CMPSCI 670: Computer Vision

Light and shading

University of Massachusetts, Amherst
September 17, 2014

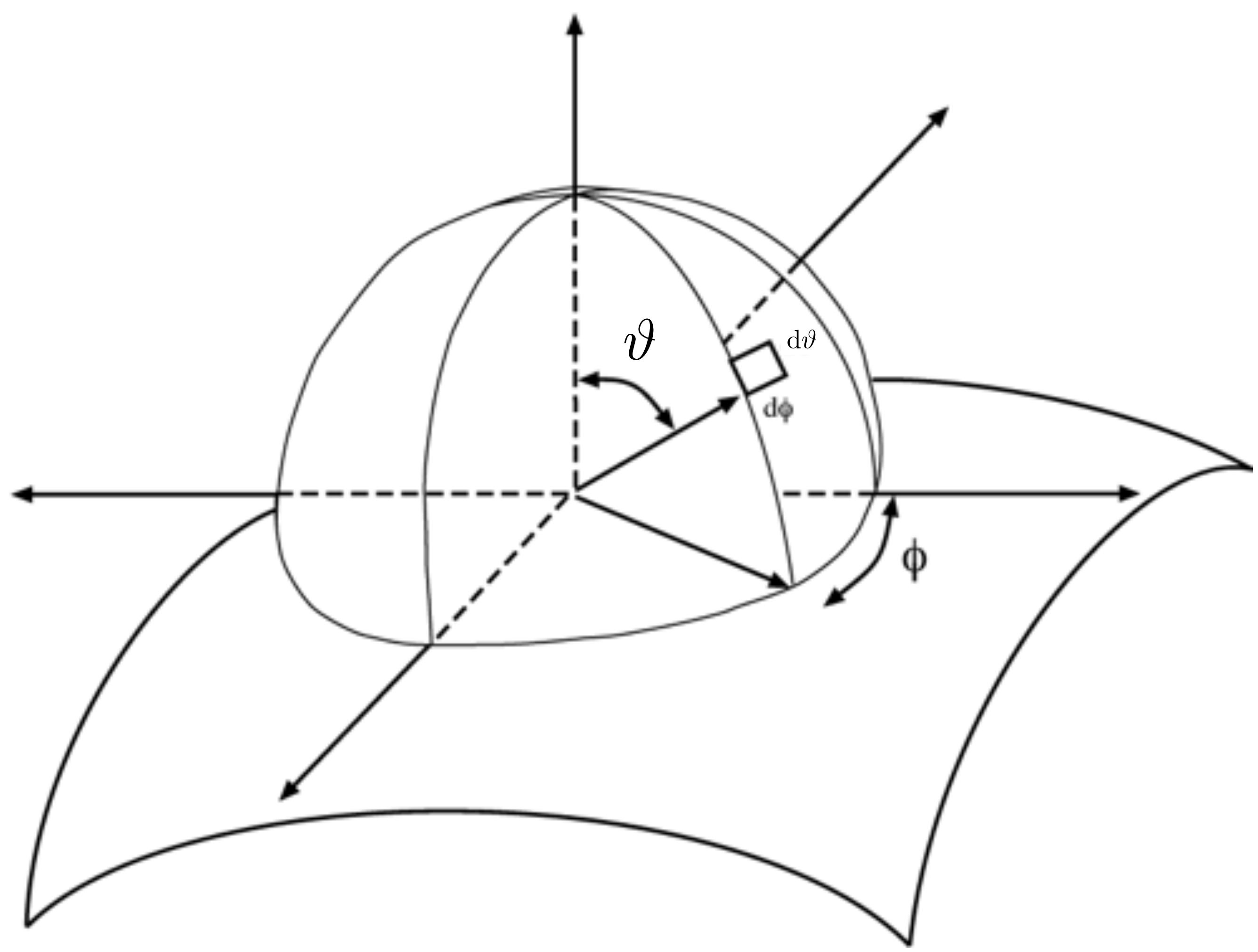
Instructor: Subhransu Maji

Administrivia

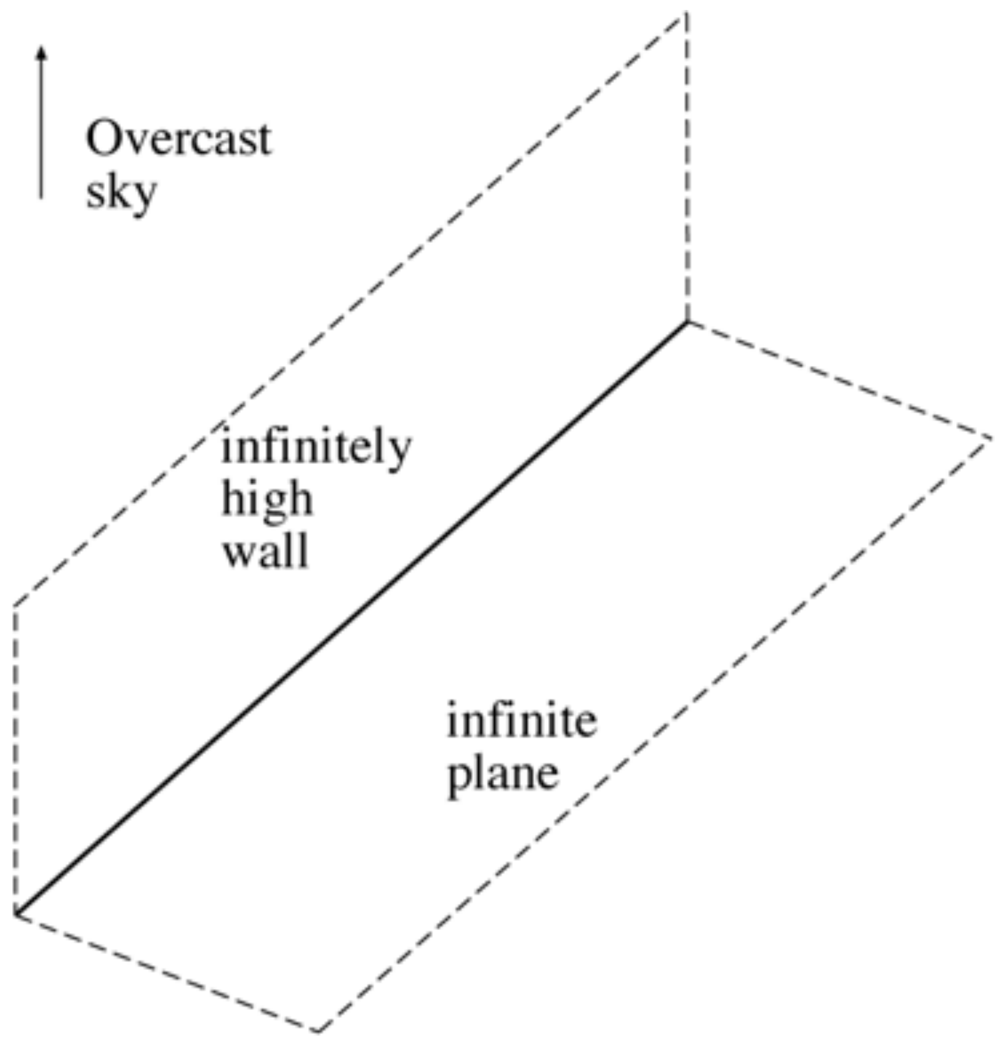
- **Homework #1** is due on Monday (Sept. 22) before class
- Submission via *edlab* accounts
  - Create a **hw1.zip** file on the top level directory
    - **/courses/cs600/cs670/<username>/hw1.zip**
    - where **hw1.zip** looks like this:
      - **alignChannels.m**
      - **demosaicImage.m**
      - **report.pdf**
- Also include additional code (e.g. for extra credit) and explain it in the report what each file does
- If all else fails email it to me before class **smaji@cs.umass.edu**

Radiometry

- Questions:
  - how “bright” will surfaces be?
  - what is “brightness”?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface around any point is a hemisphere of directions
- Simplest problems can be dealt with by reasoning about this hemisphere

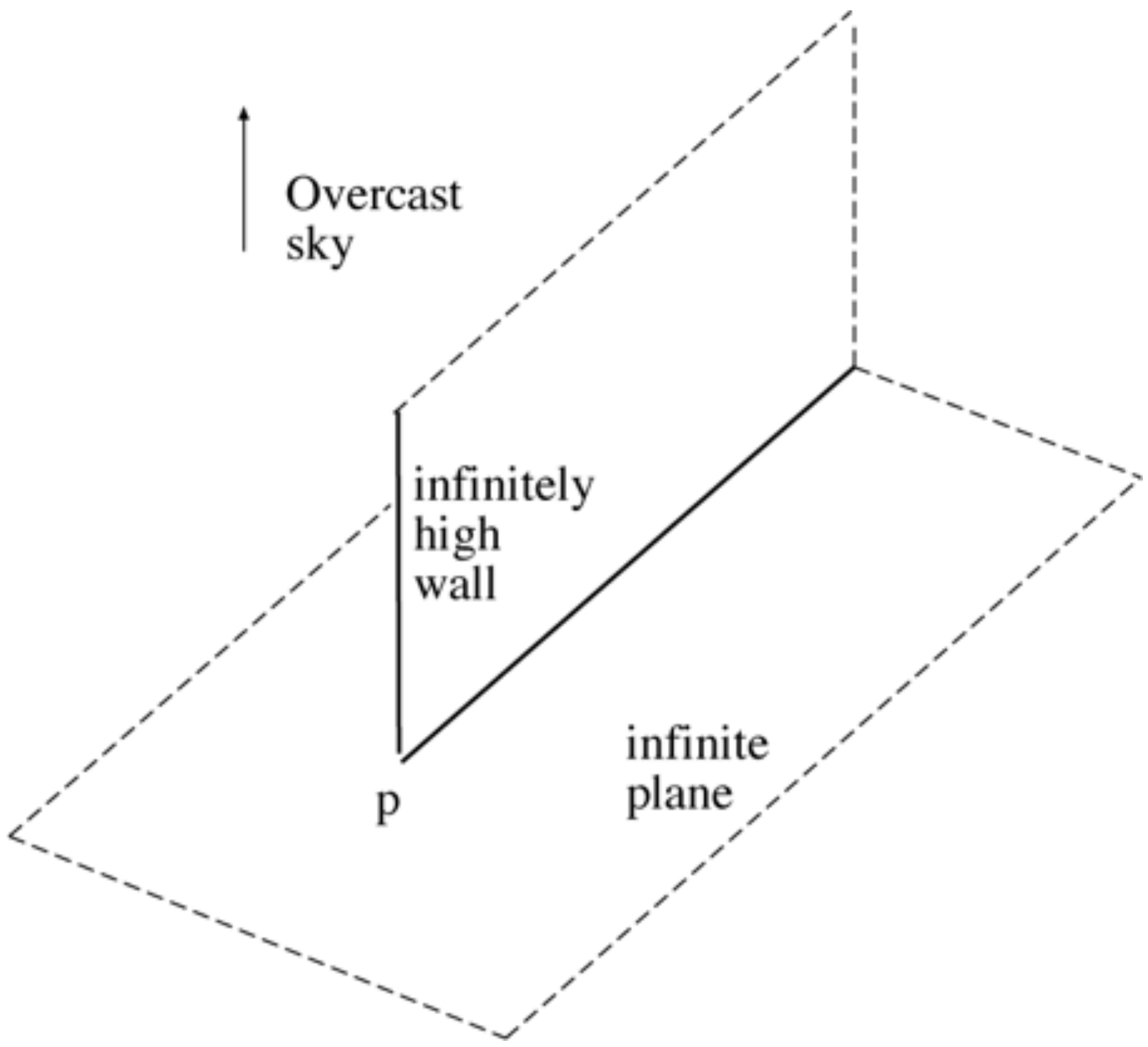


Lambert’s wall

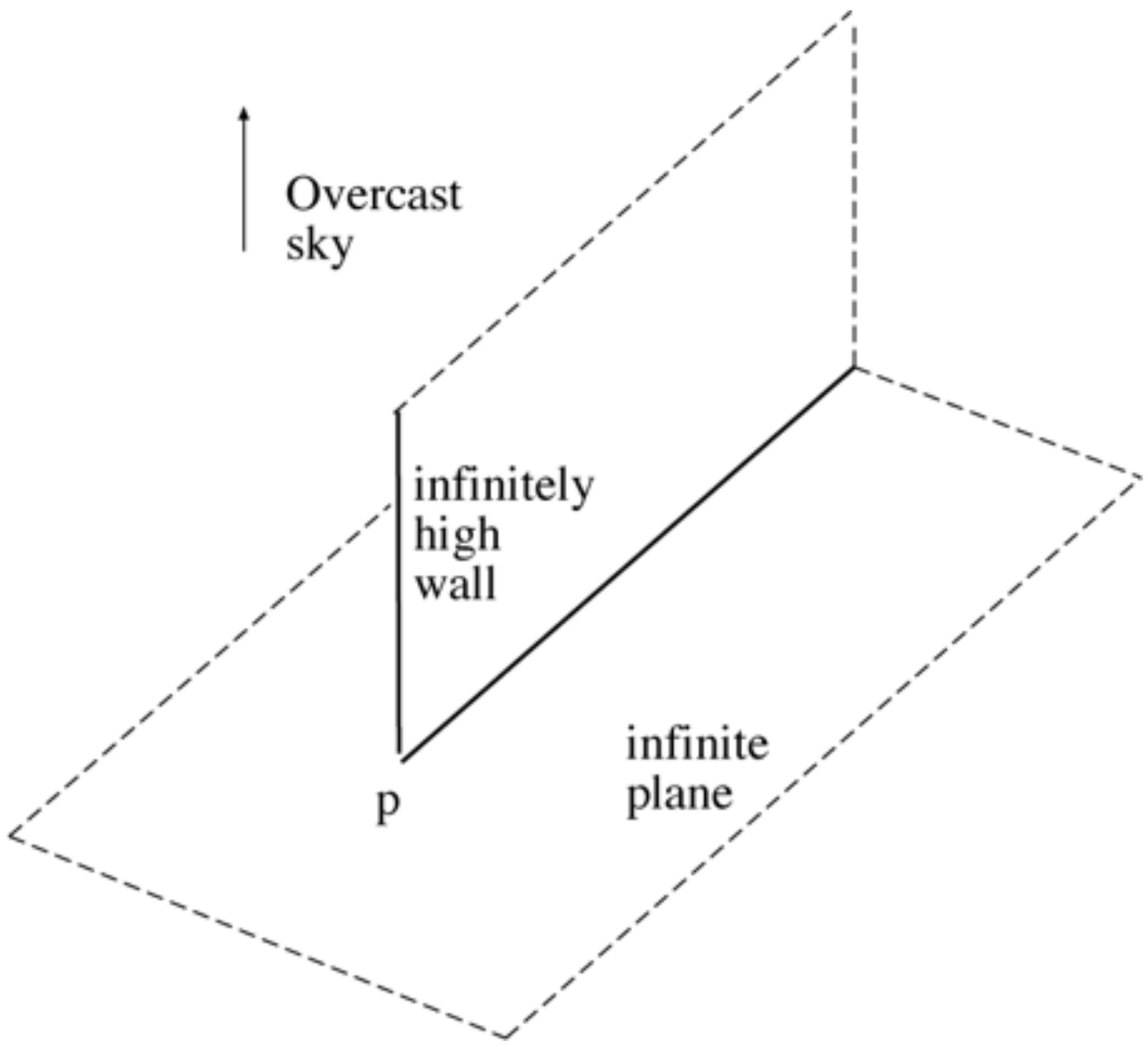


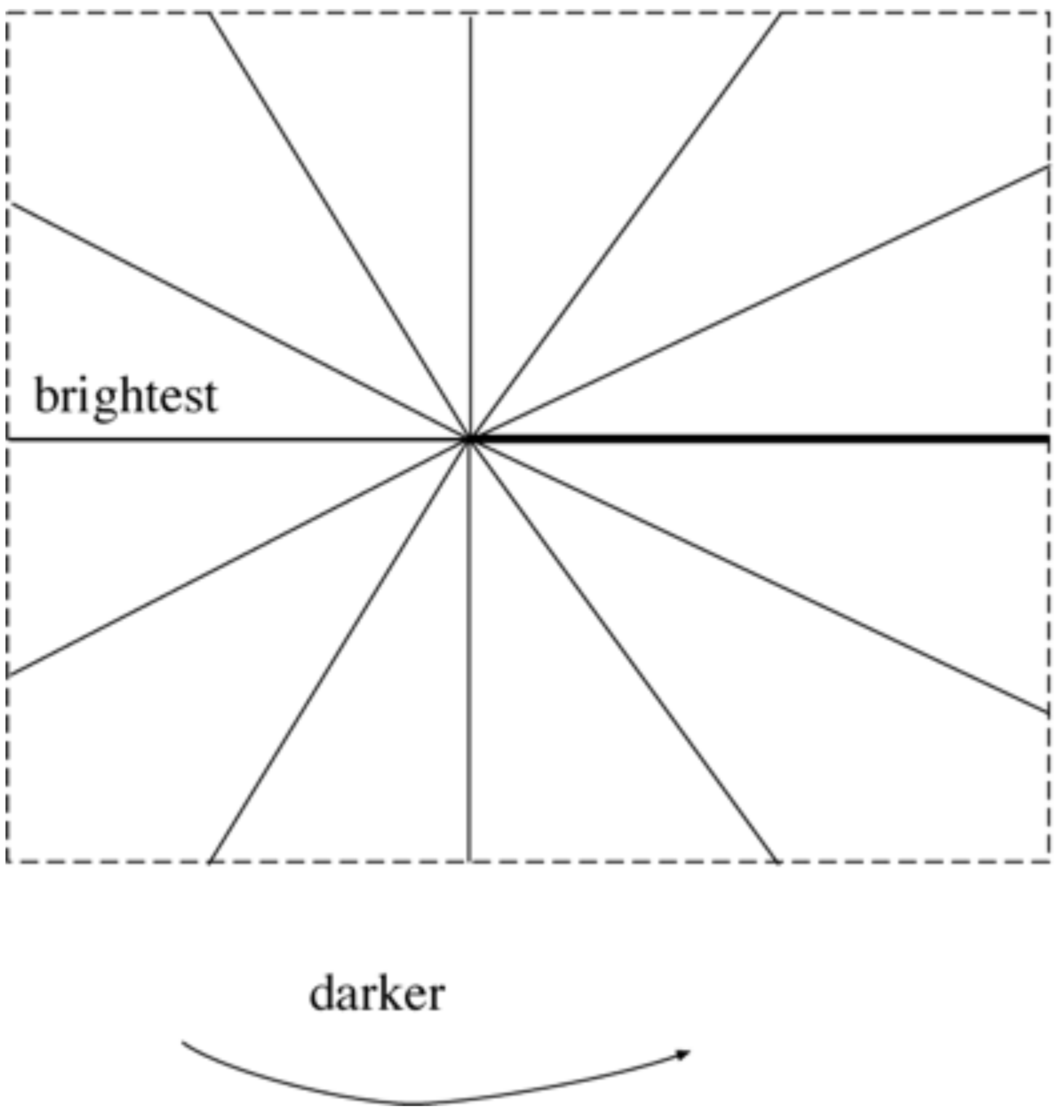
What is the distribution
of brightness on the ground?

More complex wall



More complex wall





Foreshortening

- **Principle:** two sources that look the same to a receiver must have the same effect on the receiver.
- **Principle:** two receivers that look the same to a source must receive the same amount of energy.

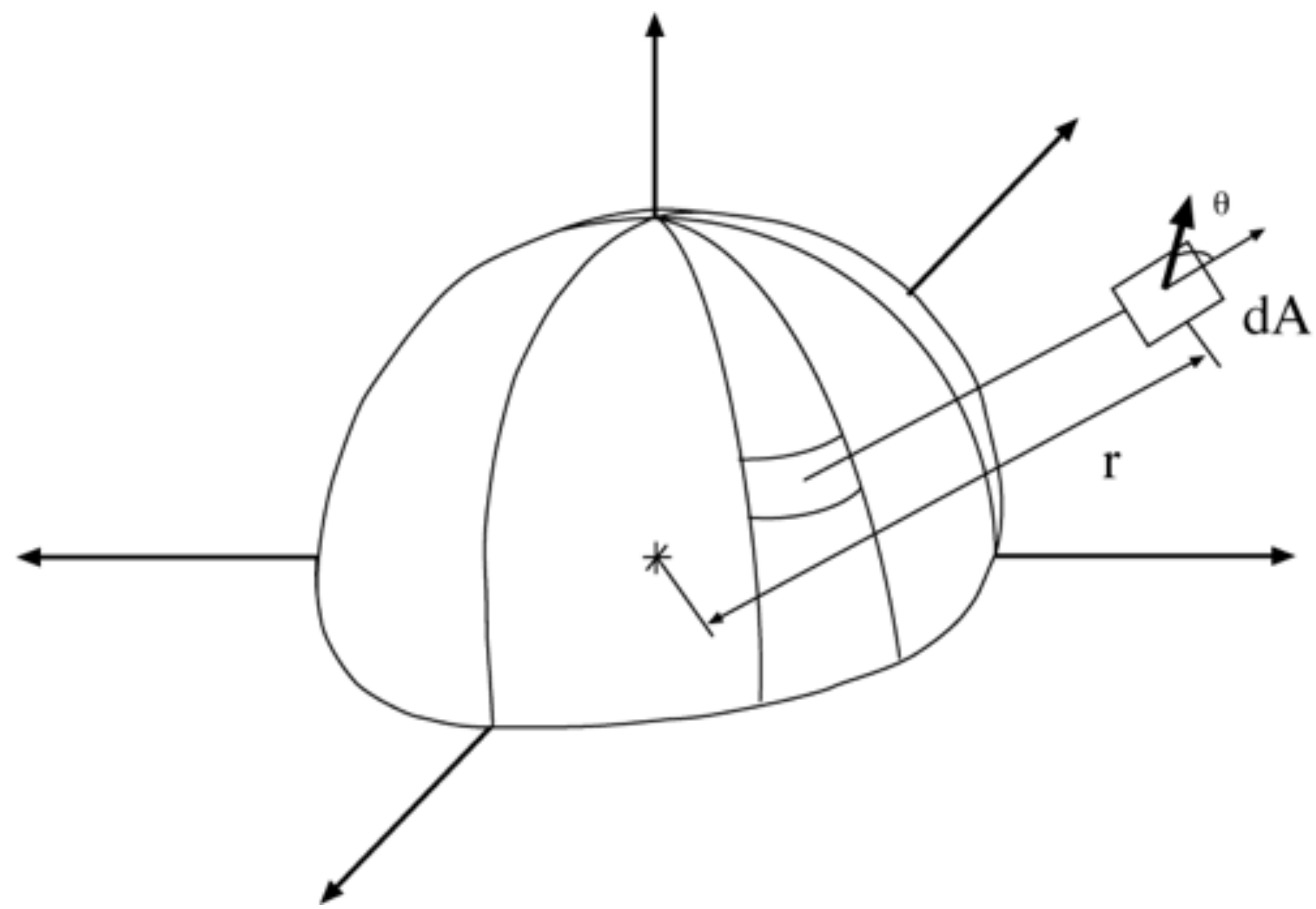
- "look the same" means produce the same input hemisphere (or output hemisphere)

- **Reason:** what else can a receiver know about a source but what appears on its input hemisphere? (ditto, swapping receiver and source)
- **Crucial consequence:** a big source (resp. receiver), viewed at a glancing angle, must produce (resp. experience) the same effect as a small source (resp. receiver) viewed frontally.

Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA\cos\theta}{r^2}$$



- Another useful expression:

$$d\omega = \sin\vartheta\left(d\vartheta\right)\left(d\phi\right)$$

Measuring Light in Free Space

- **Desirable property:** in a vacuum, the relevant unit does not go down along a straight line.
- How do we get a unit with this property? Think about the power transferred from an infinitesimal source to an infinitesimal receiver.

- We have

  total power leaving s to r =

  total power arriving at r from s

- Also:

  Power arriving at r is proportional to:

  - solid angle subtended by s at r (because if s looked bigger from r, there'd be more)
  - foreshortened area of r (because a bigger r will collect more power

Radiance

- All this suggests that the light transferred from source to receiver should be measured as:

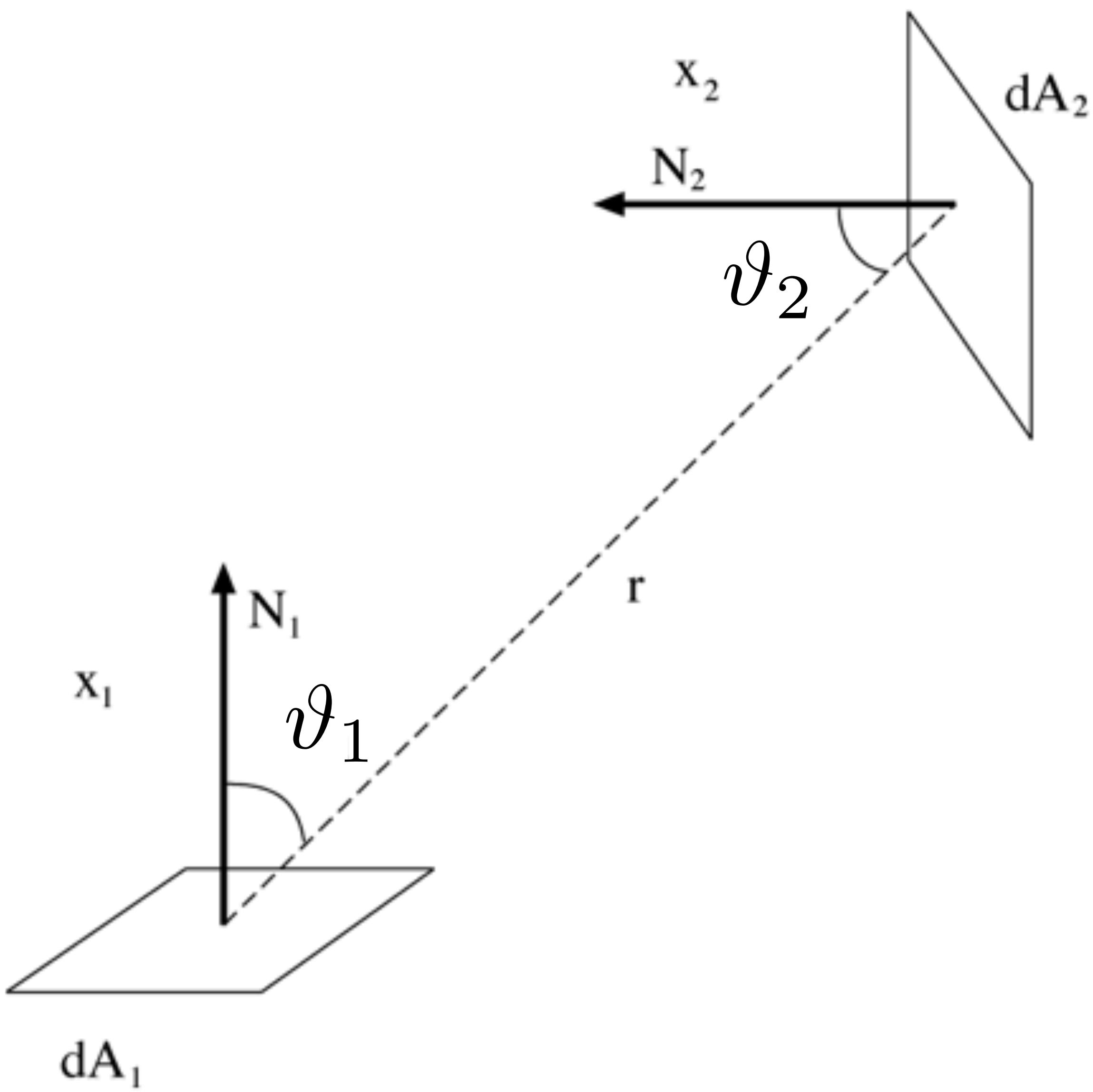
  Radiant power per unit foreshortened area per unit solid angle

- This is **radiance**
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)
- Usually written as:

$$L(\underline{x},\vartheta,\varphi)$$

- **Crucial property:** In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p
  - which was how we got to the unit

Radiance is constant along straight lines



- Power 1->2, leaving 1:

$$L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

$$L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)$$

- But these must be the same, so that the two radiances are equal

Irradiance

- How much light is arriving at a surface?
- Sensible unit is *Irradiance*
- Incident power per unit area *not foreshortened*
- This is a function of incoming angle.
- A surface experiencing radiance $L(x,\theta,\phi)$ coming in from $d\omega$ experiences irradiance

$$L(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega$$

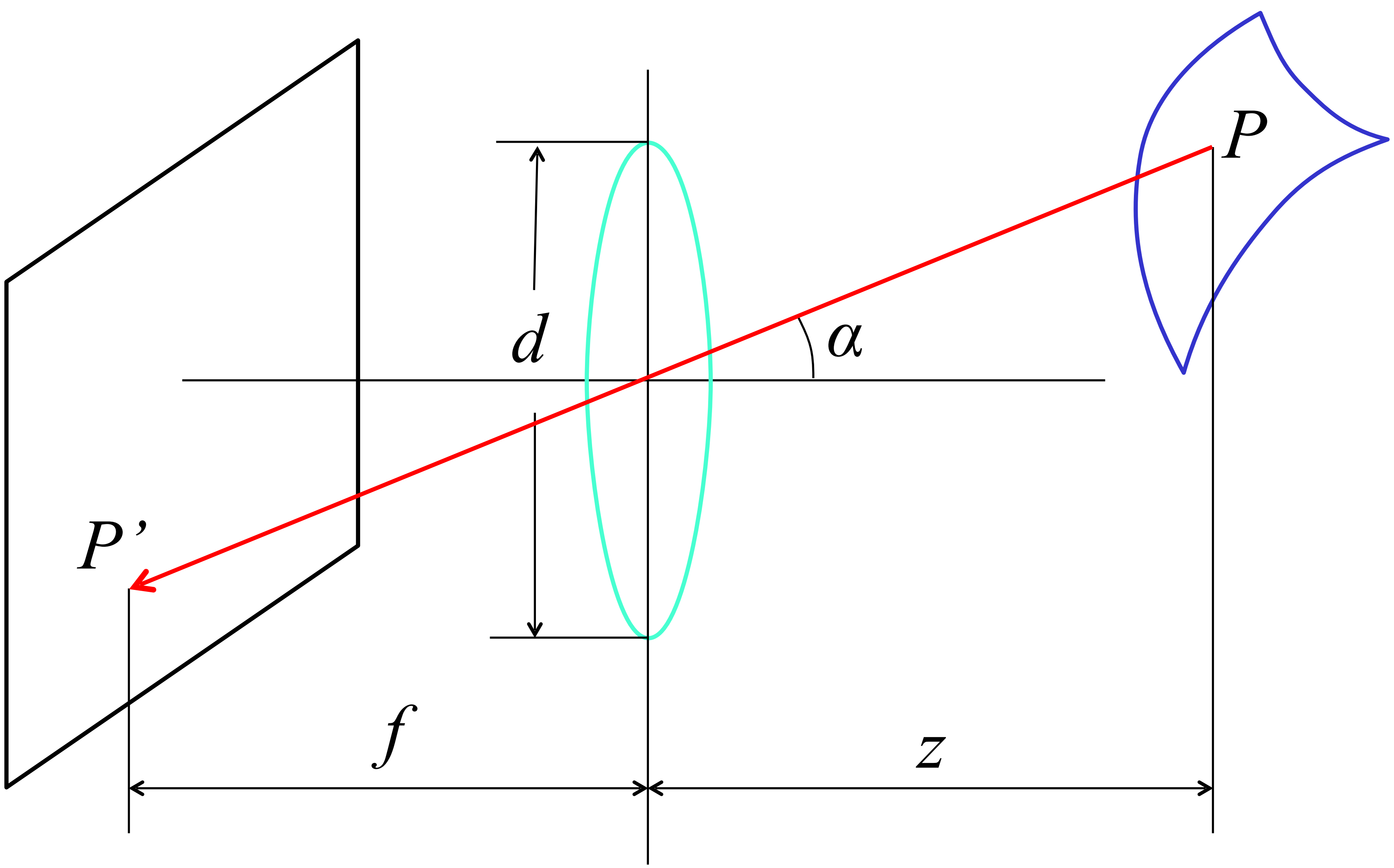
- **Crucial property:**
  Total power arriving at the surface is given by adding irradiance over all incoming angles — this is why it's a natural unit
- Total power is :

$$\int_{\Omega} L(\underline{x},\vartheta,\varphi)\cos\vartheta \sin\vartheta d\vartheta d\varphi$$

Fundamental radiometric relation

L: Radiance emitted from *P* toward *P'*

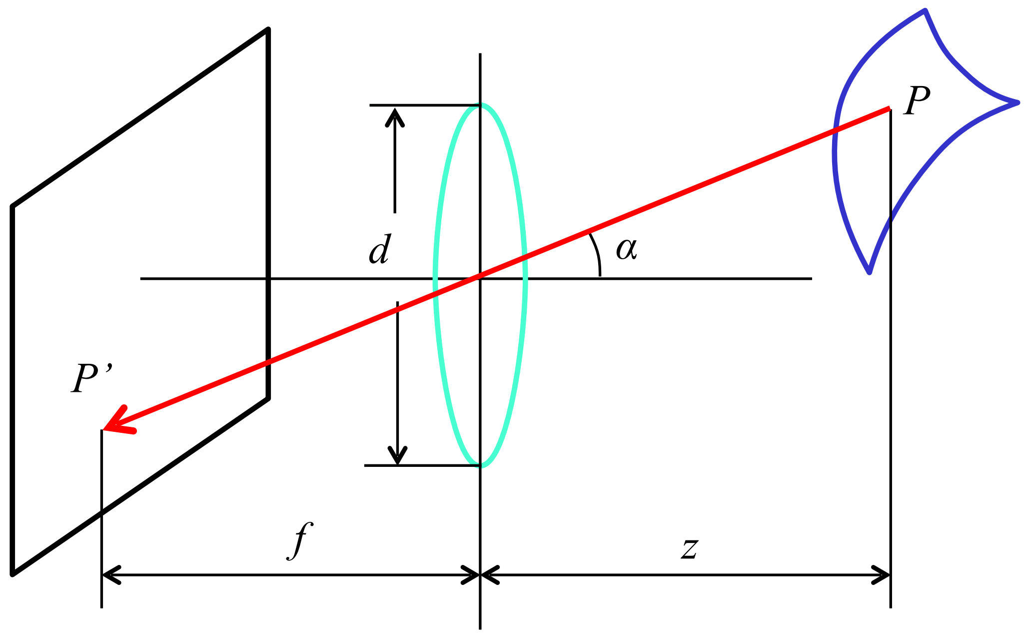
E: Irradiance falling on *P'* from the lens



What is the relationship between *E* and *L*?

Szeliski 2.2.3

Fundamental radiometric relation



$$E = \left[\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha\right] L$$

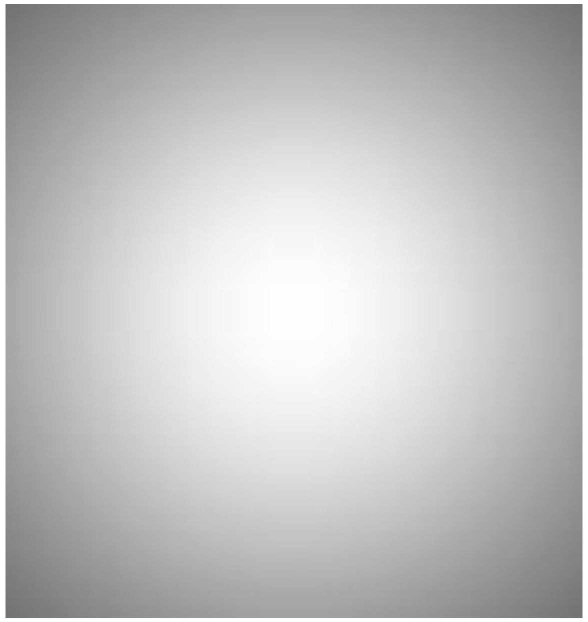
(exercise - derive this)

- Image irradiance is linearly related to scene radiance
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane
- The irradiance falls off as the angle between the viewing ray and the optical axis increases

Fundamental radiometric relation

$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

- Application:
  - S. B. Kang and R. Weiss, ***Can we calibrate a camera using an image of a flat, textureless Lambertian surface?*** ECCV 2000.



Light at surfaces

What happens when a light ray hits a point on an object?

- Some of the light gets **absorbed**
  - converted to other forms of energy (e.g., heat)
- Some gets **transmitted** through the object
  - possibly bent, through refraction
  - or scattered inside the object (subsurface scattering)
- Some gets **reflected**
  - possibly in multiple directions at once
- Really complicated things can happen
  - fluorescence

Source: Steve Seitz

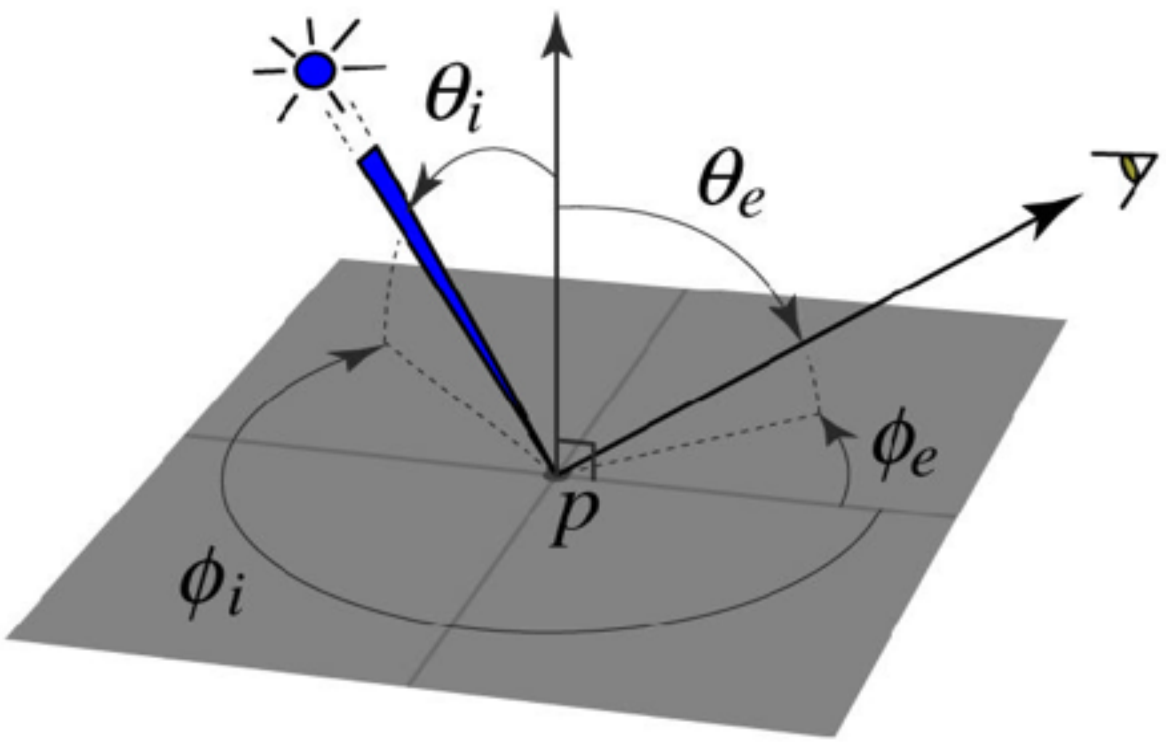
Fluorescence

Modeling surface reflectance

Bidirectional reflectance distribution function (BRDF)

- How bright a surface appears when viewed from one direction when light falls on it from another
- **Definition**: ratio of the radiance in the emitted direction to irradiance in the incident direction

$$\rho_{bd}(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i) = \frac{L_o(\underline{x},\vartheta_o,\varphi_o)}{L_i(\underline{x},\vartheta_i,\varphi_i)\cos\vartheta_i d\omega}$$



Simplifying assumptions
locality, no fluorescence, does not generate light

Gonioreflectometer

The University of Virginia spherical gantry, an example of a modern image-based gonioreflectometer

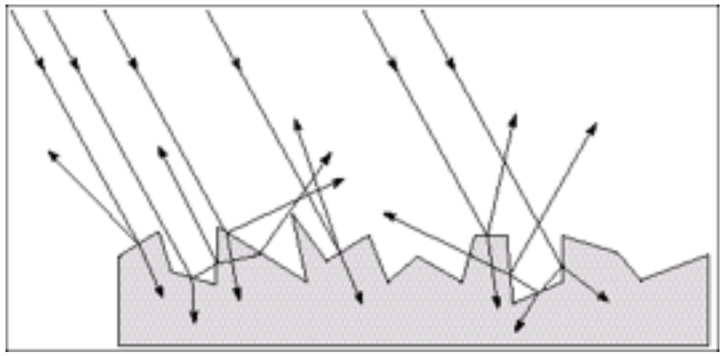
BRDFs can be incredibly complicated…

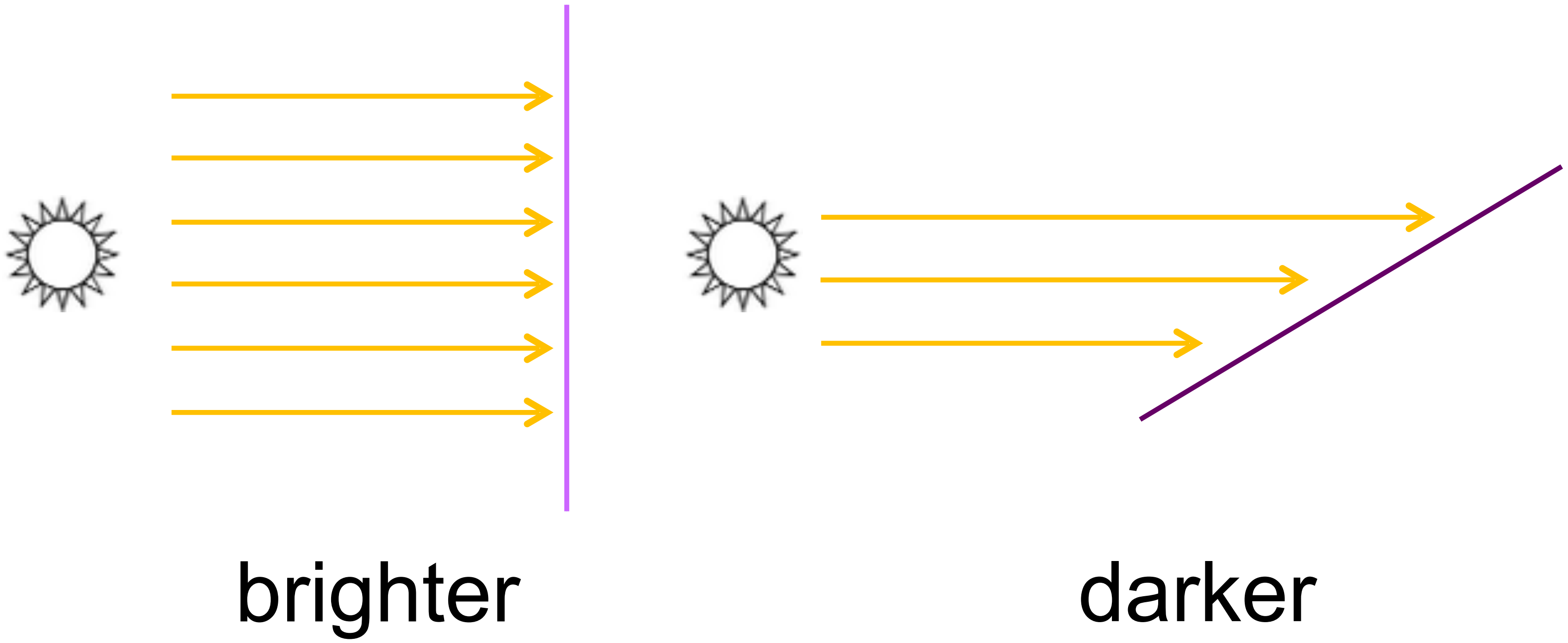
Suppressing the angles in the BRDF

- BRDF is a very general notion
  - some surfaces need it (underside of a CD; tiger eye; etc)
  - very hard to measure
    - illuminate from one direction, view from another, repeat
  - very unstable
    - minor surface damage can change the BRDF
    - e.g. ridges of oil left by contact with the skin can act as lenses
- for many surfaces, light leaving the surface is largely independent of exit angle
  - surface roughness is one source of this property

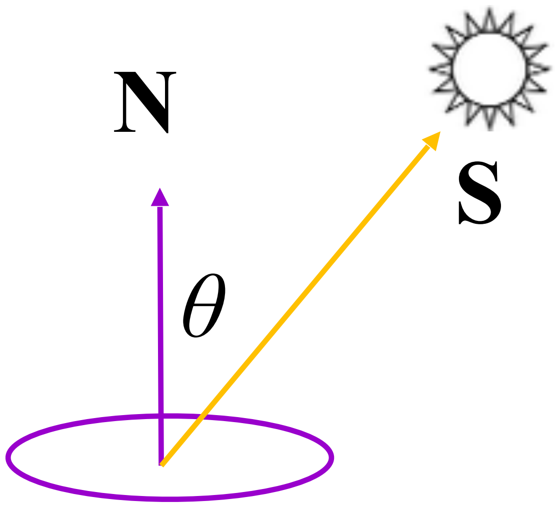
Special cases: Diffuse reflection

- Light is reflected equally in all directions
  - Dull, matte surfaces like chalk or cotton cloth
  - Microfacets scatter incoming light randomly
  - Effect is that light is reflected (approximately) equally in all directions
- Brightness of the surface depends on the incidence of illumination

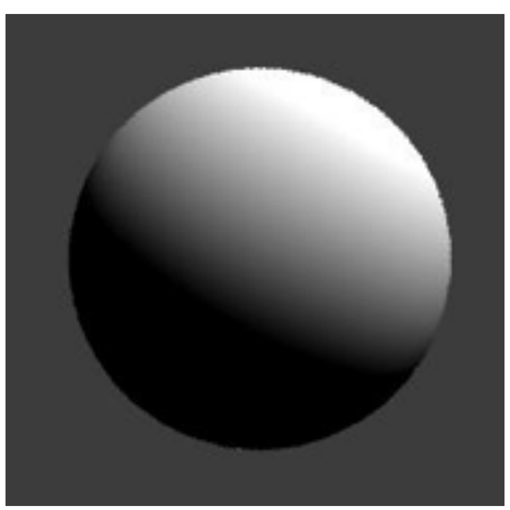




Diffuse reflection: Lambert's law



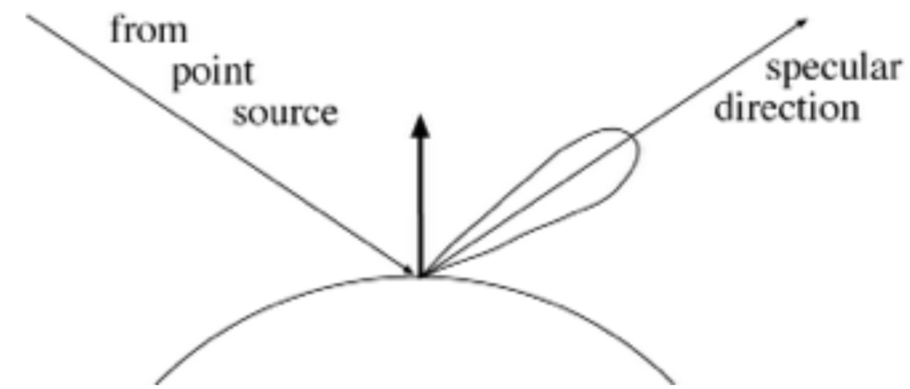
$$B = \rho(\mathbf{N} \cdot \mathbf{S})$$
$$= \rho \left\| \mathbf{S} \right\| \cos\theta$$

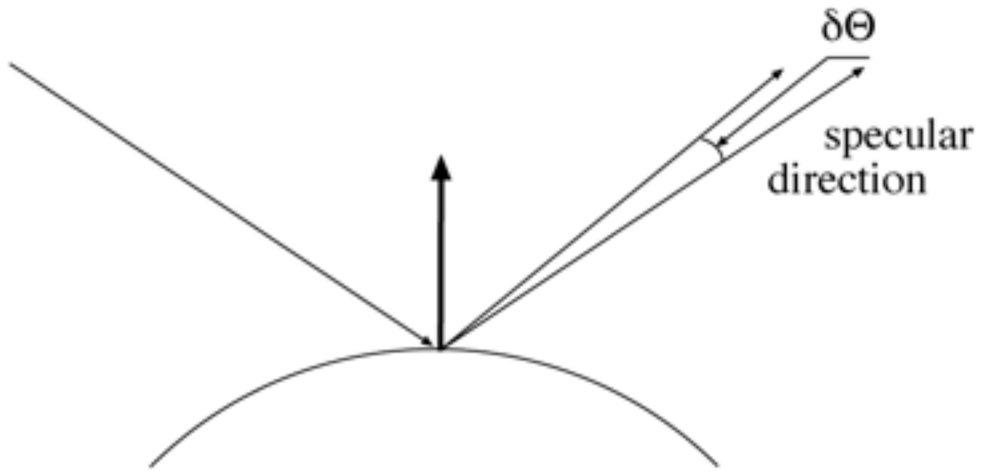


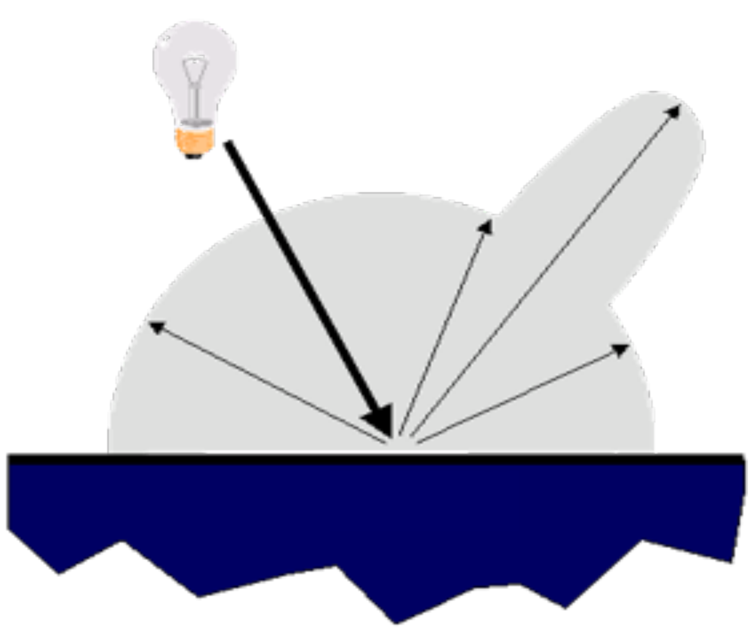
B: radiosity (total power leaving the surface per unit area)
ρ: albedo (fraction of incident irradiance reflected by the surface)
N: unit normal
S: source vector (magnitude proportional to intensity of the source)

Specular reflection

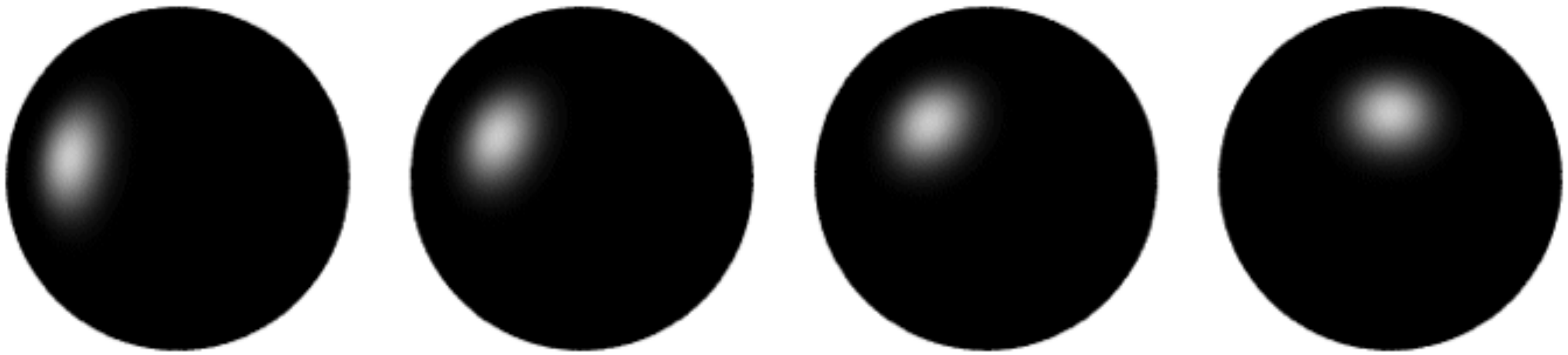
- Radiation arriving along a source direction leaves along the specular direction (source direction reflected about normal)
- Some fraction is absorbed, some reflected
- On real surfaces, energy usually goes into a lobe of directions
- Phong model: reflected energy falls of with $\cos^n(\delta\theta)$
- **Lambertian + specular model**: sum of diffuse and specular term
    - a reasonable approximation to lot of surfaces we see



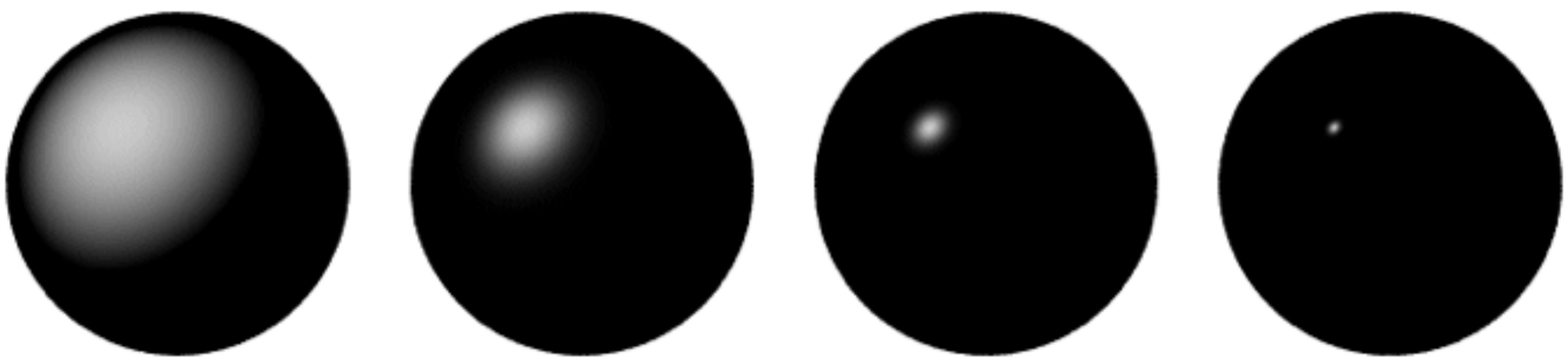




Specular reflection

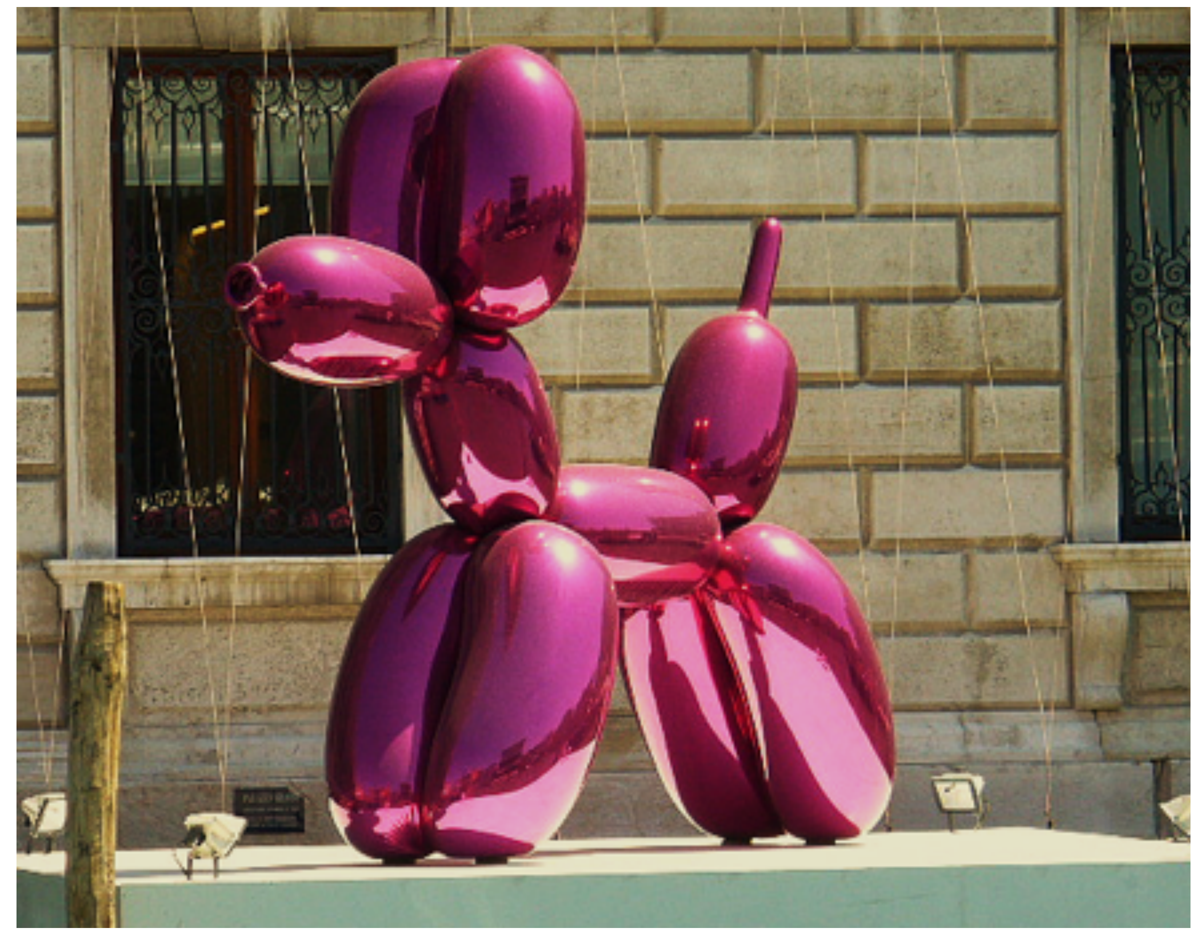


Moving the light source



Changing the exponent

Role of specularity in computer vision



Photometric stereo (shape from shading)

- Can we reconstruct the shape of an object based on shading cues?

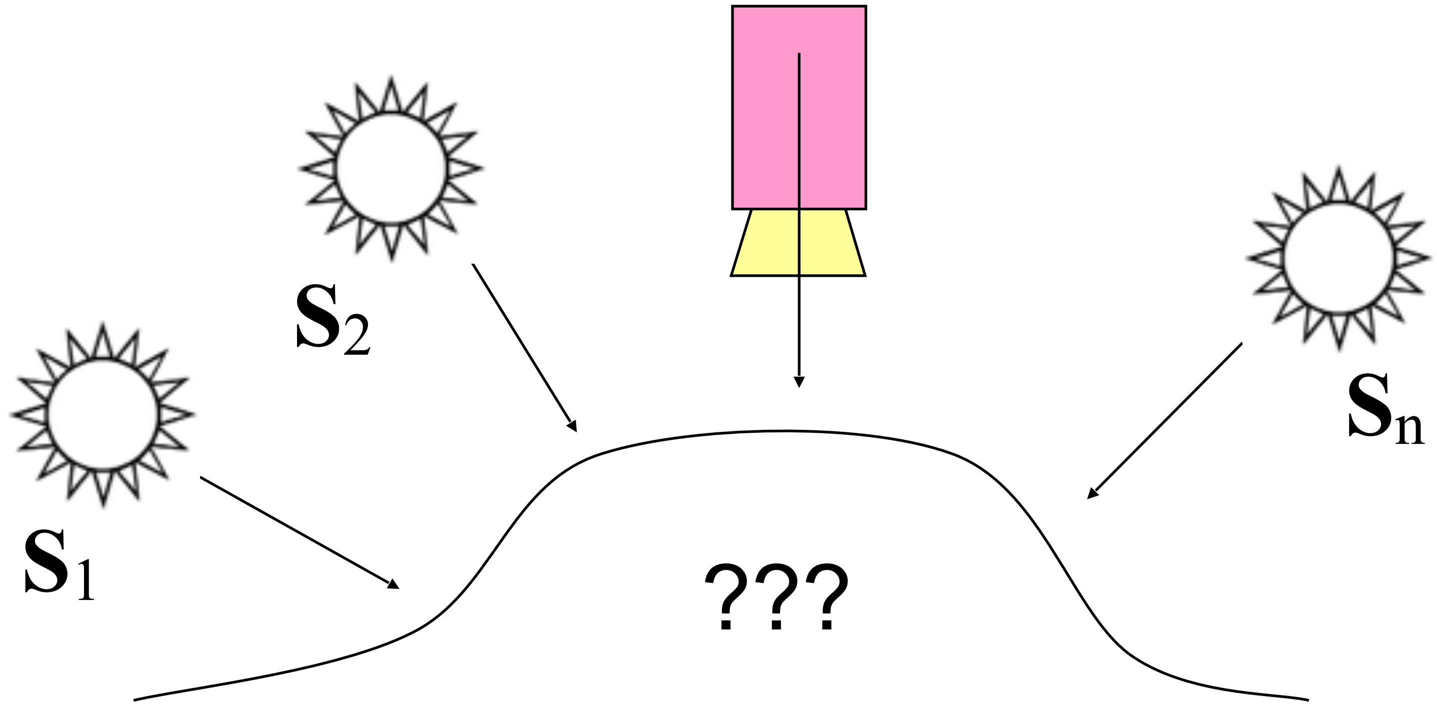
Luca della Robbia, *Cantoria*, 1438

Photometric stereo

Assume:

- A Lambertian object
- A ***local shading model*** (each point on a surface receives light only from sources visible at that point)
- A set of ***known*** light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
- Orthographic projection

Goal: reconstruct object shape and albedo



F&P 2nd ed., sec. 2.2.4

Surface model: Monge patch

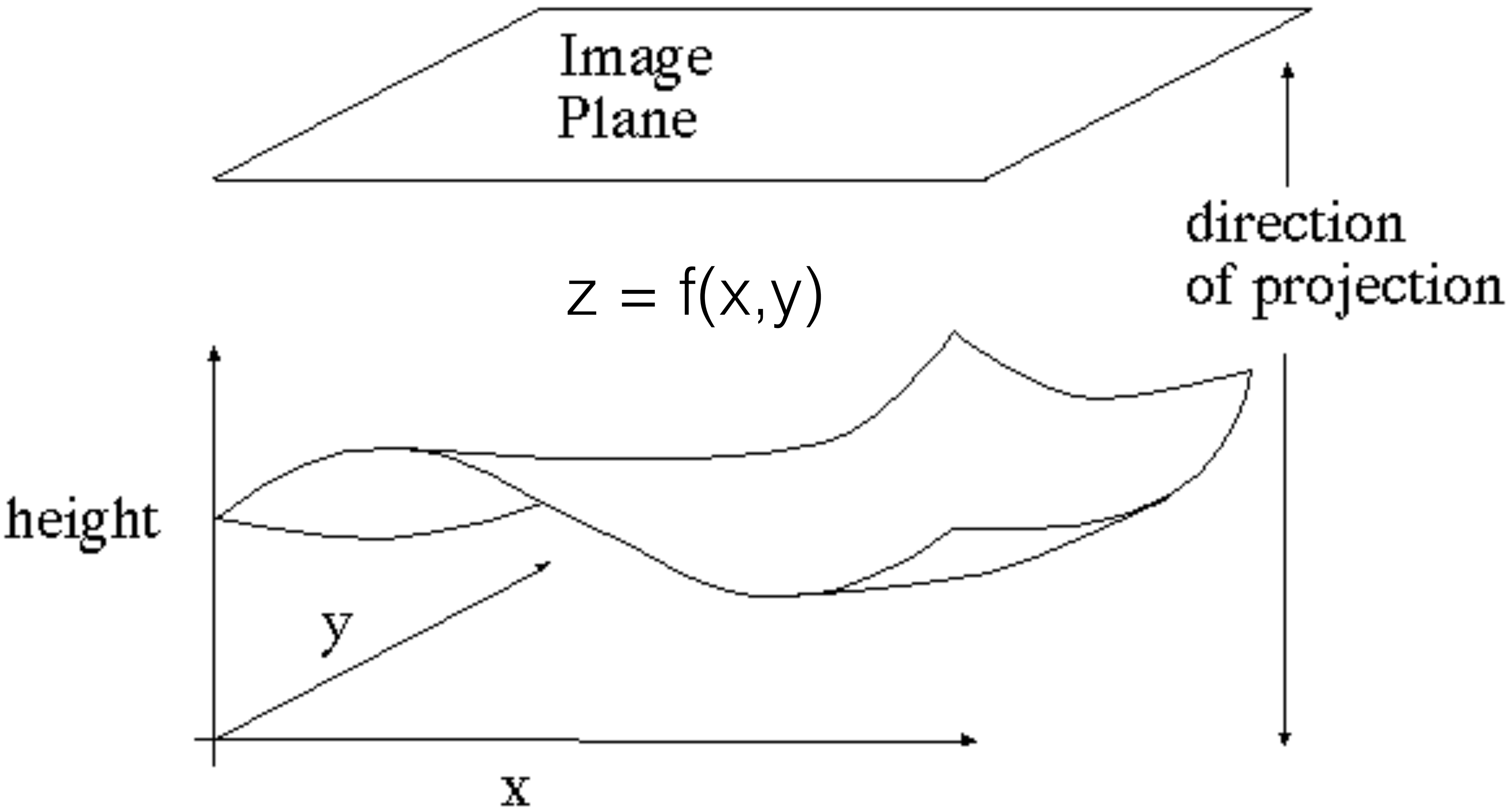


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Image model

- **Known:** source vectors $\mathbf{S}_j$ and pixel values $I_j(x,y)$
- **Unknown:** surface normal $\mathbf{N}(x,y)$ and albedo $\rho(x,y)$
- Assume that the response function of the camera is a linear scaling by a factor of k
- Lambert's law:

$$
\begin{aligned}
I_j(x,y) &= k\rho(x,y)\left(\mathbf{N}(x,y)\cdot\mathbf{S}_j\right) \\
&= \left(\rho(x,y)\mathbf{N}(x,y)\right)\cdot(k\mathbf{S}_j) \\
&= \mathbf{g}(x,y)\cdot\mathbf{V}_j
\end{aligned}
$$

F&P 2nd ed., sec. 2.2.4

Least squares problem

- For each pixel, set up a linear system:

$$\begin{bmatrix} I_1(x,y) \\ I_2(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \vdots \\ \mathbf{V}_n^T \end{bmatrix} \mathbf{g}(x,y)$$

$(n \times 1)$ known — $(n \times 3)$ known — (3×1) unknown

- Obtain least-squares solution for $\mathbf{g}(x,y)$ (which we defined as $\mathbf{N}(x,y)\ \rho(x,y)$)
- Since $\mathbf{N}(x,y)$ is the unit normal, $\rho(x,y)$ is given by the magnitude of $\mathbf{g}(x,y)$
- Finally, $\mathbf{N}(x,y) = \mathbf{g}(x,y) / \rho(x,y)$

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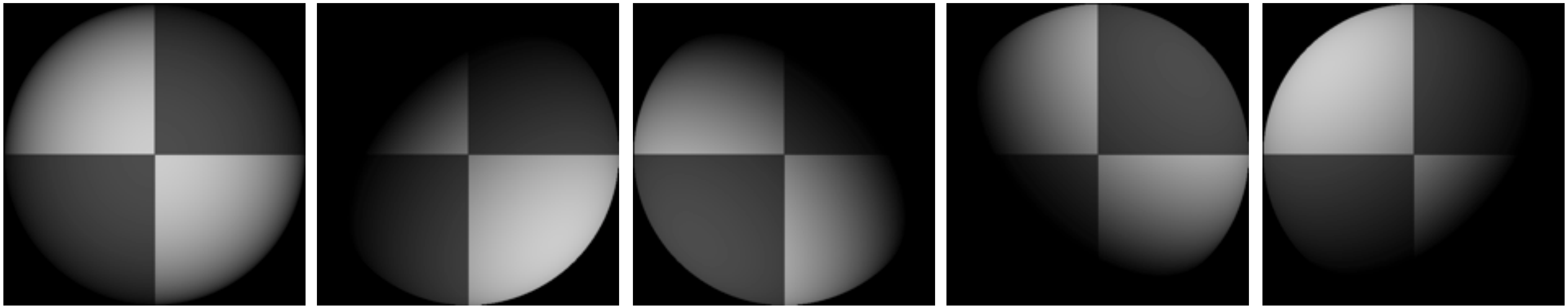
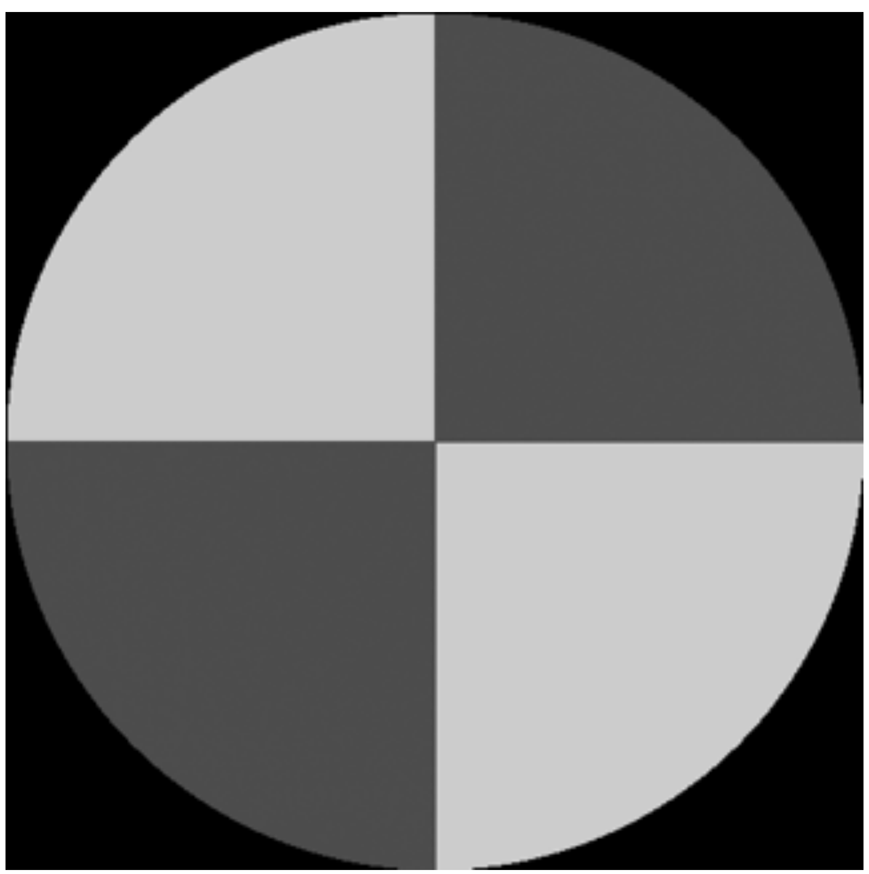
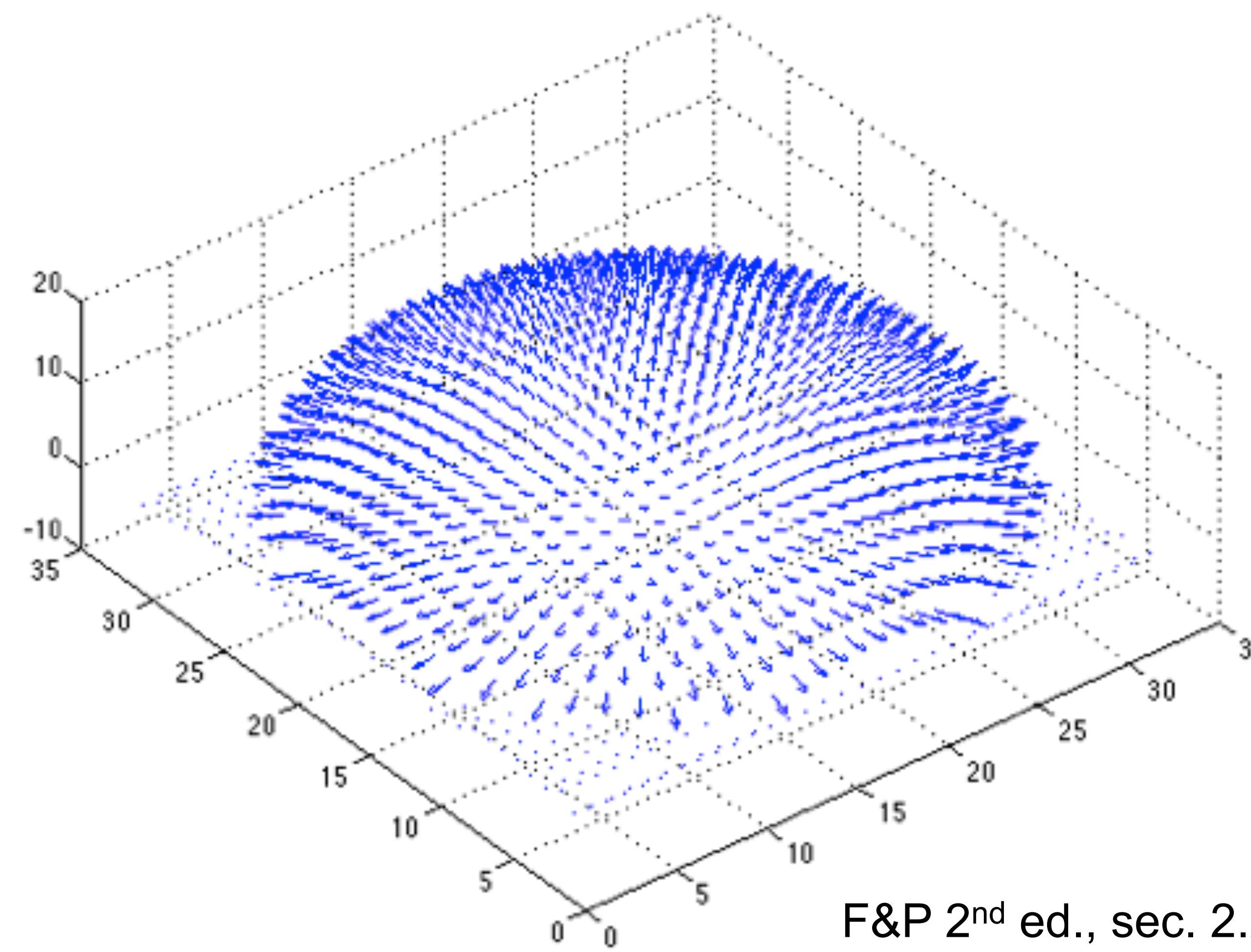
Example



Recovered albedo



Recovered normal field



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Recovering a surface from normals

Recall the surface is written as

$$(x, y, f(x, y))$$

This means the normal has the form:

$$\mathbf{N}(x, y) = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \begin{pmatrix} f_x \\ f_y \\ 1 \end{pmatrix}$$

If we write the estimated vector *g* as

$$\mathbf{g}(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}$$

Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = g_1(x, y) / g_3(x, y)$$
$$f_y(x, y) = g_2(x, y) / g_3(x, y)$$

Recovering a surface from normals

Integrability: for the surface f to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial}{\partial y}(g_1(x,y)/g_3(x,y)) =$$

$$\frac{\partial}{\partial x}(g_2(x,y)/g_3(x,y))$$

(in practice, they should at least be similar)

We can now recover the surface height at any point by integration along some path, e.g.

$$f(x,y) = \int_0^x f_x(s,y)ds +$$

$$\int_0^y f_y(x,t)dt + C$$

(for robustness, should take integrals over many different paths and average the results)

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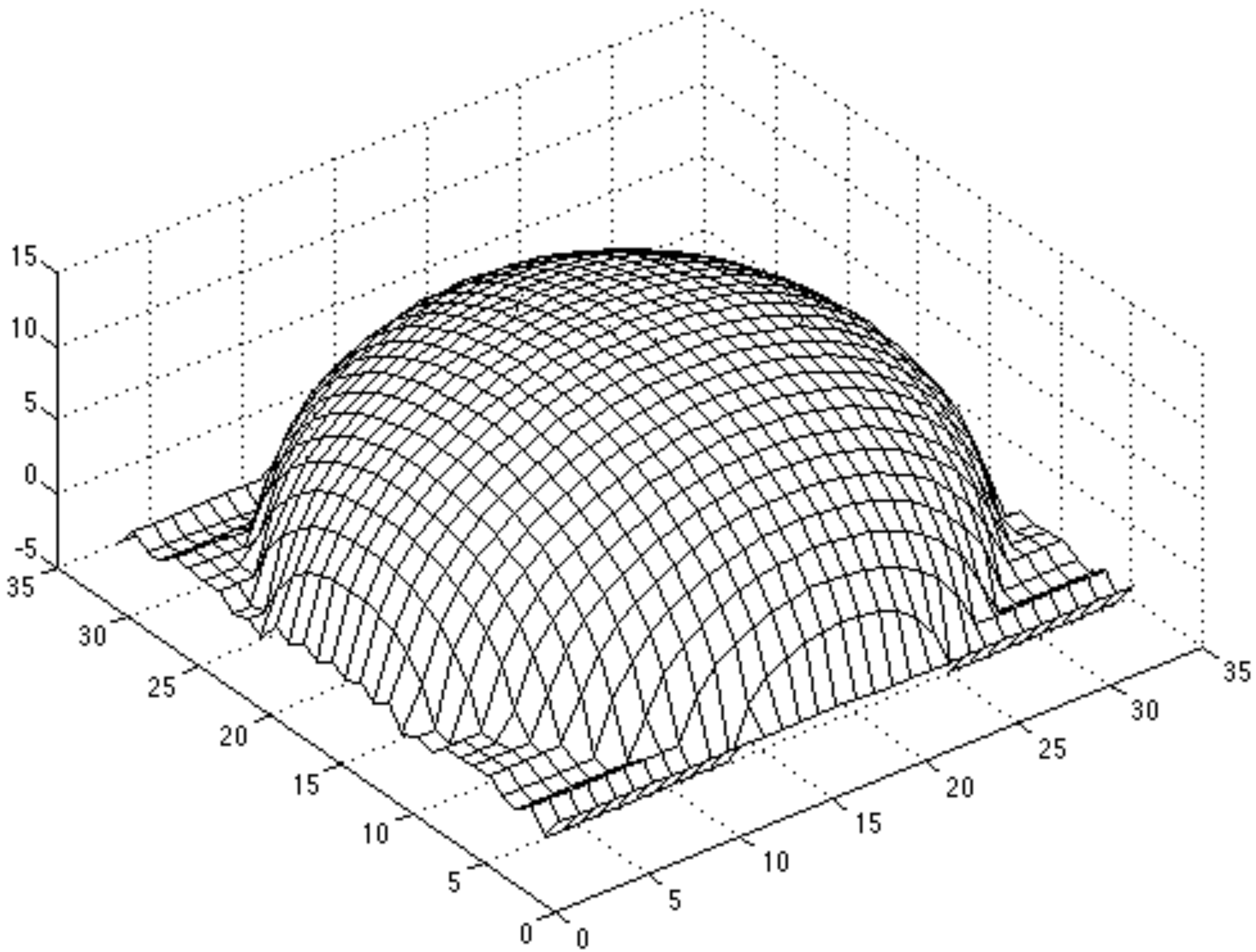
Surface recovered by integration



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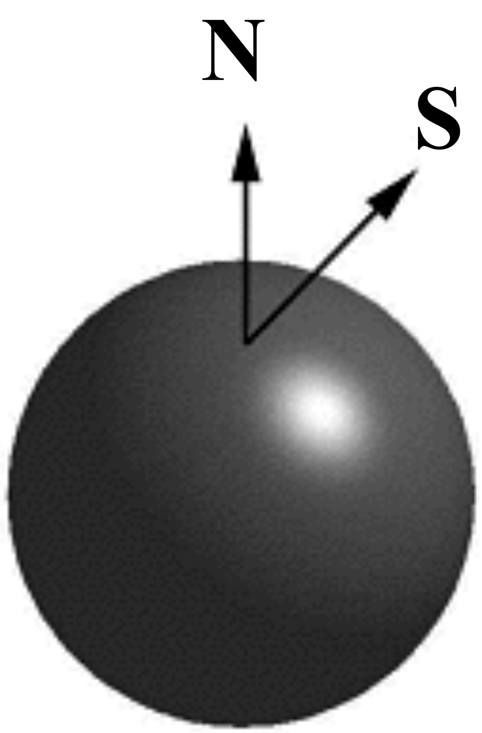
Limitations

- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

Finding the direction of the light source

$$I(x,y) = \mathbf{N}(x,y) \cdot \mathbf{S}(x,y) + A$$



Full 3D case:

$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & N_z(x_1,y_1) & 1 \\ N_x(x_2,y_2) & N_y(x_2,y_2) & N_z(x_2,y_2) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & N_z(x_n,y_n) & 1 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \\ A \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

For points on the *occluding contour*:

$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & 1 \\ N_x(x_2,y_2) & N_y(x_2,y_2) & 1 \\ \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & 1 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ A \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Finding the direction of the light source

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Application: Detecting composite photos

Fake photo

Real photo

M. K. Johnson and H. Farid, **Exposing Digital Forgeries by Detecting Inconsistencies in Lighting**, ACM Multimedia and Security Workshop, 2005.

More readings and thoughts …

- Derive the fundamental radiometric relation in lenses:

$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

- Derive the formula for the BRDF for a mirror
- People can perceive reflectance
  - **Surface reflectance estimation and natural illumination statistics**, R.O. Dror, E.H. Adelson, and A.S. Willsky, Workshop on Statistical and Computational Theories of Vision 2001
- HDR photography
  - **Recovering High Dynamic Range Radiance Maps from Photographs**, Paul E. Devebec and Jitendra Malik, SIGGRAPH 1997