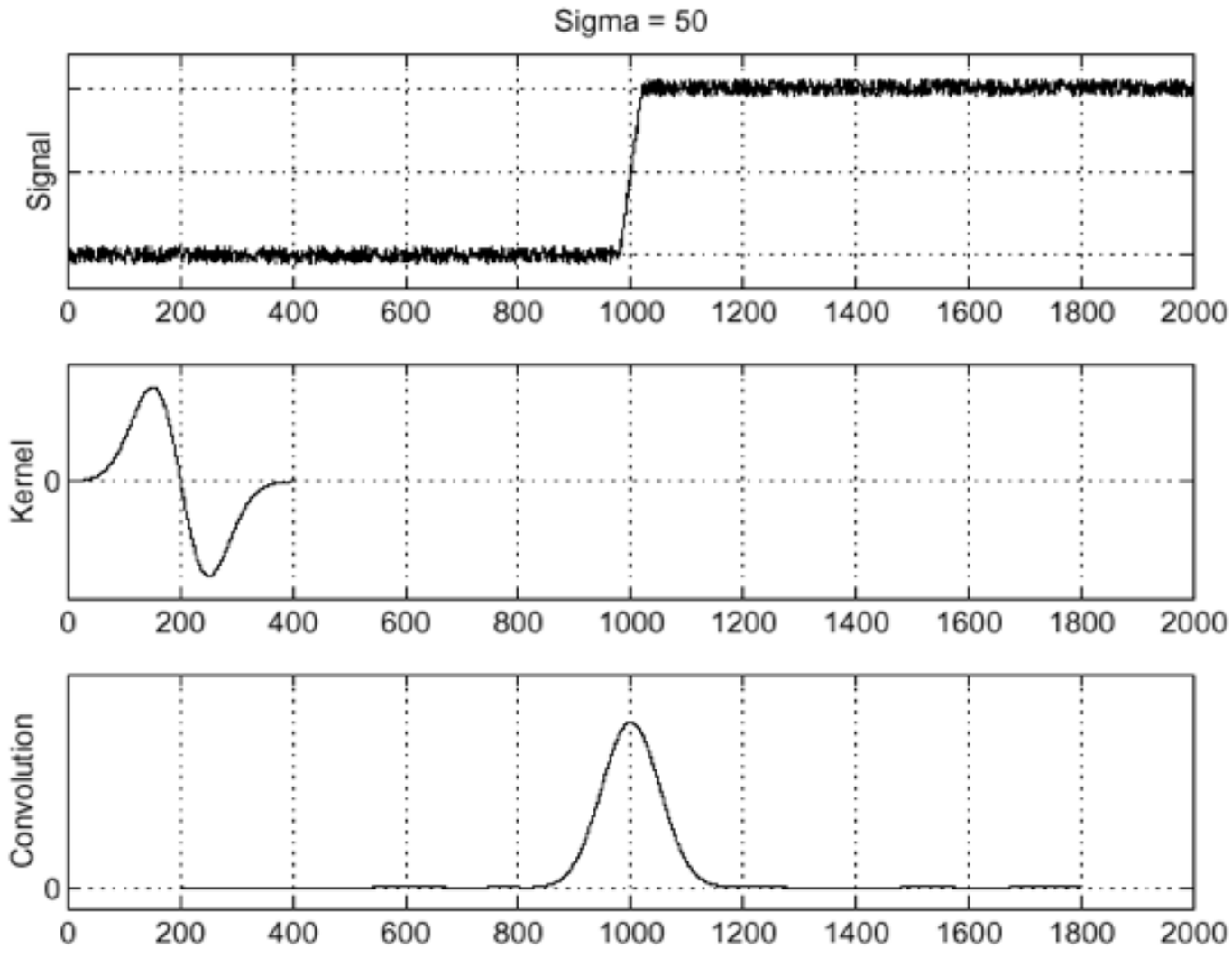
$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

- This saves us one operation:

f

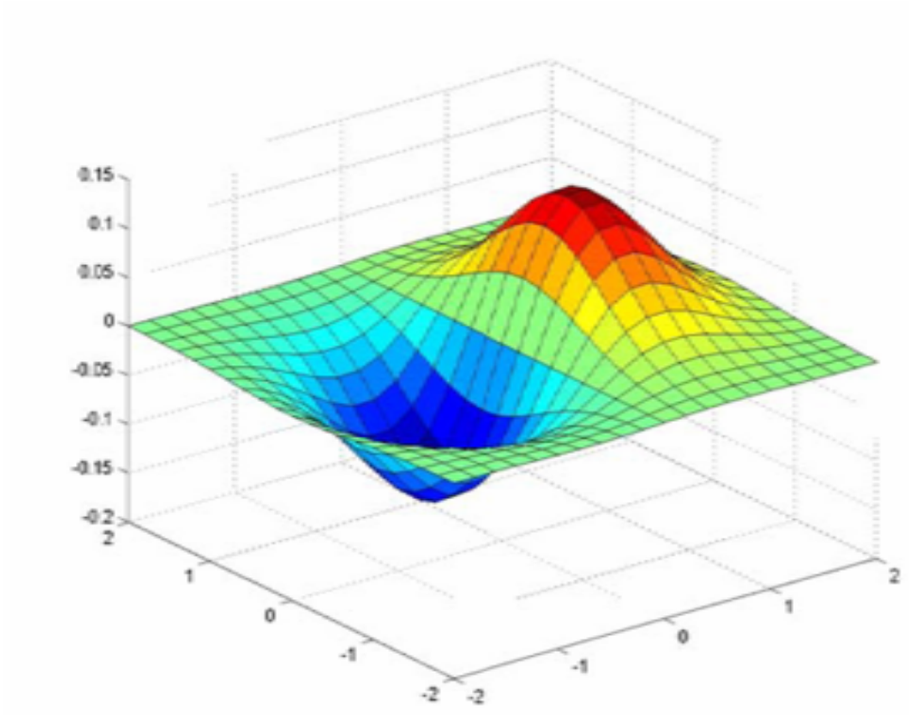
$\frac{d}{dx} g$

$f * \frac{d}{dx} g$

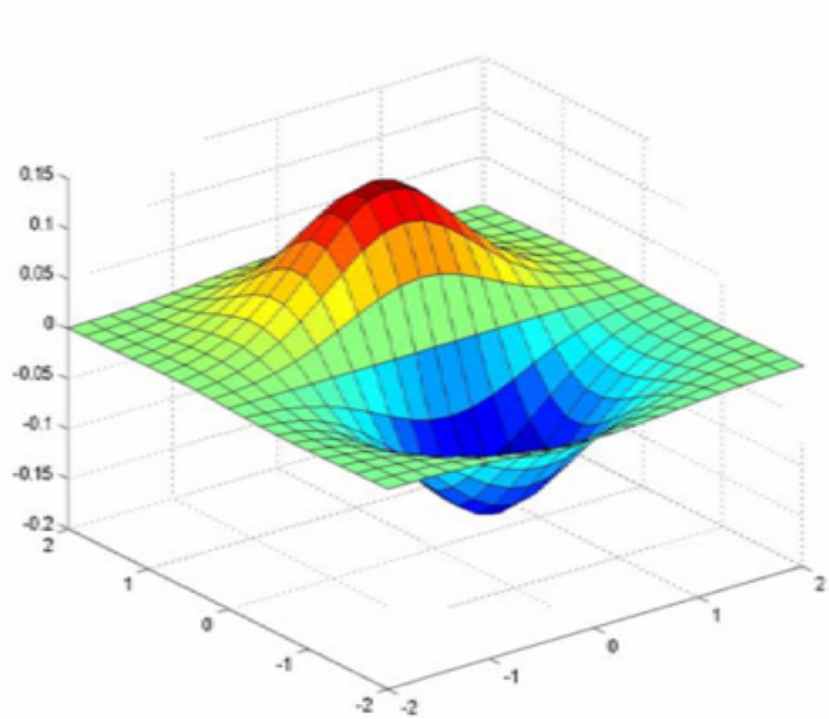


Source: S. Seitz

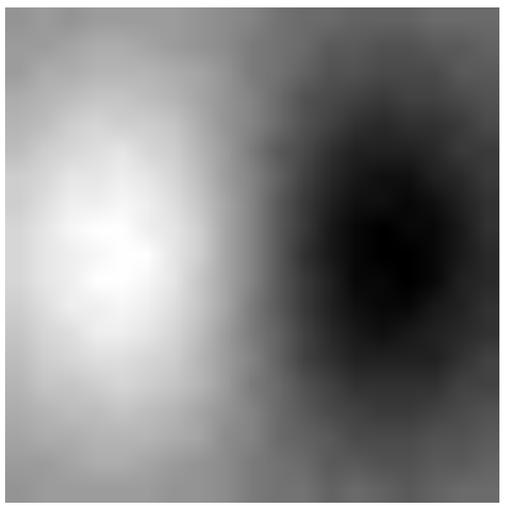
Derivative of Gaussian filters

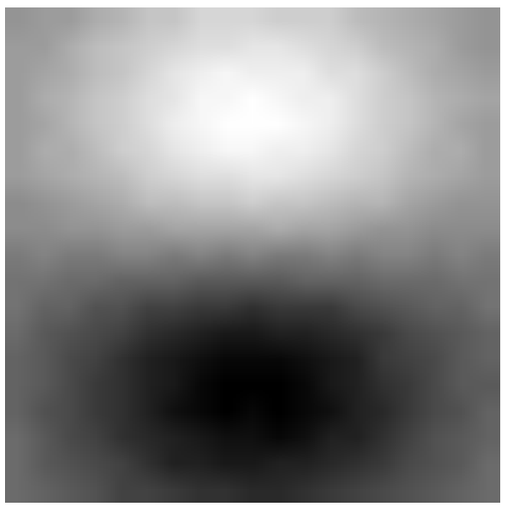


x-direction



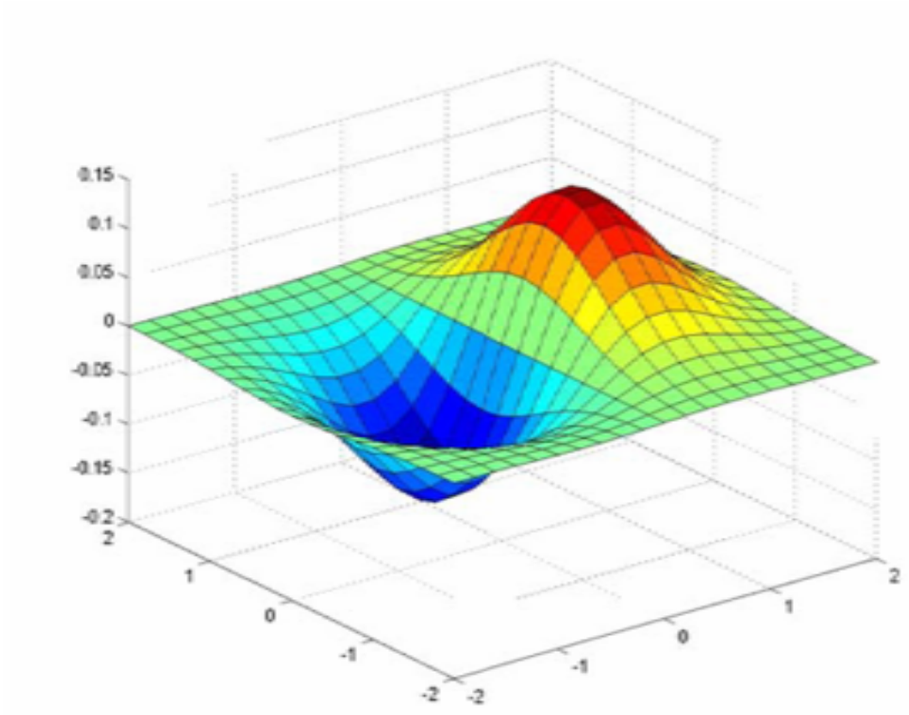
y-direction



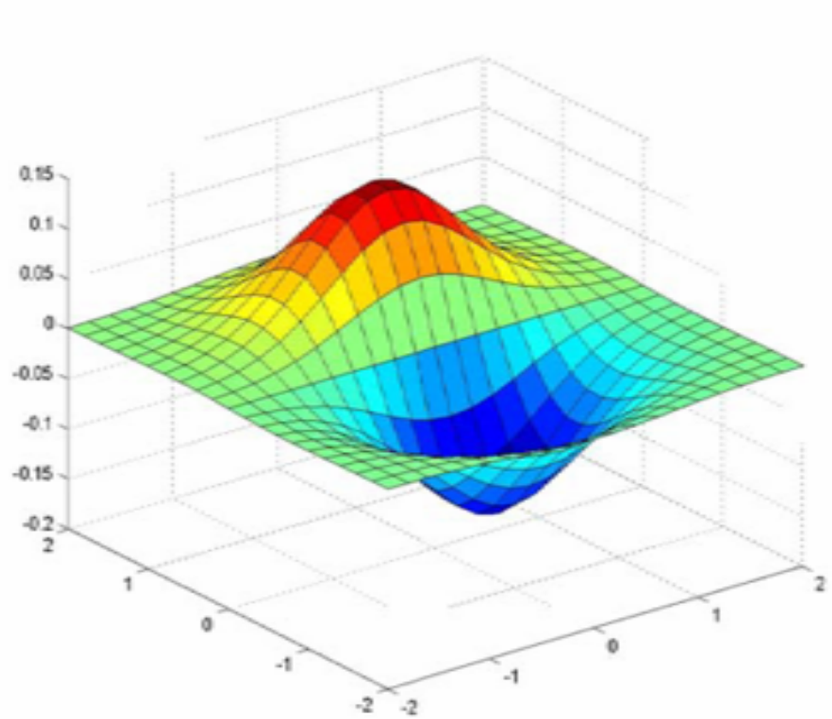


Which one finds horizontal/vertical edges?

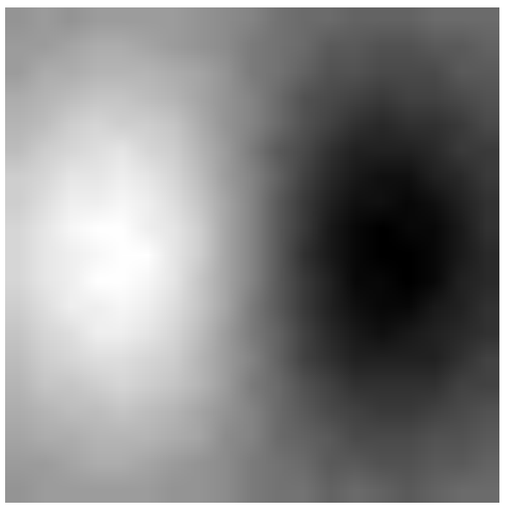
Derivative of Gaussian filters

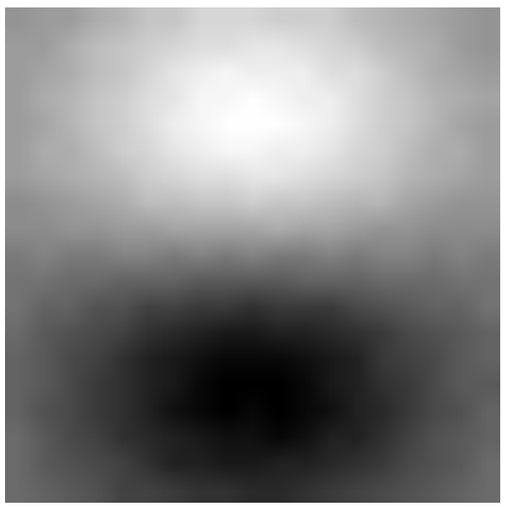


x-direction



y-direction





Are these filters separable?

Recall: Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

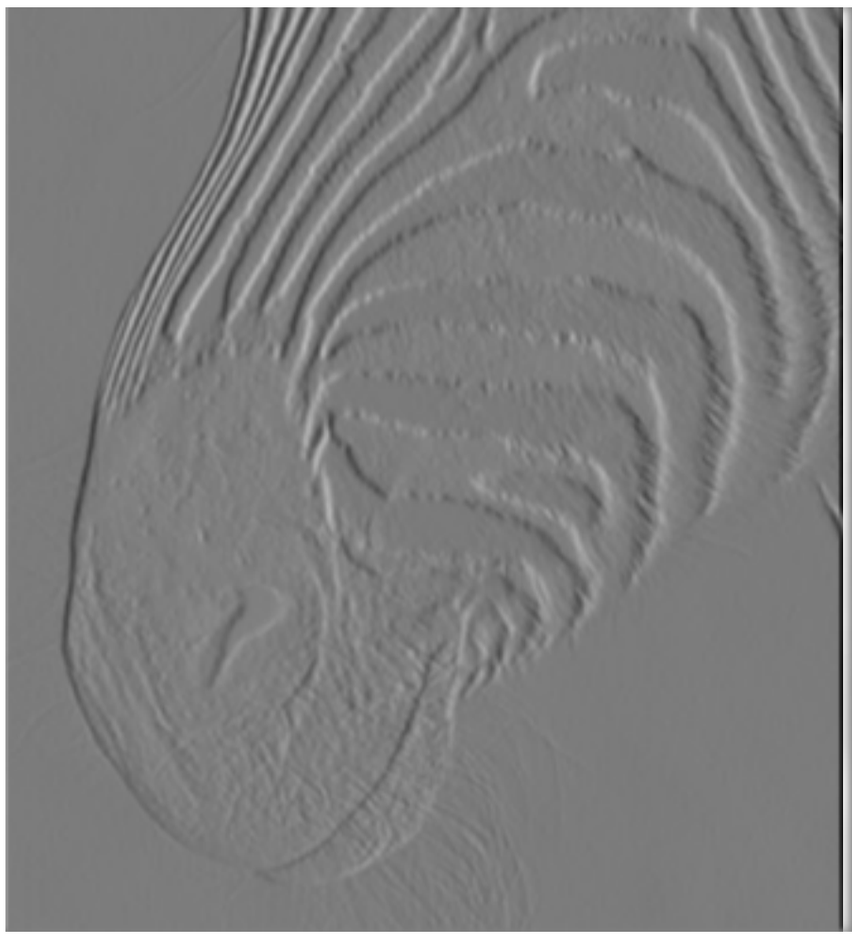
$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

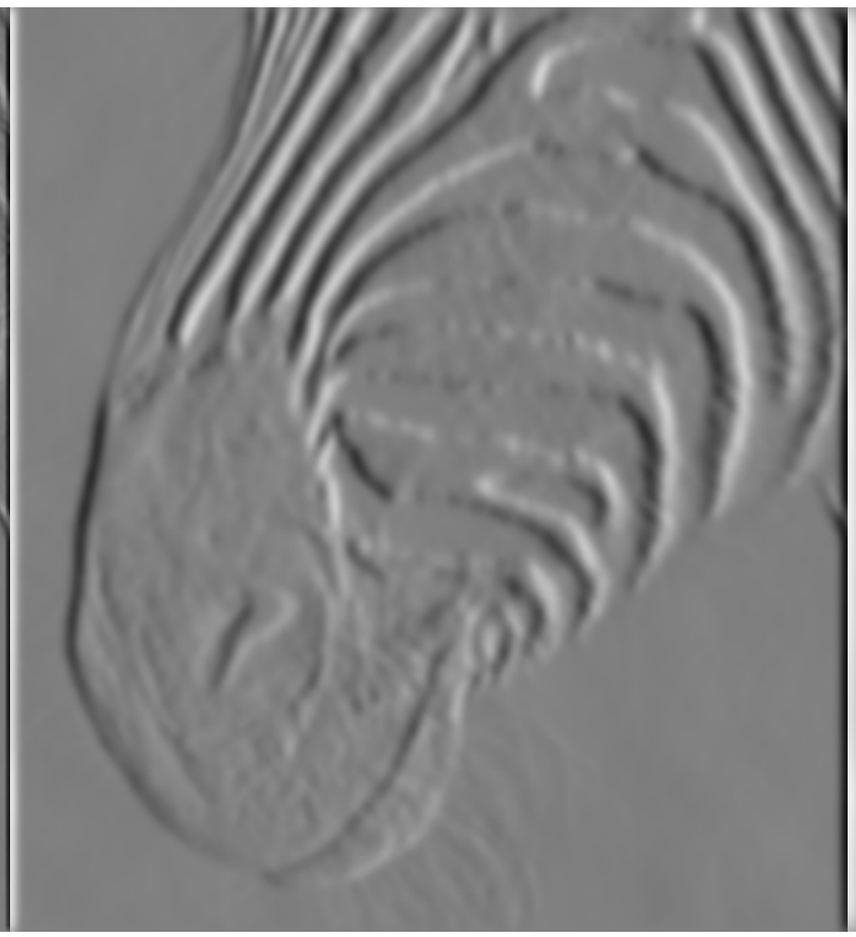
In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

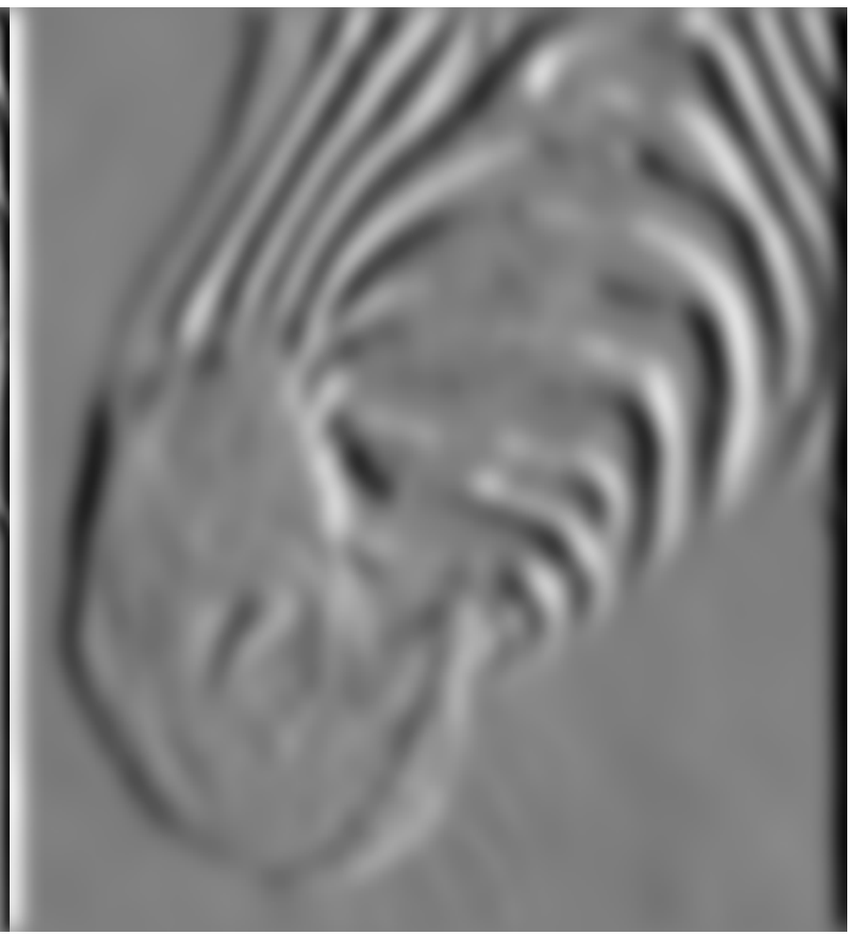
Scale of Gaussian derivative filter



1 pixel



3 pixels



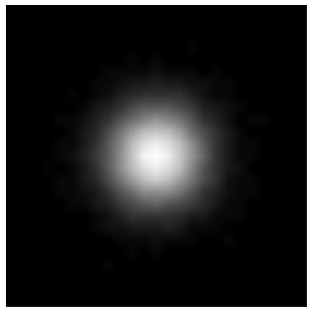
7 pixels

Smoothed derivative removes noise, but blurs edge. Also finds edges at different “scales”

Source: D. Forsyth

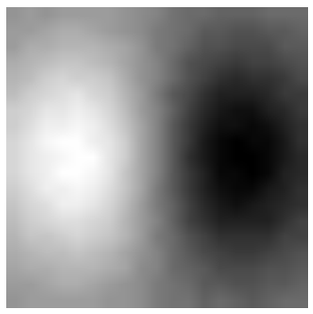
Review: Smoothing vs. derivative filters

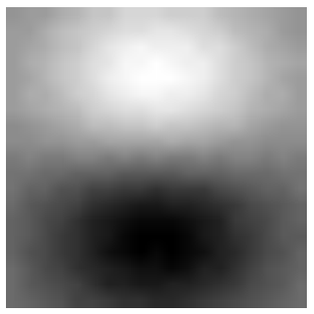
Smoothing filters



- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter

Derivative filters





- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions
- High absolute value at points of high contrast

The Canny edge detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   - Thin wide "ridges" down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

MATLAB: `edge(image, 'canny');`

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

The Canny edge detector

original image

Slide credit: Steve Seitz

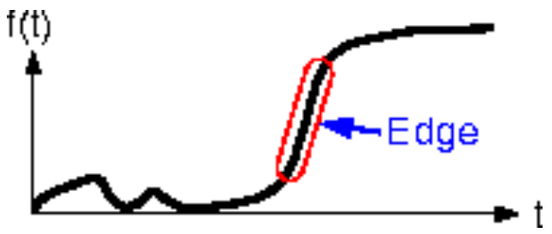
The Canny edge detector

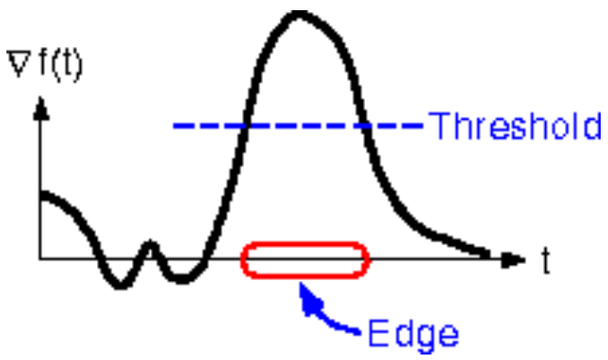
norm of the gradient

The Canny edge detector

thresholding

The Canny edge detector

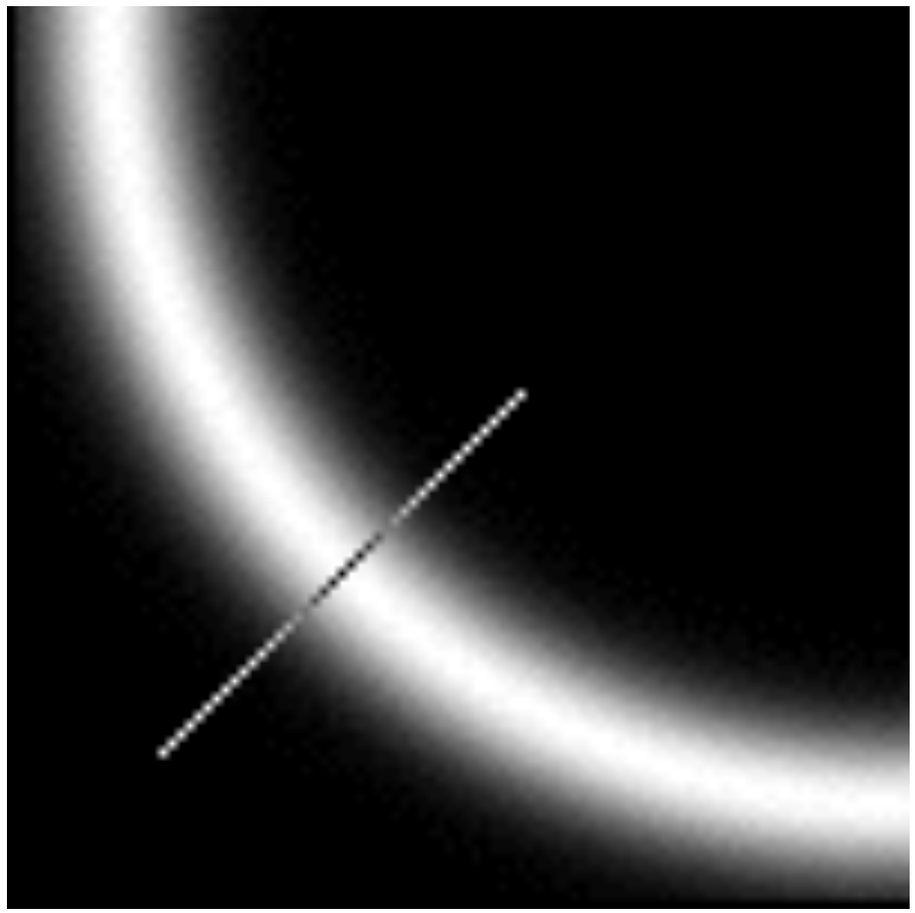


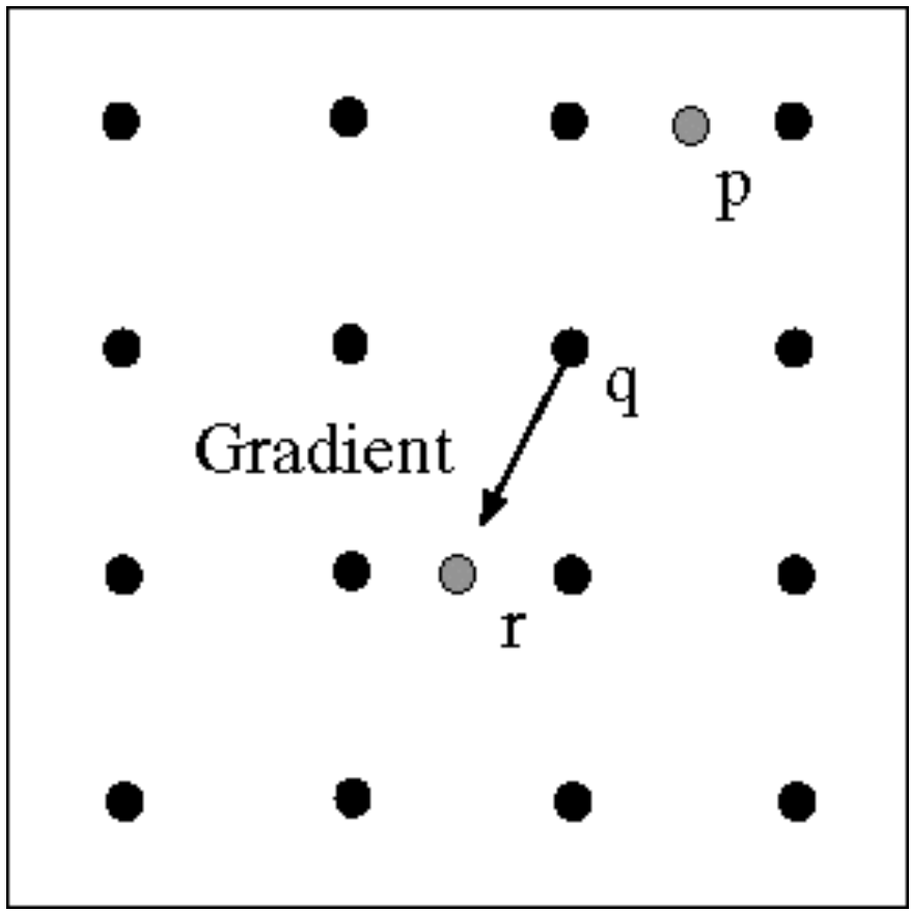


How to turn these thick regions of the gradient into curves?

thresholding

Non-maximum suppression





Check if pixel is local maximum along gradient direction, select single max across width of the edge

- requires checking interpolated pixels p and r

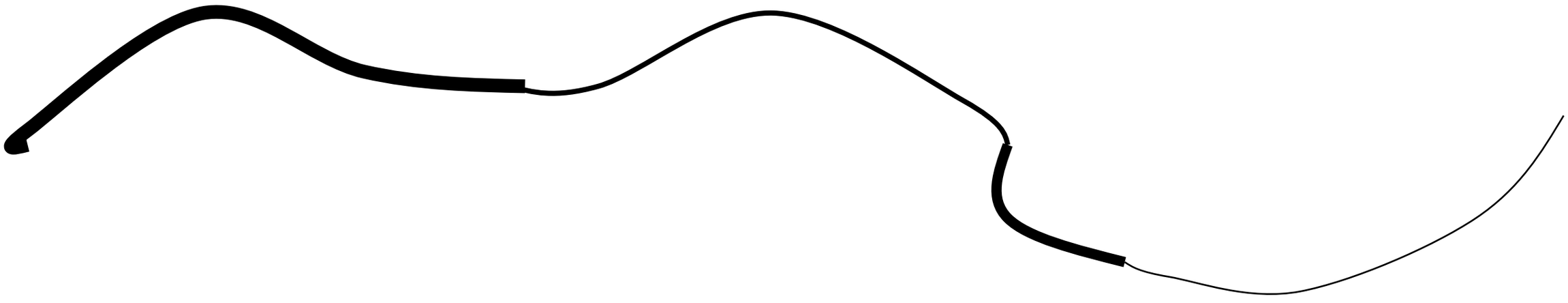
The Canny edge detector

Problem:
pixels along
this edge
didn’t survive
the
thresholding

thinning
(non-maximum suppression)

Hysteresis thresholding

Use a high threshold to start edge curves, and a low threshold to continue them.



Source: Steve Seitz

Hysteresis thresholding

original image

high threshold (strong edges)

low threshold (weak edges)

hysteresis threshold

Source: L. Fei-Fei

Recap: Canny edge detector

1. Compute x and y gradient images
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   - Thin wide "ridges" down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

MATLAB: `edge(image, 'canny');`

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Further thoughts and readings …

- Hybrid images project
  - **http://cvcl.mit.edu/hybridimage.htm**
- Canny edge detector
  - **www.limsi.fr/Individu/vezien/PAPIERS_ACS/canny1986.pdf**
- Bilateral filtering for image denoising (and other application)
  - **http://people.csail.mit.edu/sparis/bf_course/**

- If all else fails **www.xkcd.com**