CMPSCI 670: Computer Vision

Image formation

University of Massachusetts, Amherst

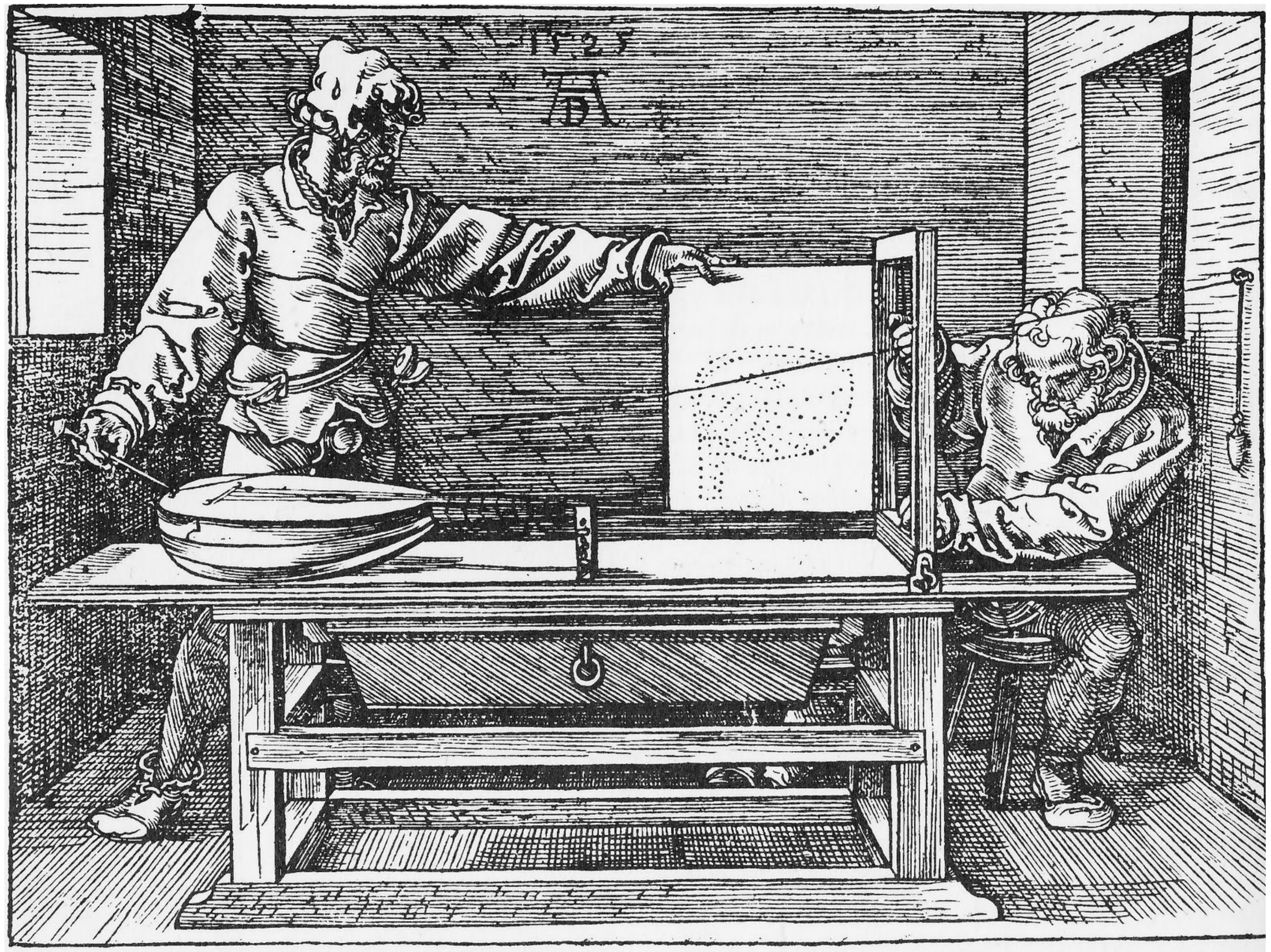
September 8, 2014

Instructor: Subhransu Maji

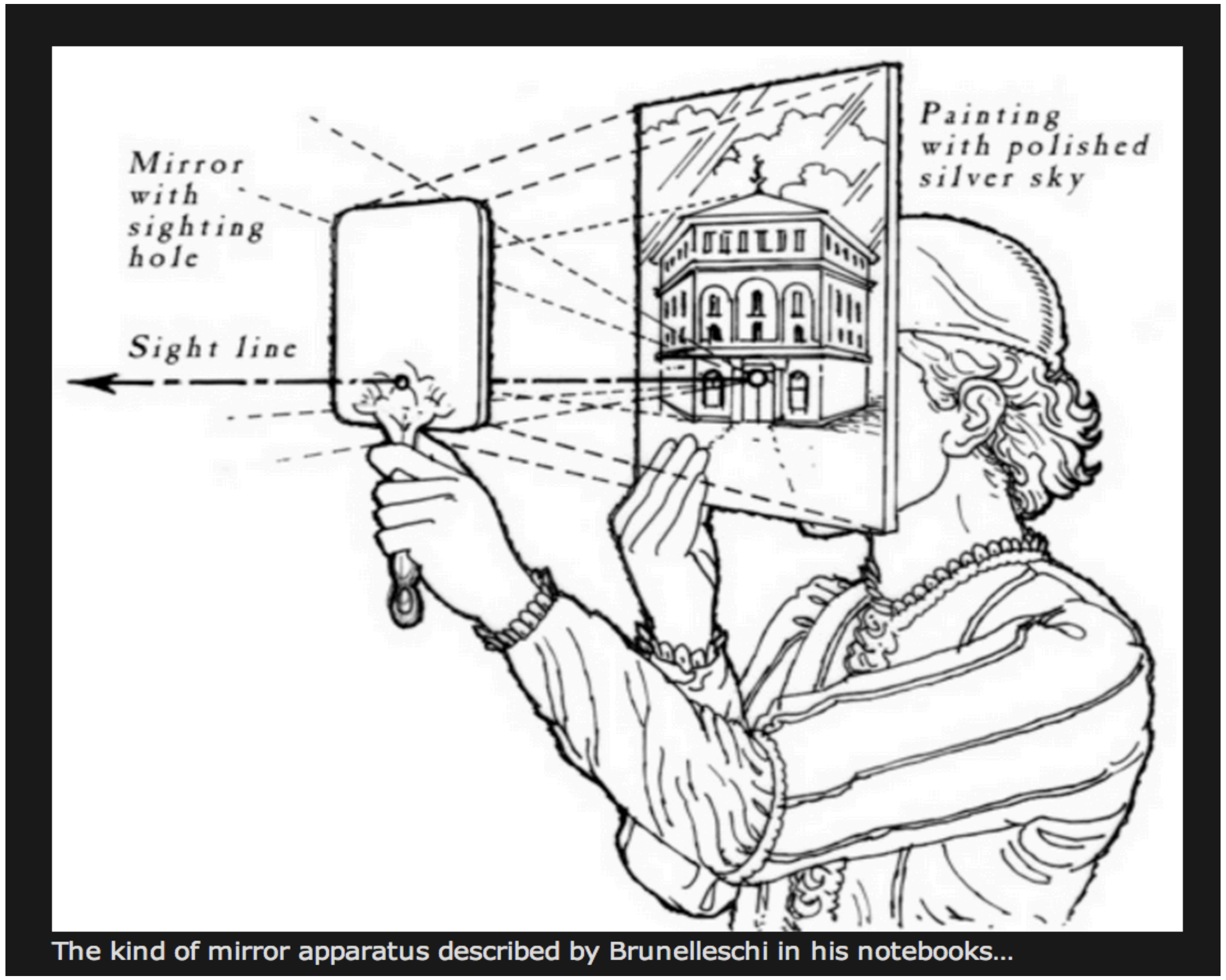
Administrivia

- MATLAB setup and tutorial
  - Does everyone have access to MATLAB yet?
- EdLab accounts have been created
  - **http://edlab-www.cs.umass.edu**
- Homework 1 is up on the course webpage
  - Due September 22 before the start of the class
  - Submission instructions will the posted soon
- Lecture 1 slides posted
  - Do you also want 2 slides/page, 4 slides/page versions?
- Last day of class is December 3 (expect a mid-point report of your projects). Final project reports will be due on December 12.

Cameras



Albrecht Dürer early 1500s



Brunelleschi, early 1400s

Overview of the next two lectures

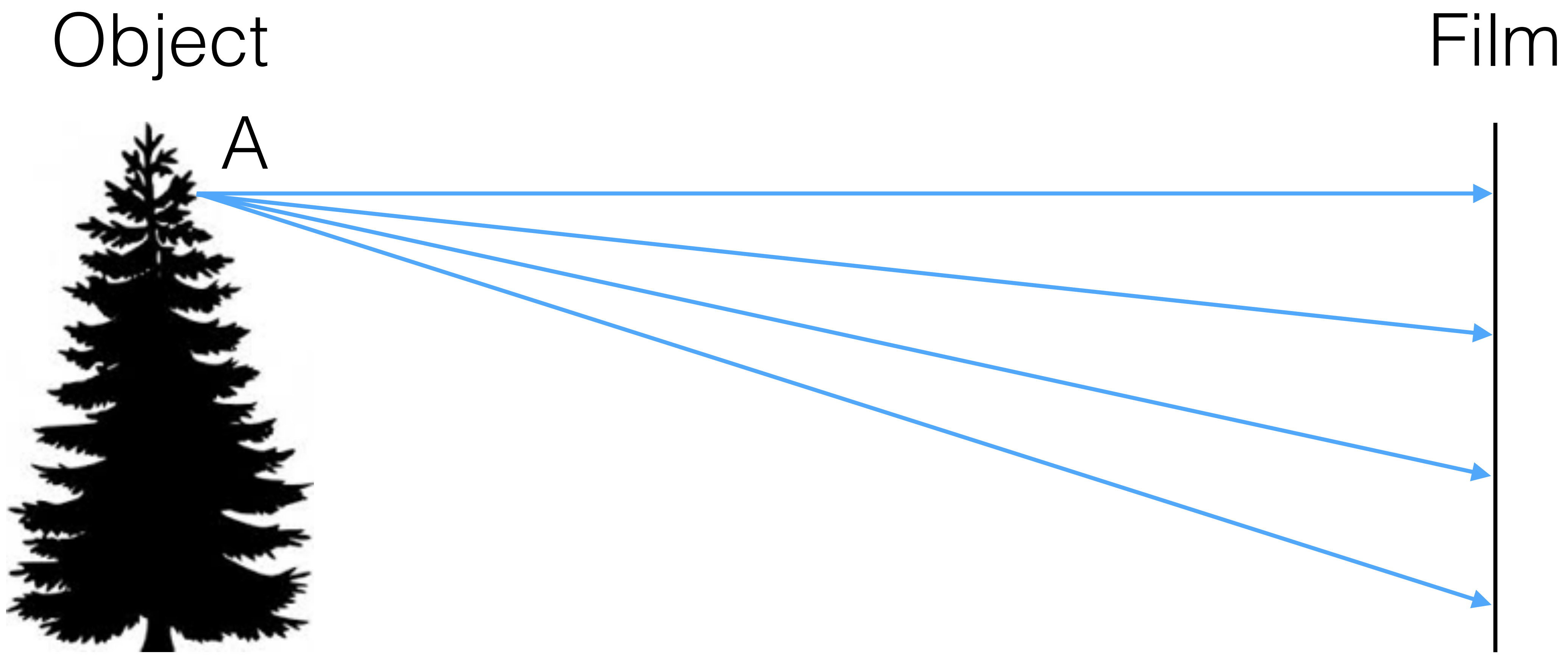
- **The pinhole projection model**
  - qualitative properties
  - perspective projection matrix
- **Cameras with lenses**
  - Depth of focus
  - Field of view
  - Lens aberrations
- **Digital cameras**
  - Sensors
  - Colors
  - Artifacts

Lets design a camera

Object Film

Idea 1: Lets put a film in front of an object
Do we get a reasonable image?

Lets design a camera



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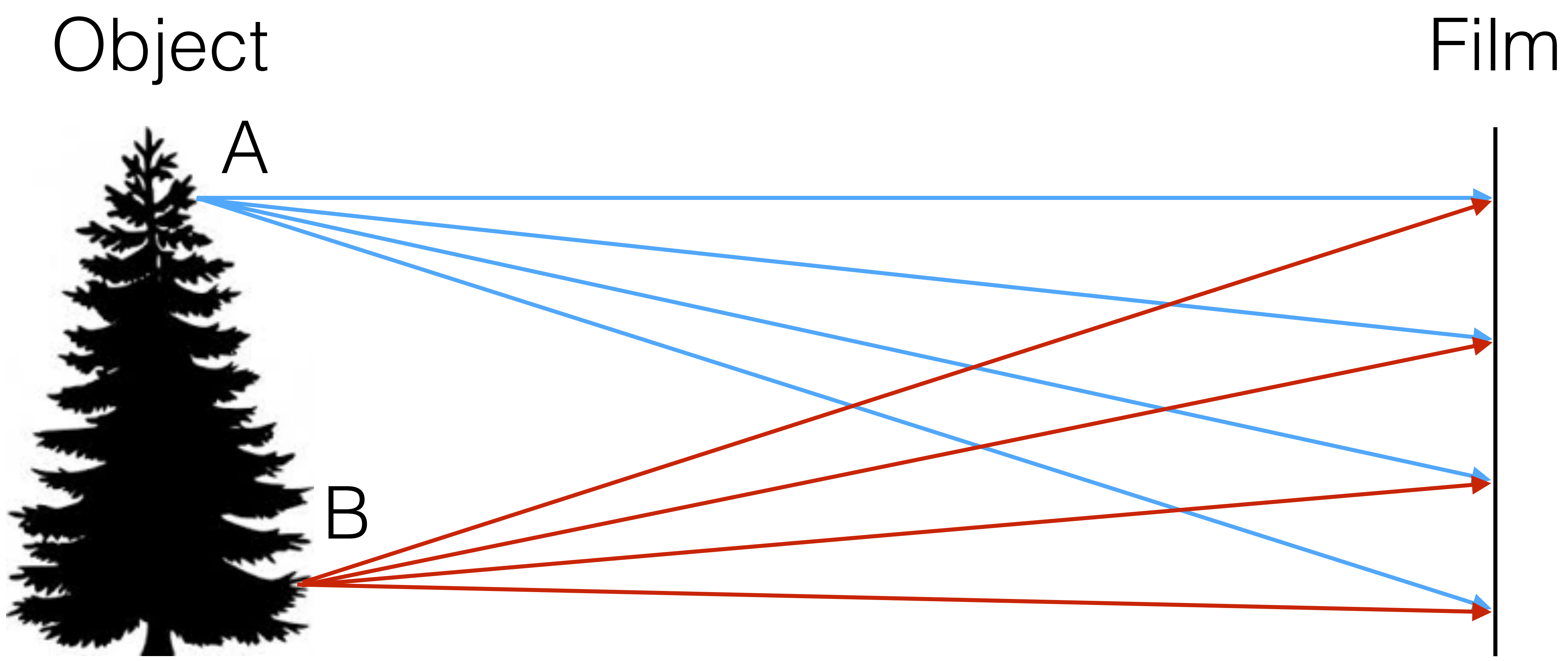
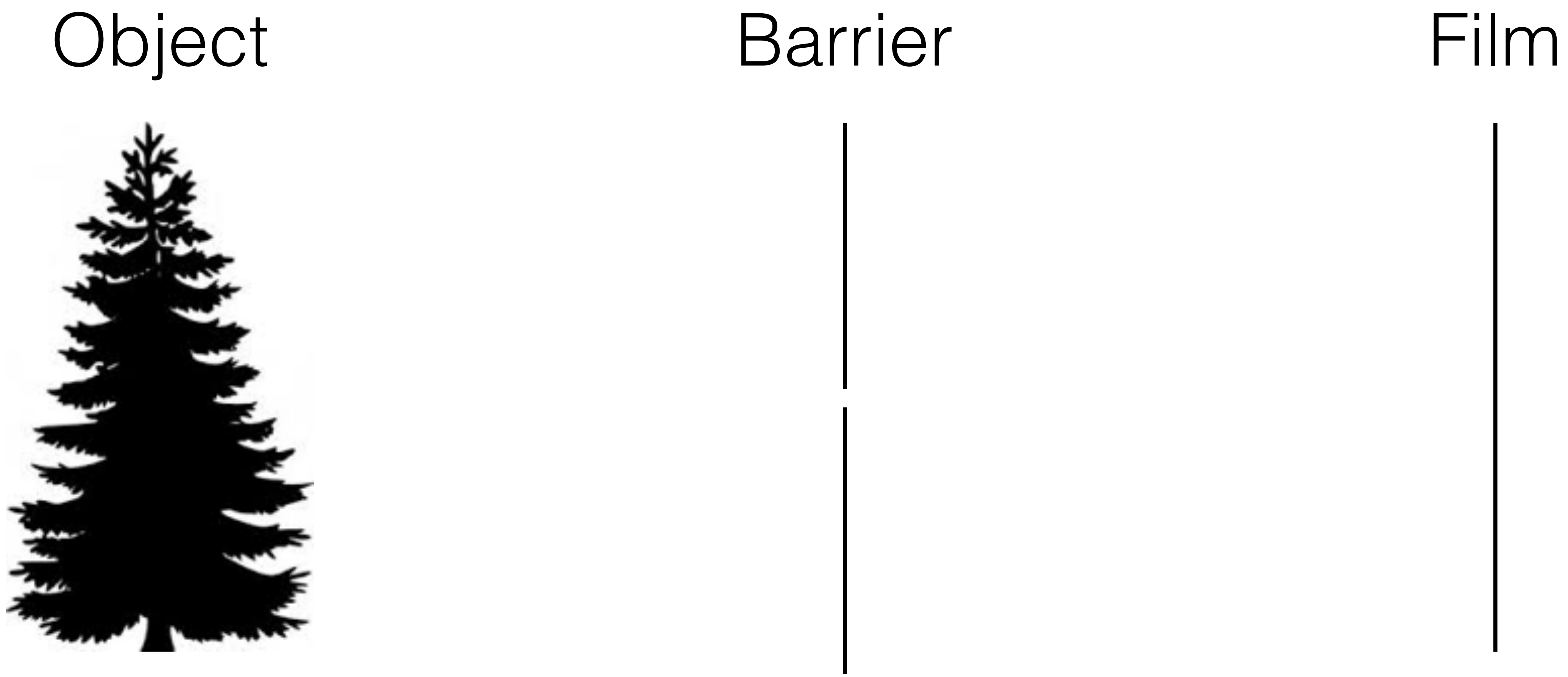


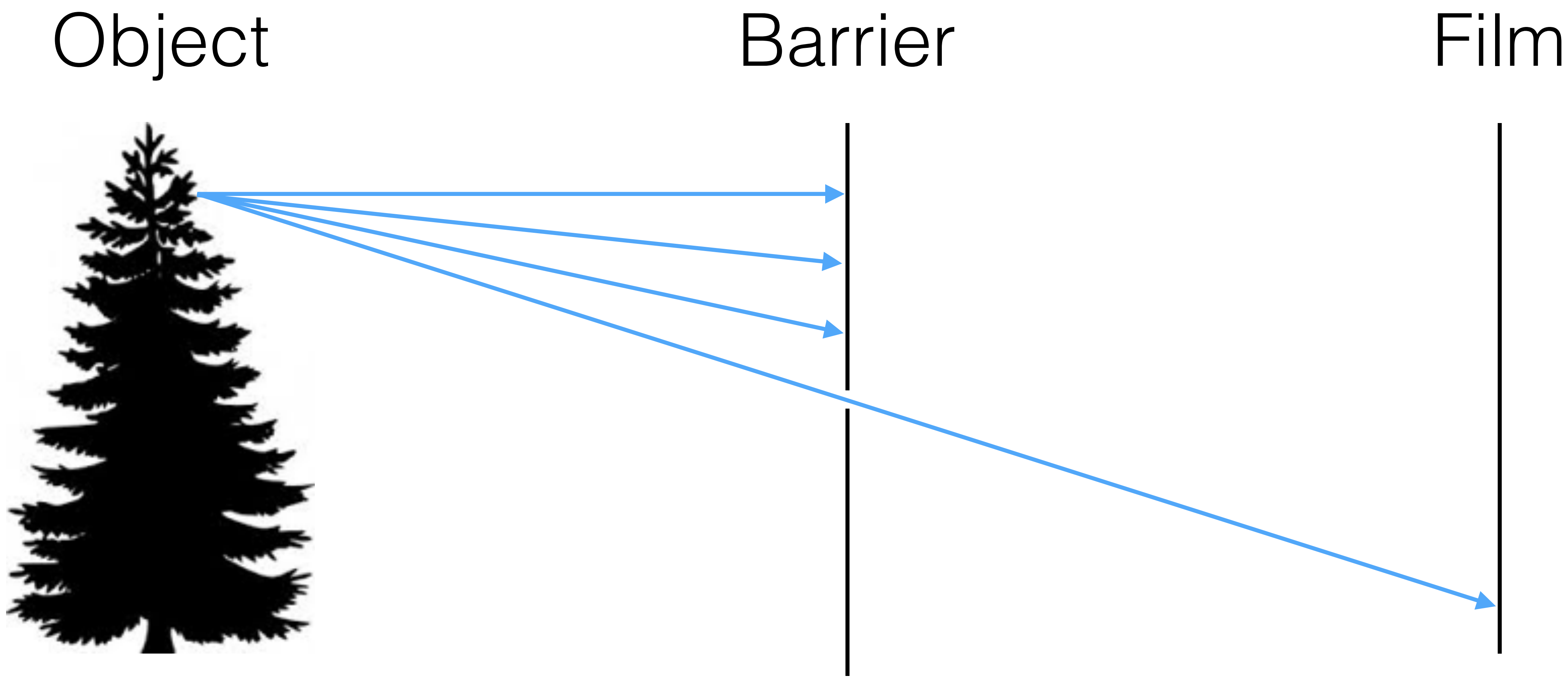
Idea 1: Lets put a film in front of an object
Do we get a reasonable image?

Pinhole camera



Add a barrier to block of most rays

Pinhole camera



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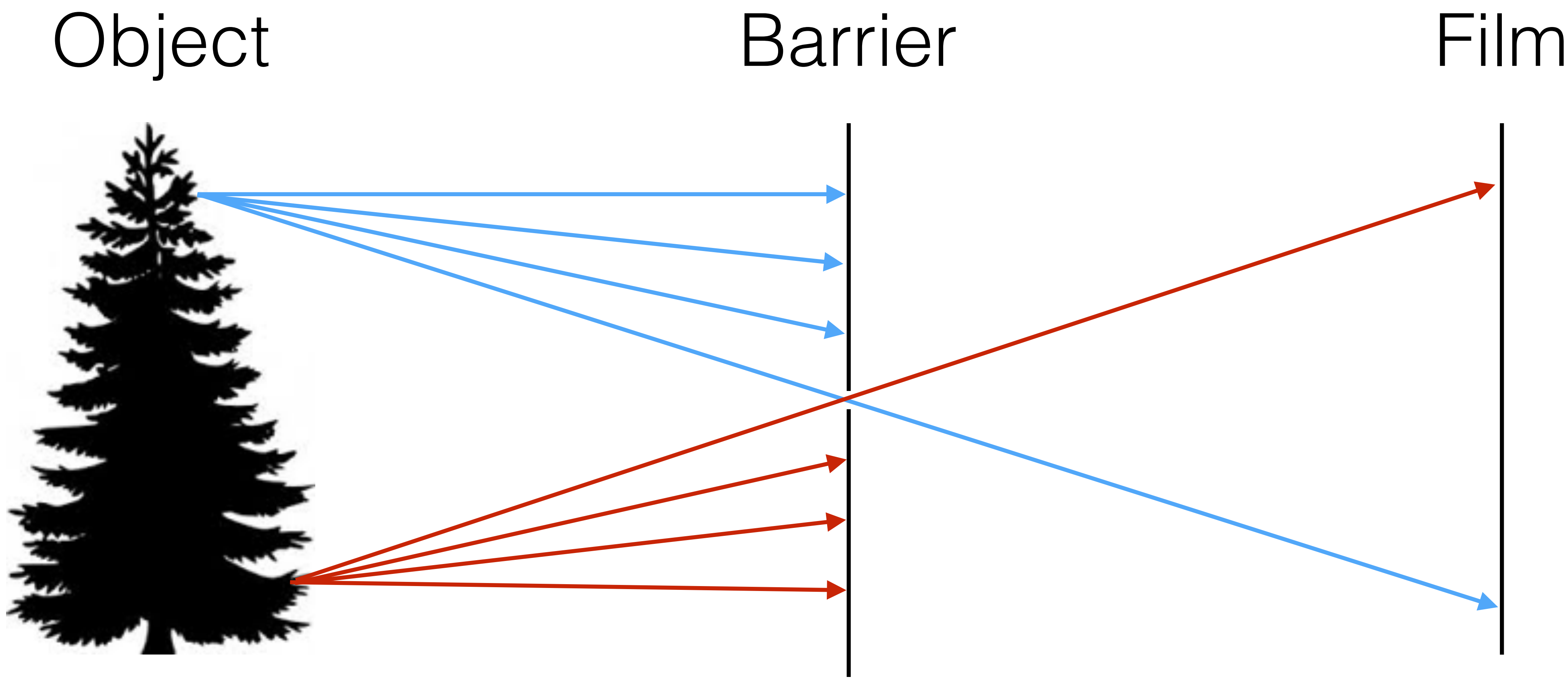
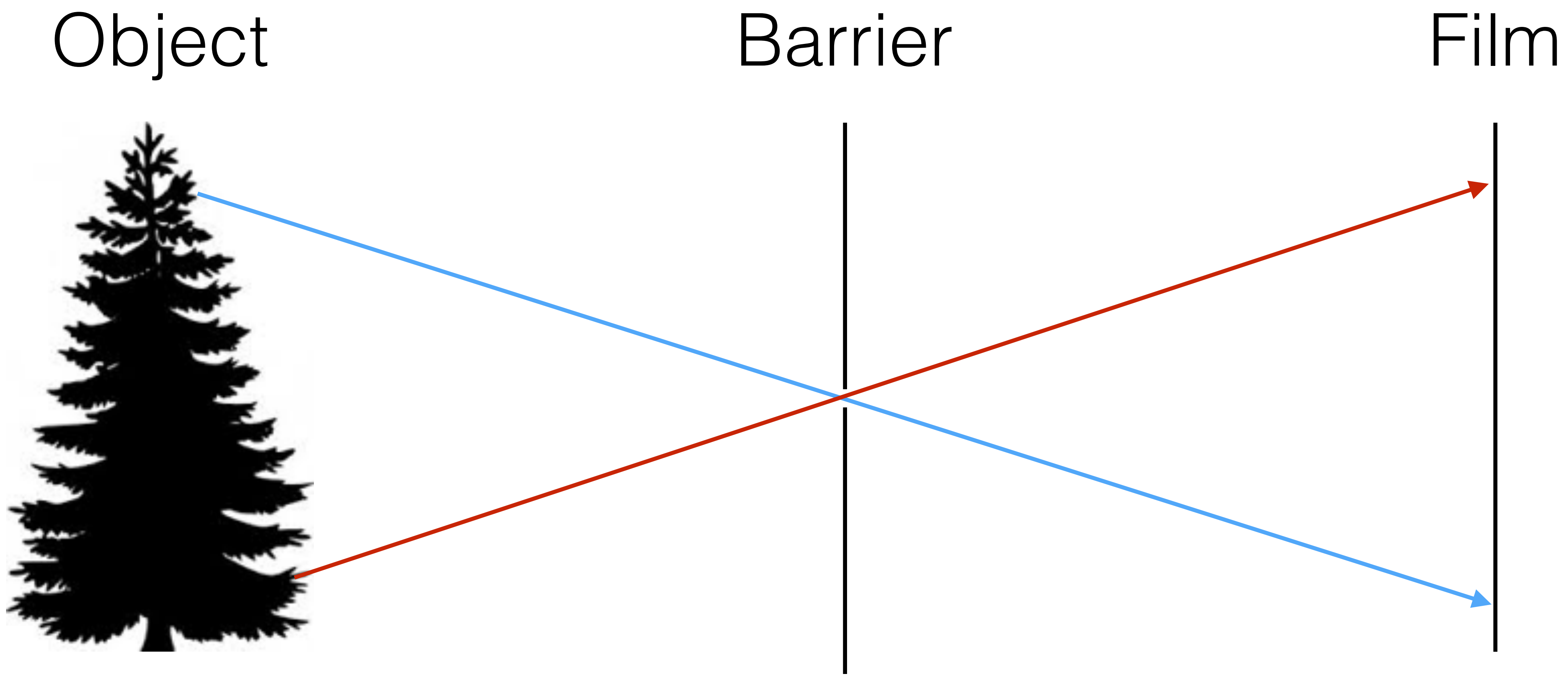


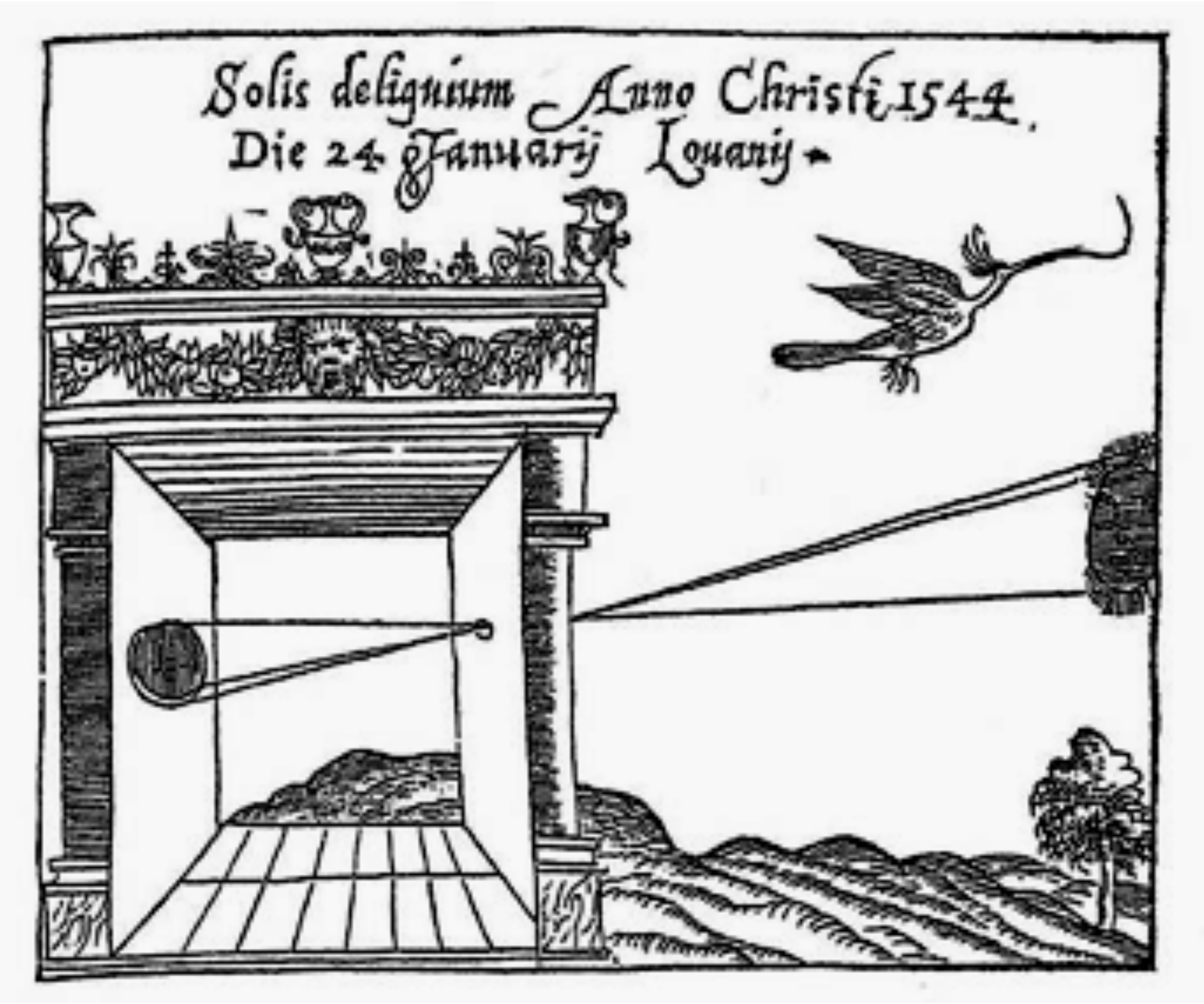
Add a barrier to block of most rays

Pinhole camera



- Captures **pencil of rays** - all rays through a single point: **aperture, center of projection, focal point, camera center**
- The image is formed on the **image plane**

Camera obscura



Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aids for artists: described by Leonardo Da Vinci (1452-1519 AD)

"Camera obscure" Latin for "darkened room"

Pinhole cameras are everywhere

Tree shadow during a solar eclipse
photo credit: Nils van der Burg
http://www.physicstogo.org/index.cfm

Accidental pinhole cameras

My hotel room, contrast enhanced.

The view from my window

Accidental pinholes produce images that are unnoticed or misinterpreted as shadows

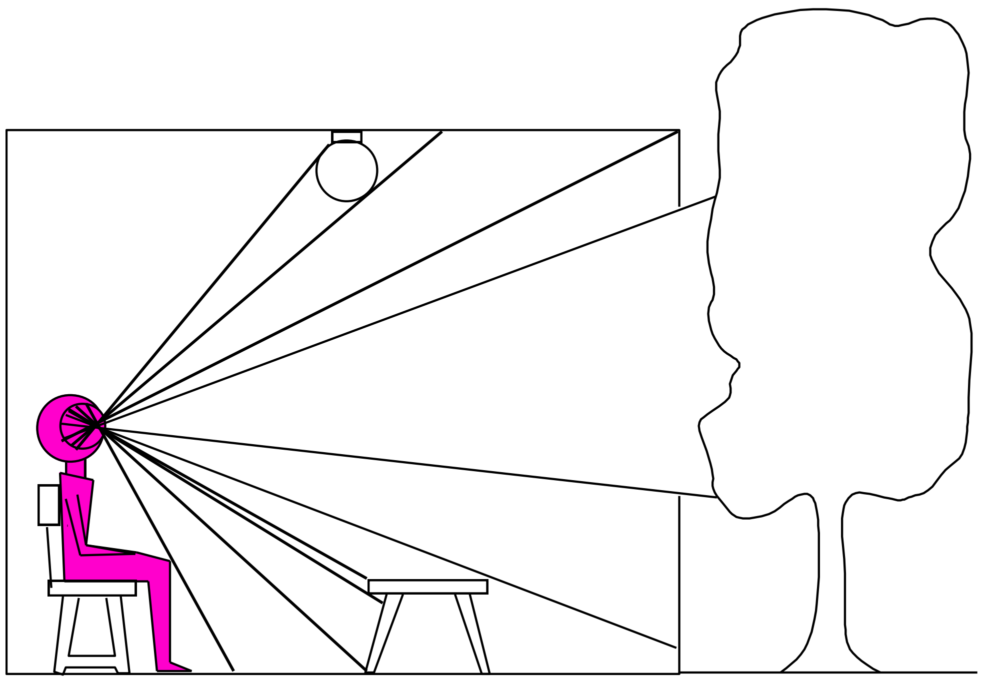
A. Torralba and W. Freeman, **Accidental Pinhole and Pinspeck Cameras**, CVPR 2012

Home-made pinhole camera

http://www.pauldebevec.com/Pinhole

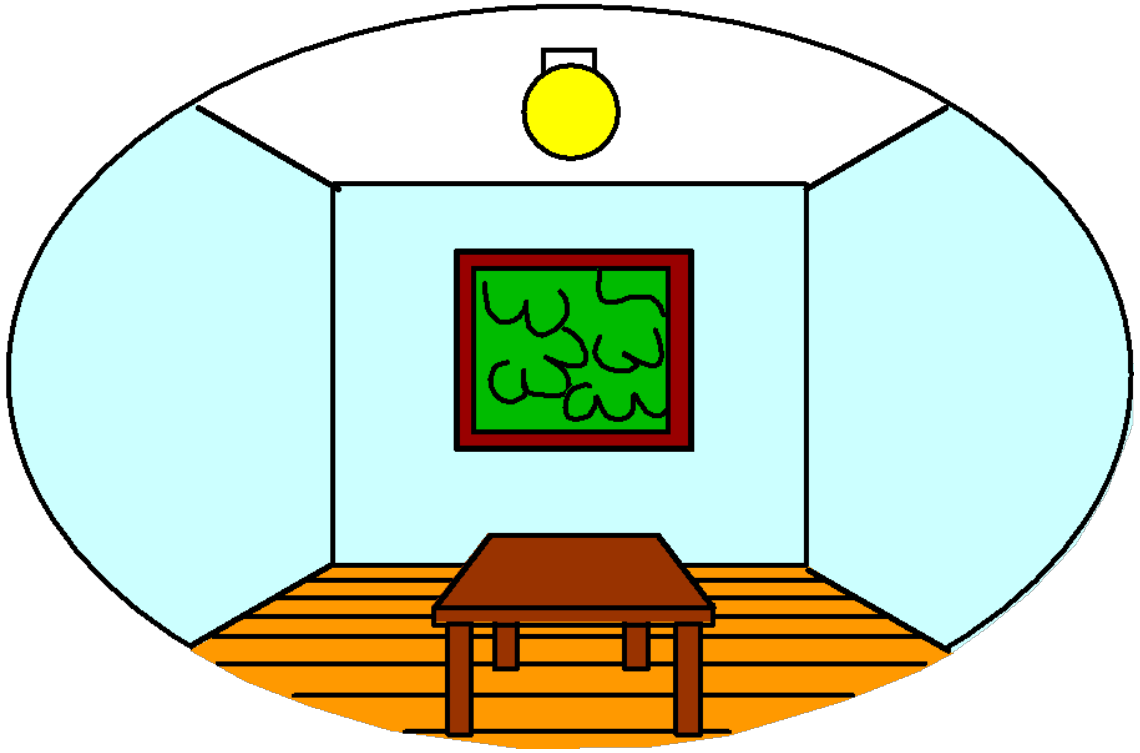
Dimensionality reduction: 3D to 2D

3D world



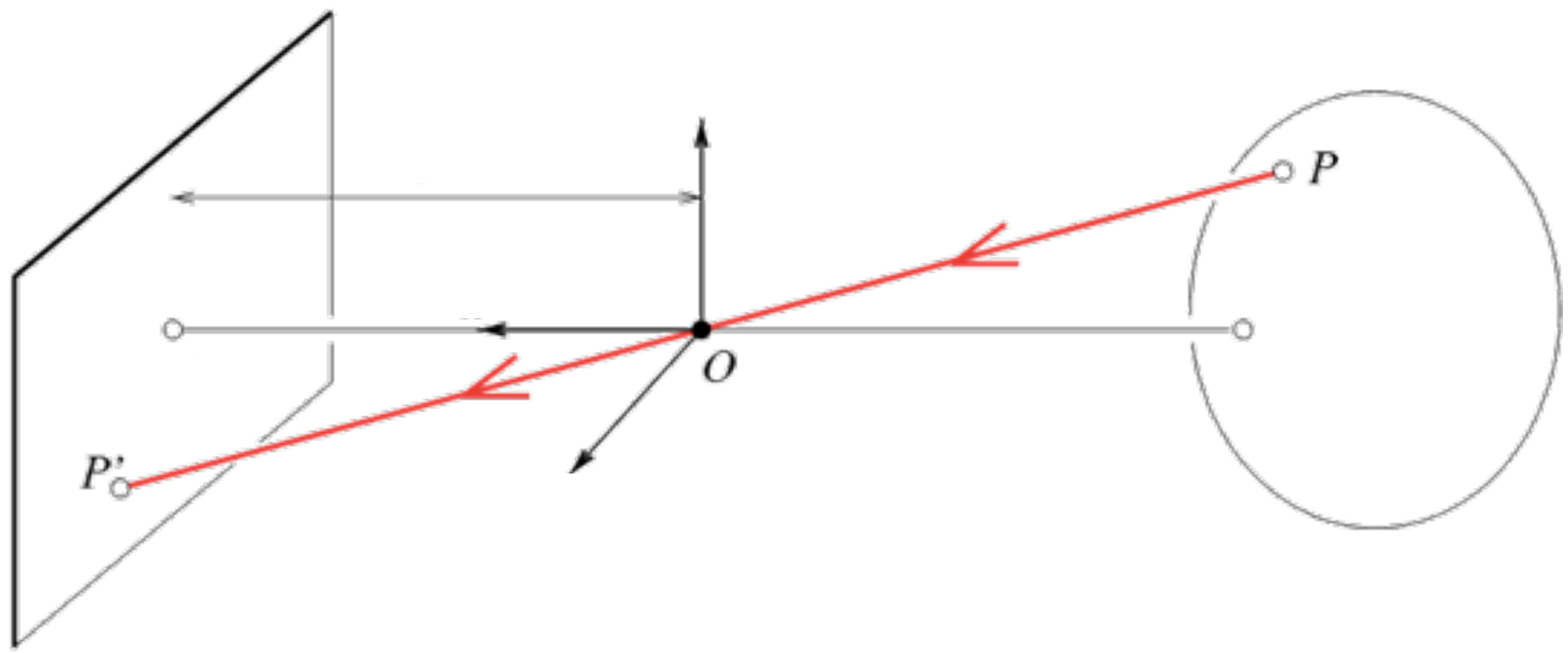
Point of observation

2D image



- **What is preserved?**
  - Straight lines, incidence
- **What is not preserved?**
  - Angles, lengths

Modeling projection



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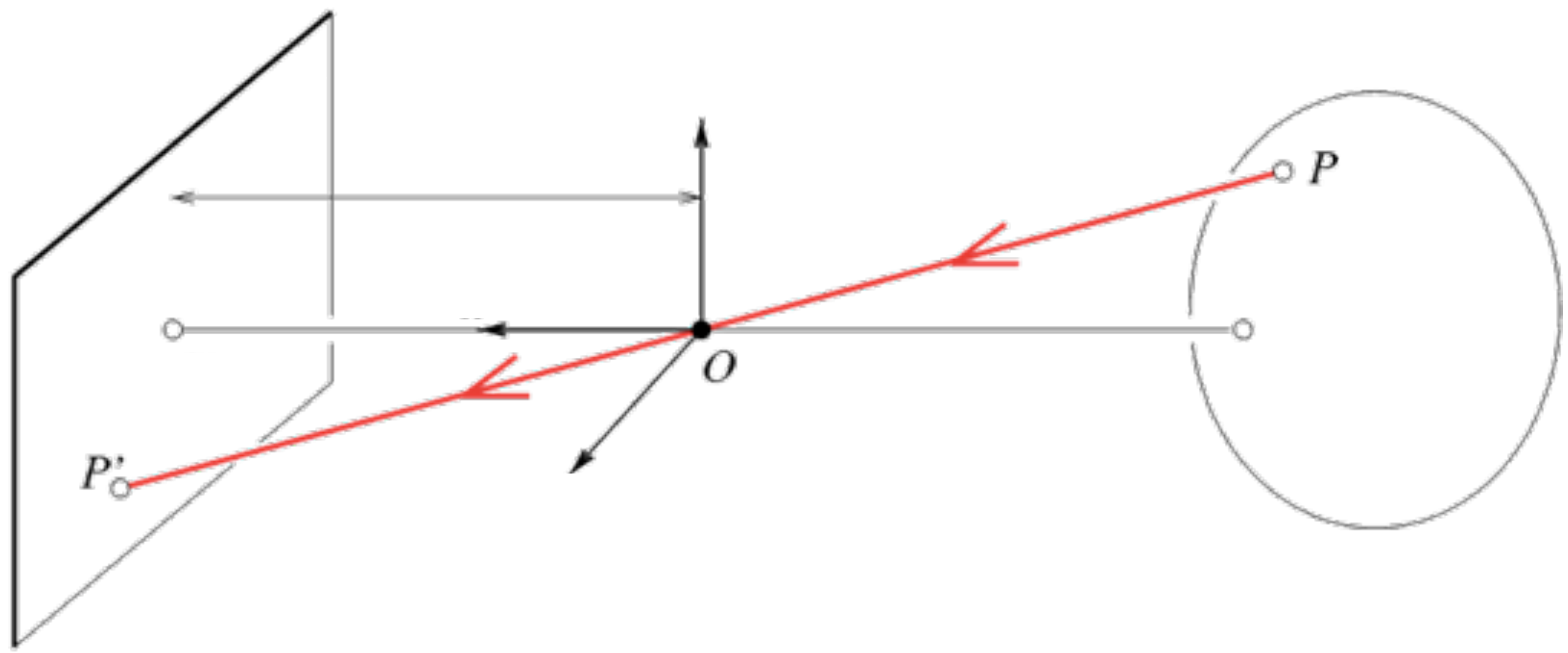
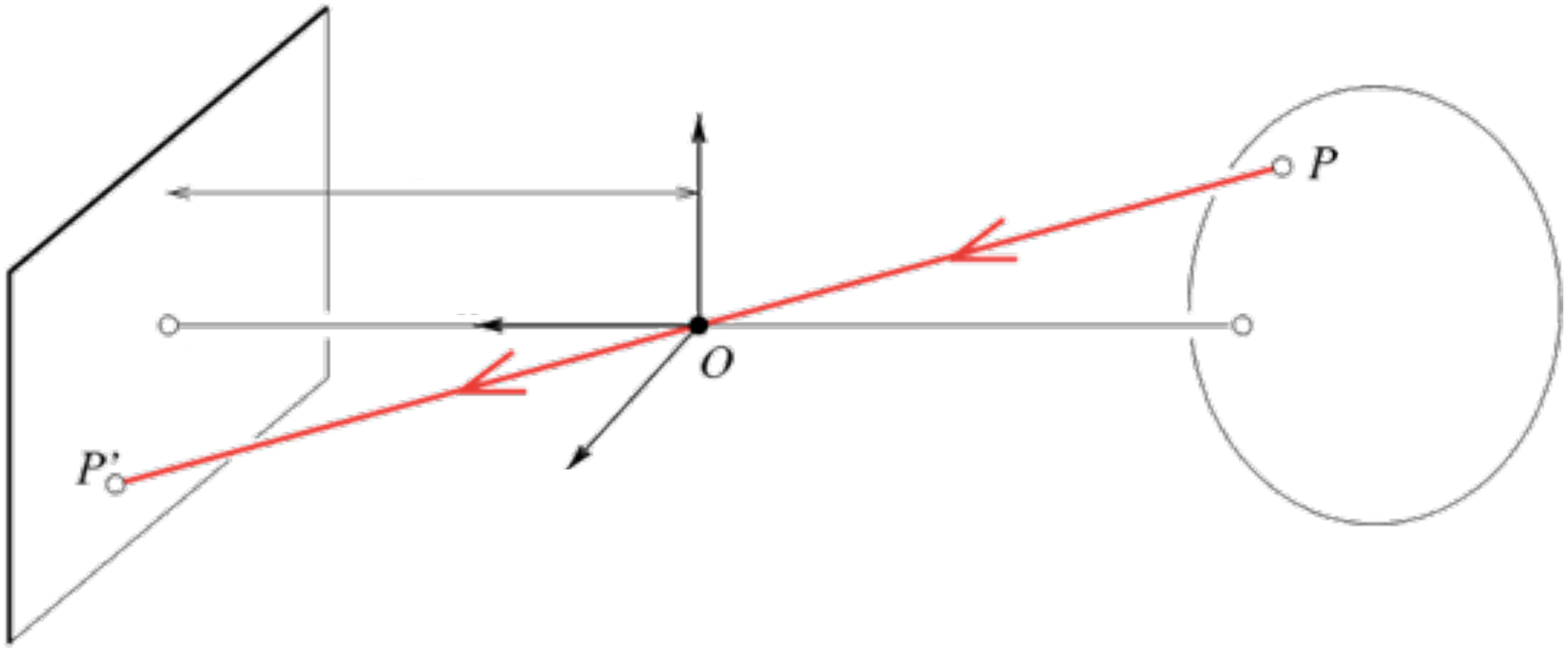


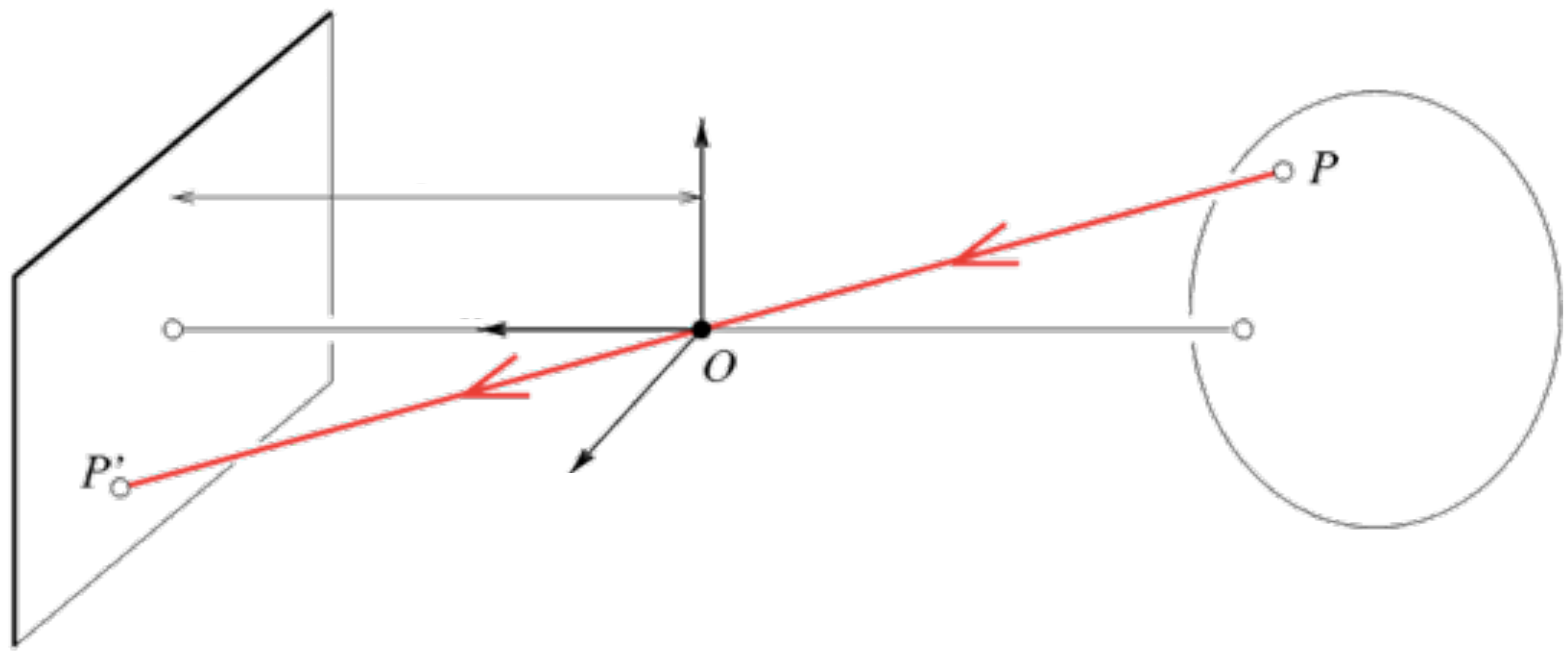
- To compute the projection *P'* of a scene point *P*, form a **visual ray** connection P to the camera center *O* and find where it intersects the image plane

Modeling projection



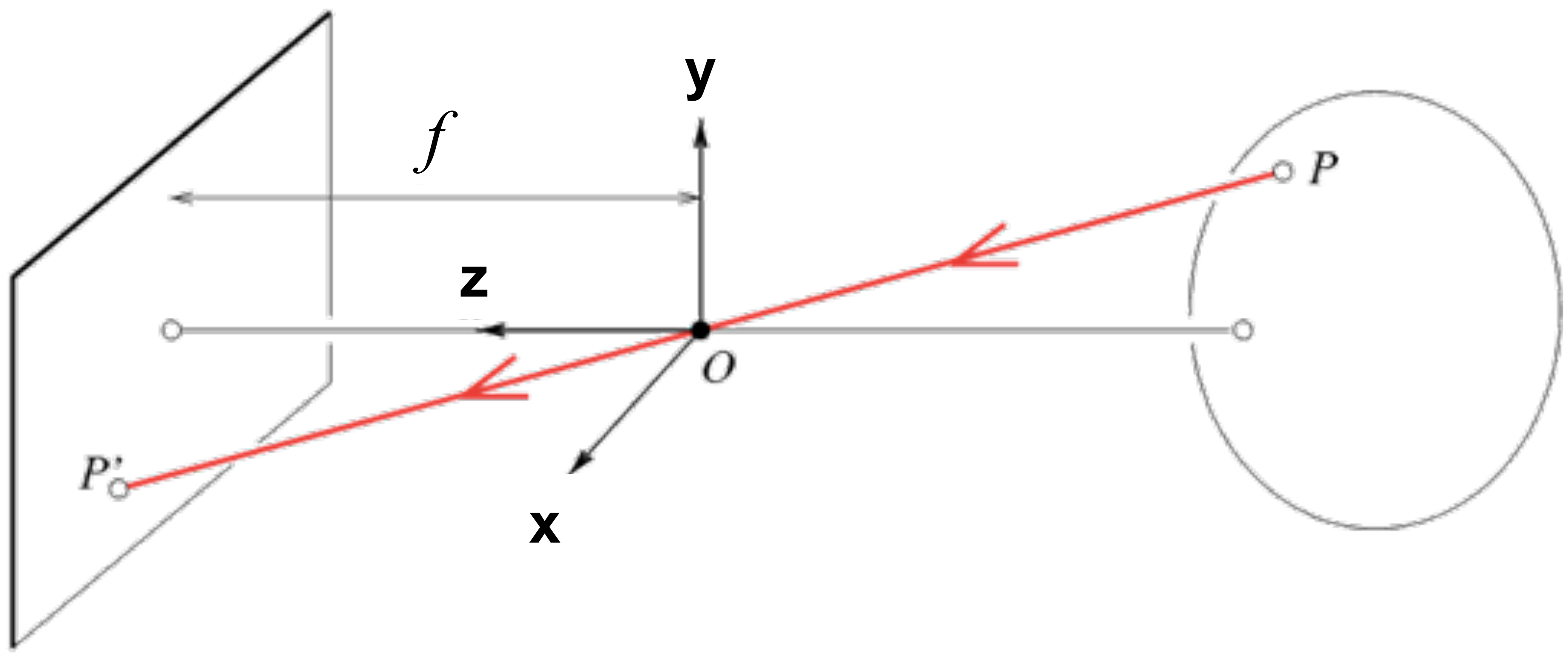
- To compute the projection *P'* of a scene point *P*, form a **visual ray** connection P to the camera center *O* and find where it intersects the image plane
    - All scene points that lie on this visual ray have the same projection on the image

Modeling projection

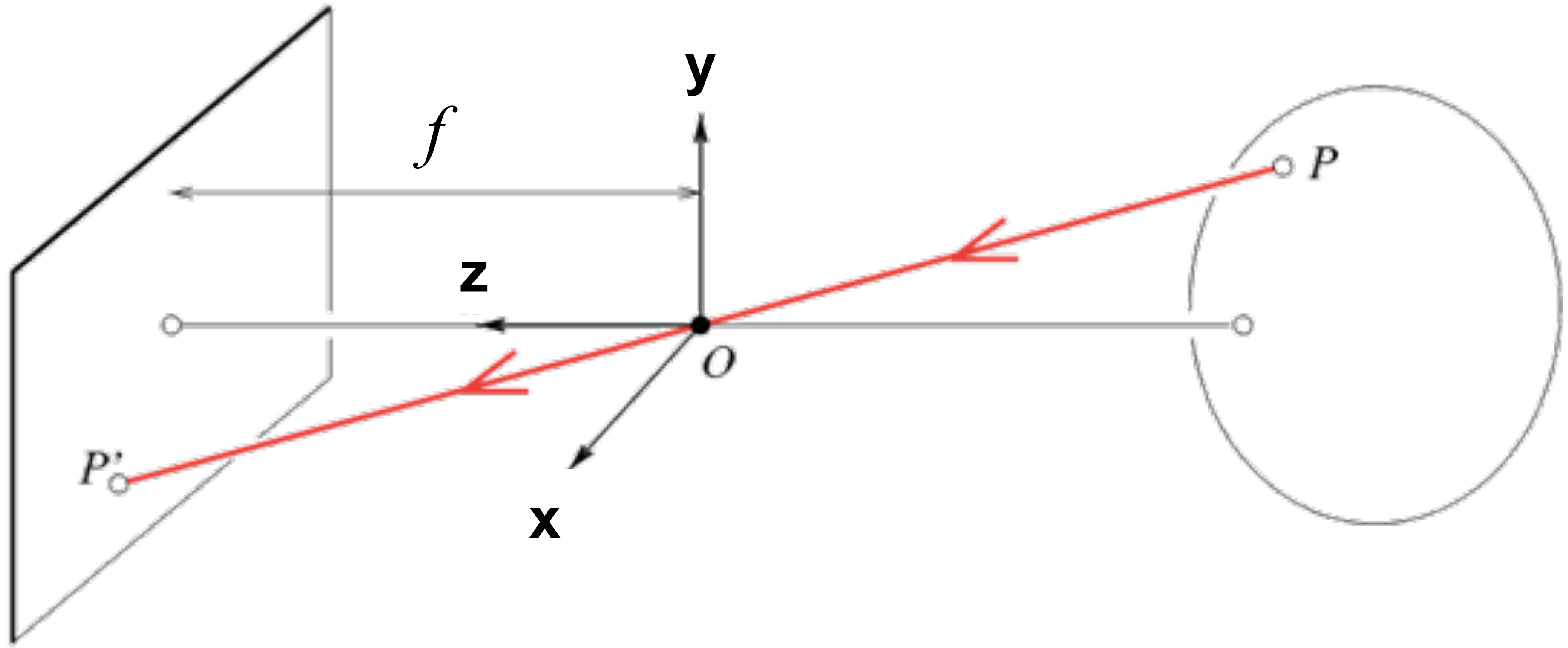


- To compute the projection *P'* of a scene point *P*, form a **visual ray** connection P to the camera center *O* and find where it intersects the image plane
  - All scene points that lie on this visual ray have the same projection on the image
  - Are there points for which this projection is not defined?

Modeling projection

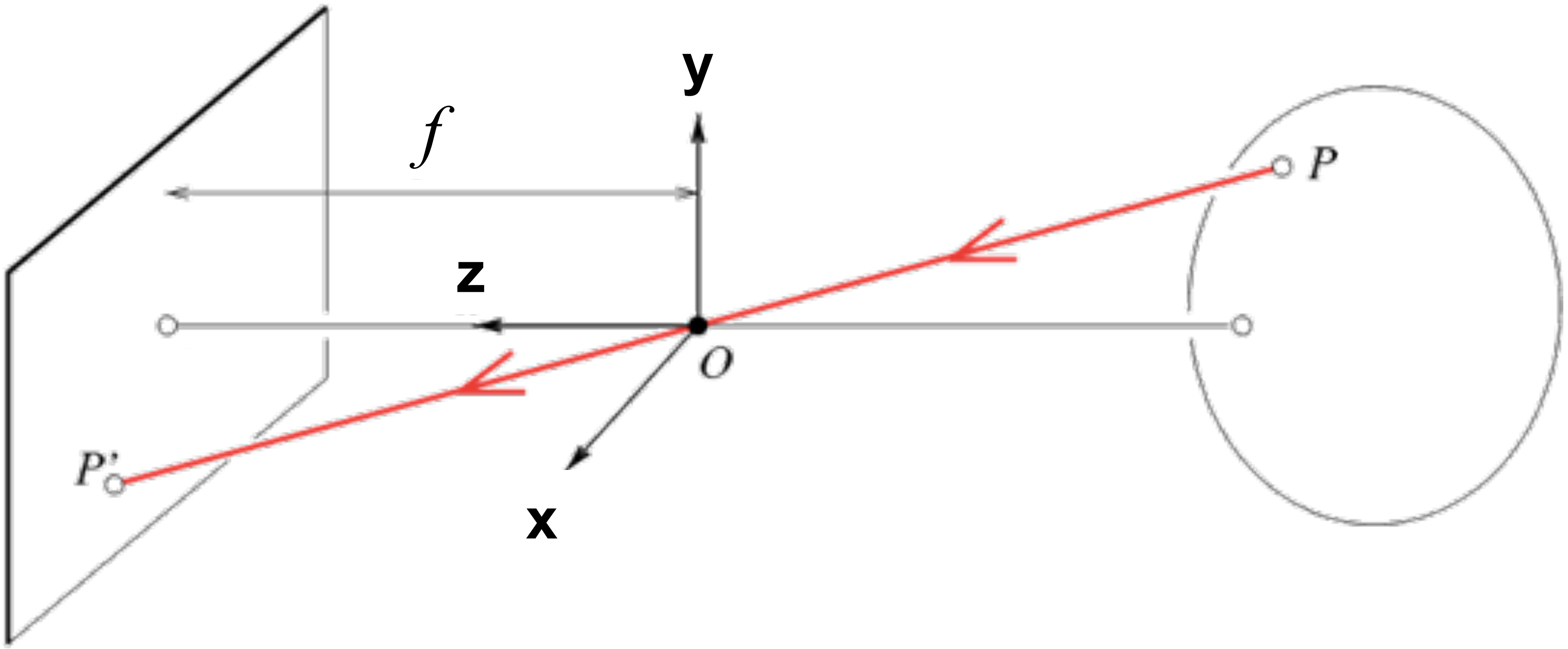


Modeling projection



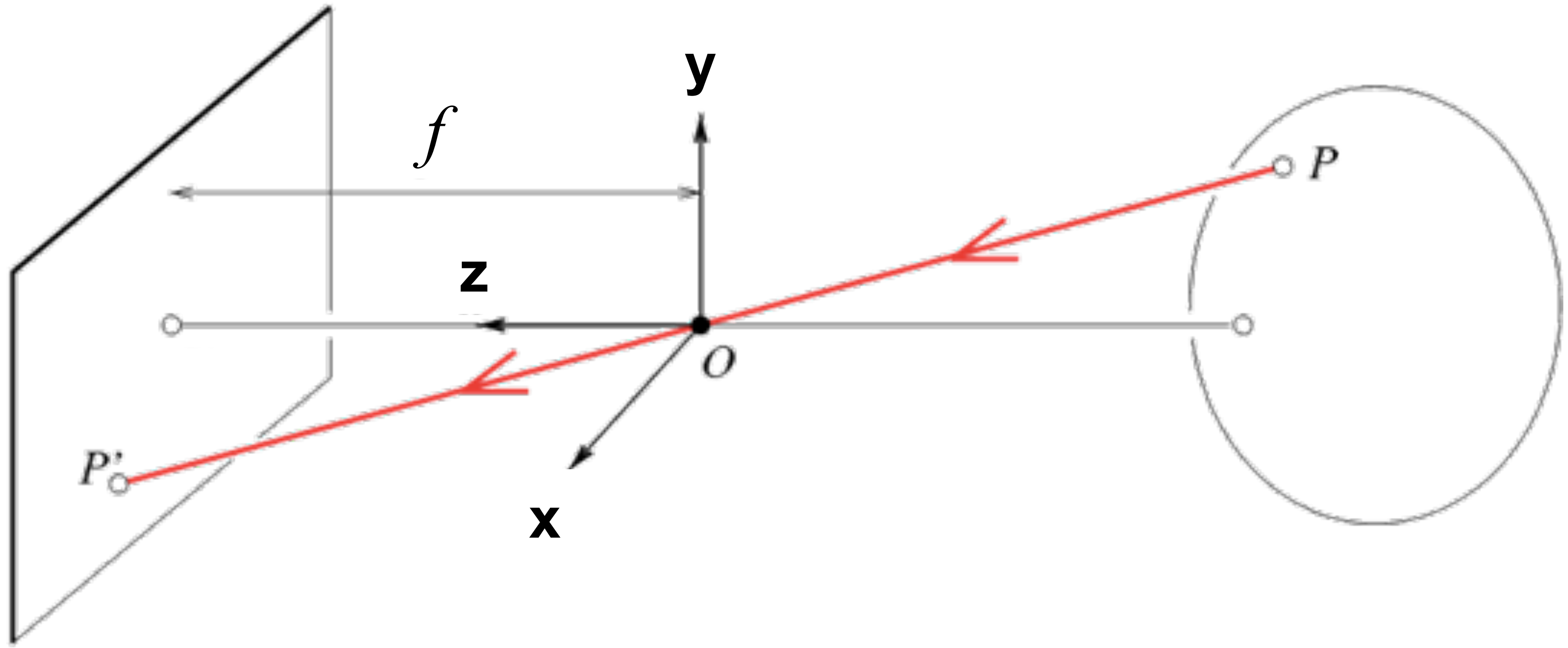
- **The coordinate system**

Modeling projection



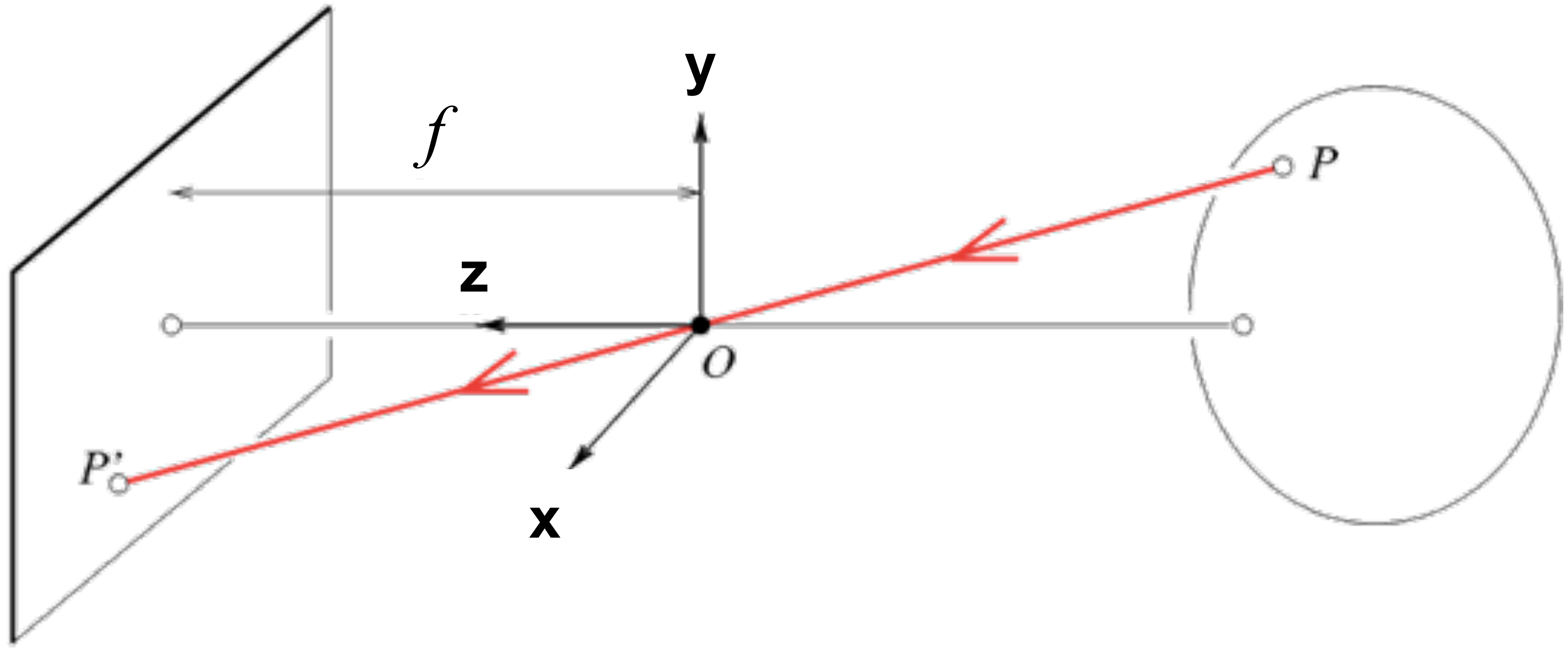
- **The coordinate system**
    - The optical center (***O***) is at the origin

Modeling projection



- **The coordinate system**
  - The optical center (***O***) is at the origin
  - The image plane is parallel to the xy-plane (perpendicular to the z axis)

Modeling projection



- **The coordinate system**
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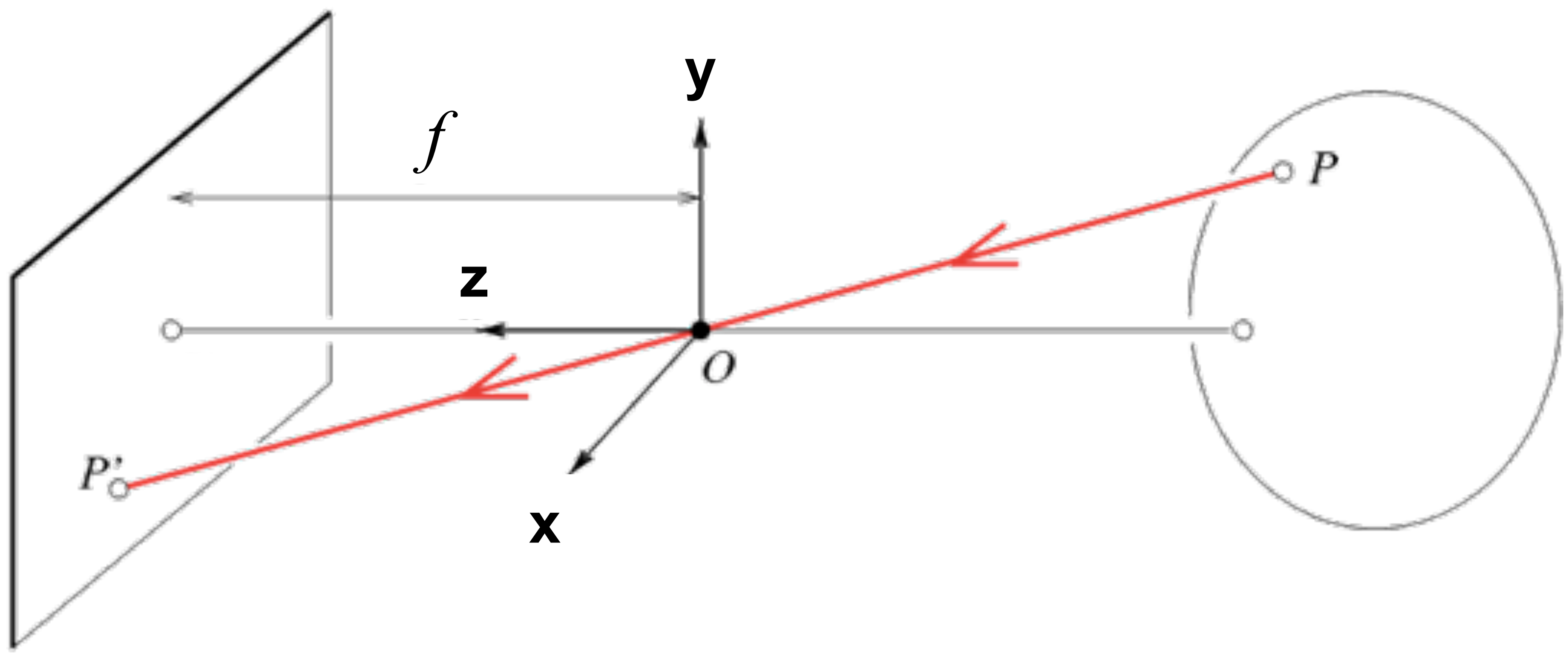
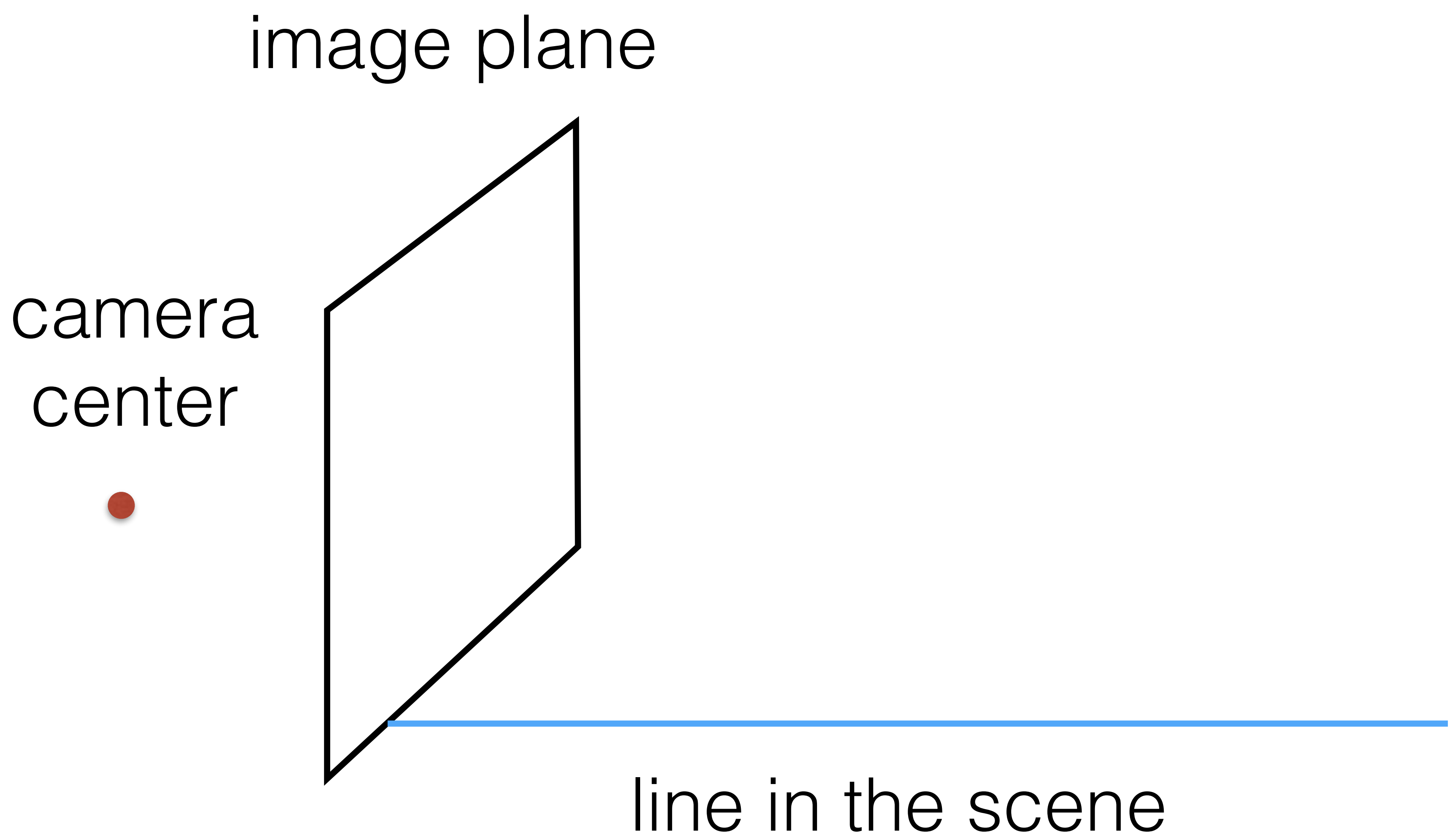
- **Projection equations**

Modeling projection



- **The coordinate system**
  - The optical center (***O***) is at the origin
  - The image plane is parallel to the xy-plane (perpendicular to the z axis)
- **Projection equations**
  - Derive using similar triangles $(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$

Projection of a line



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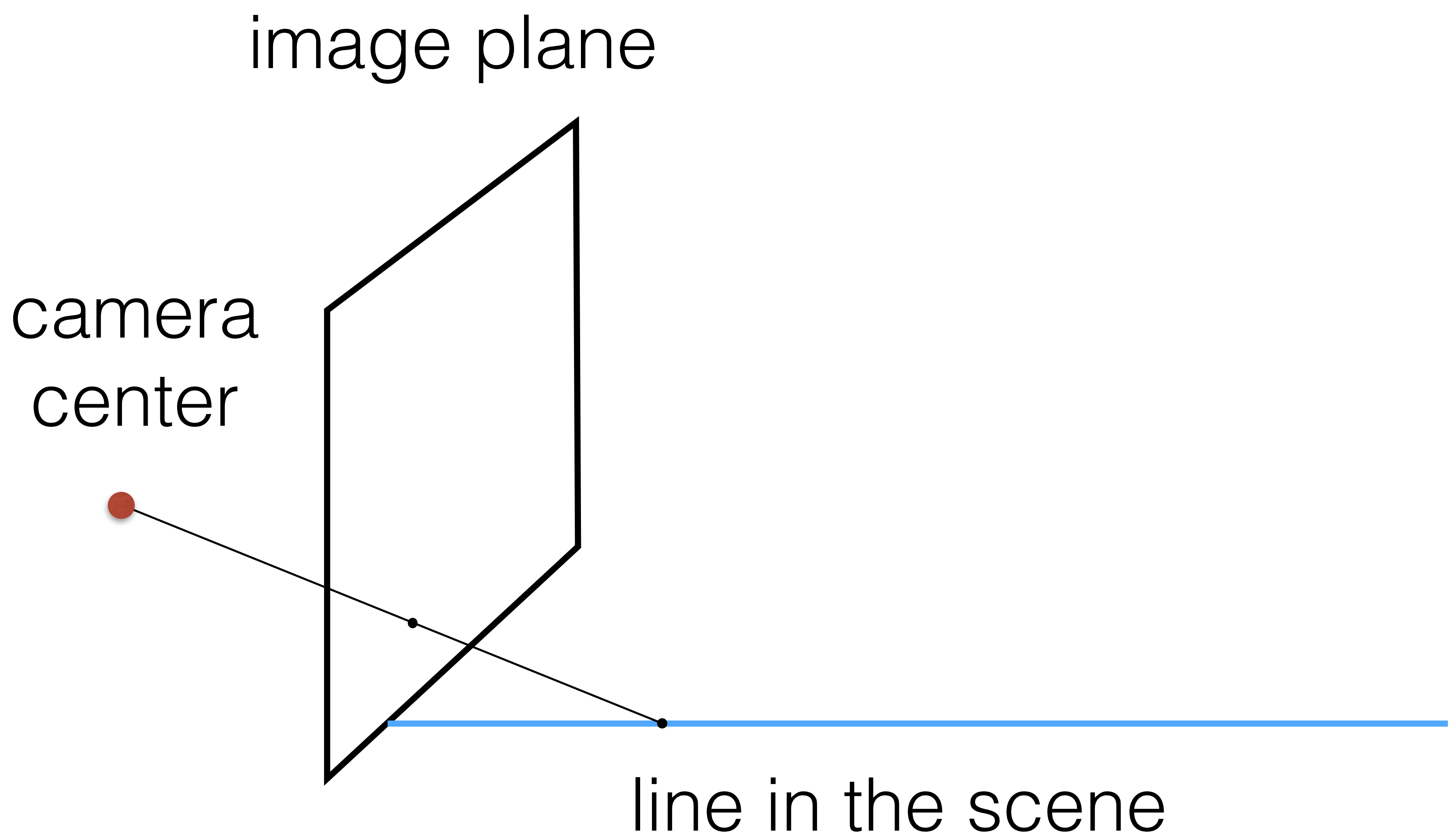

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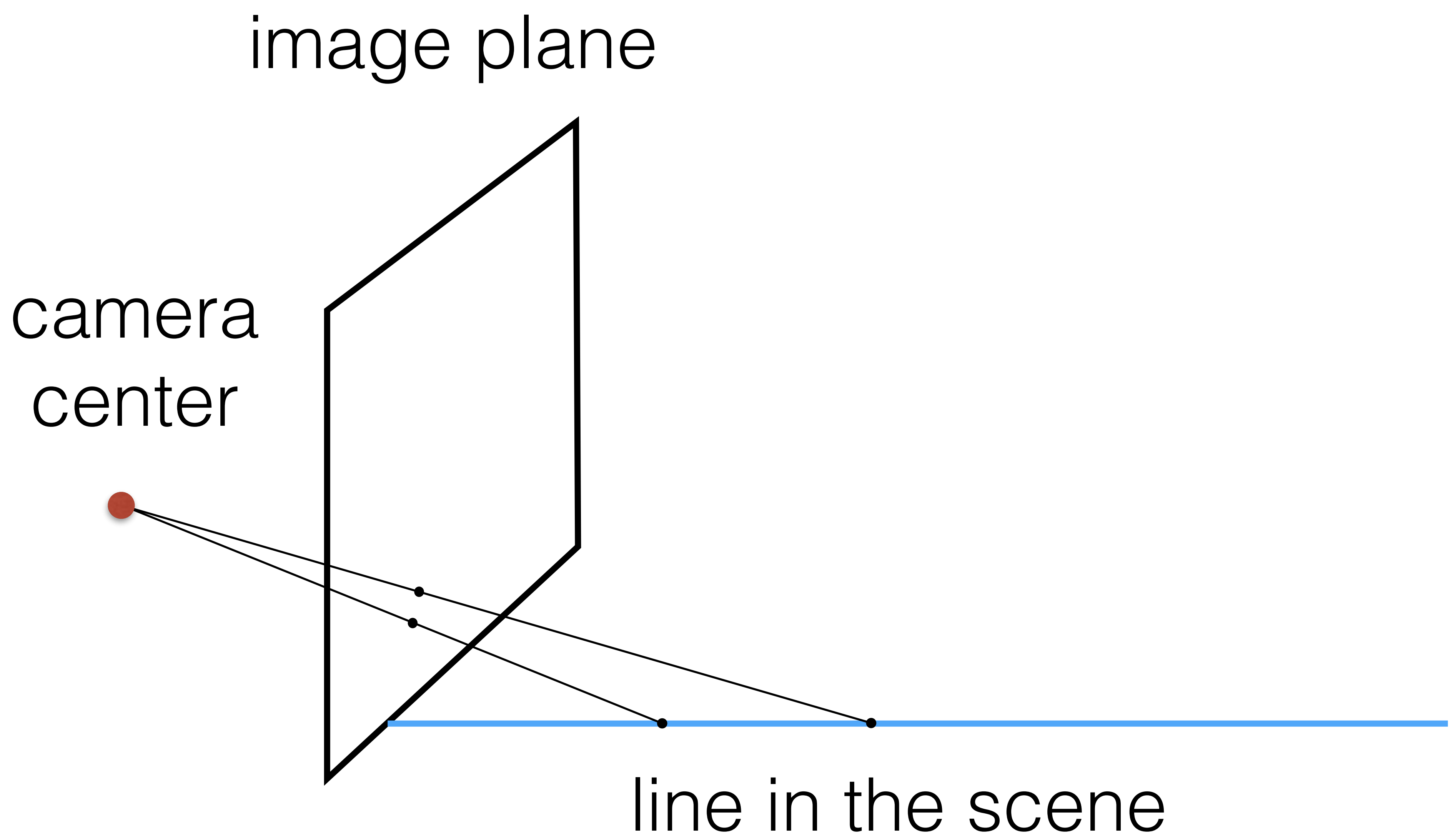

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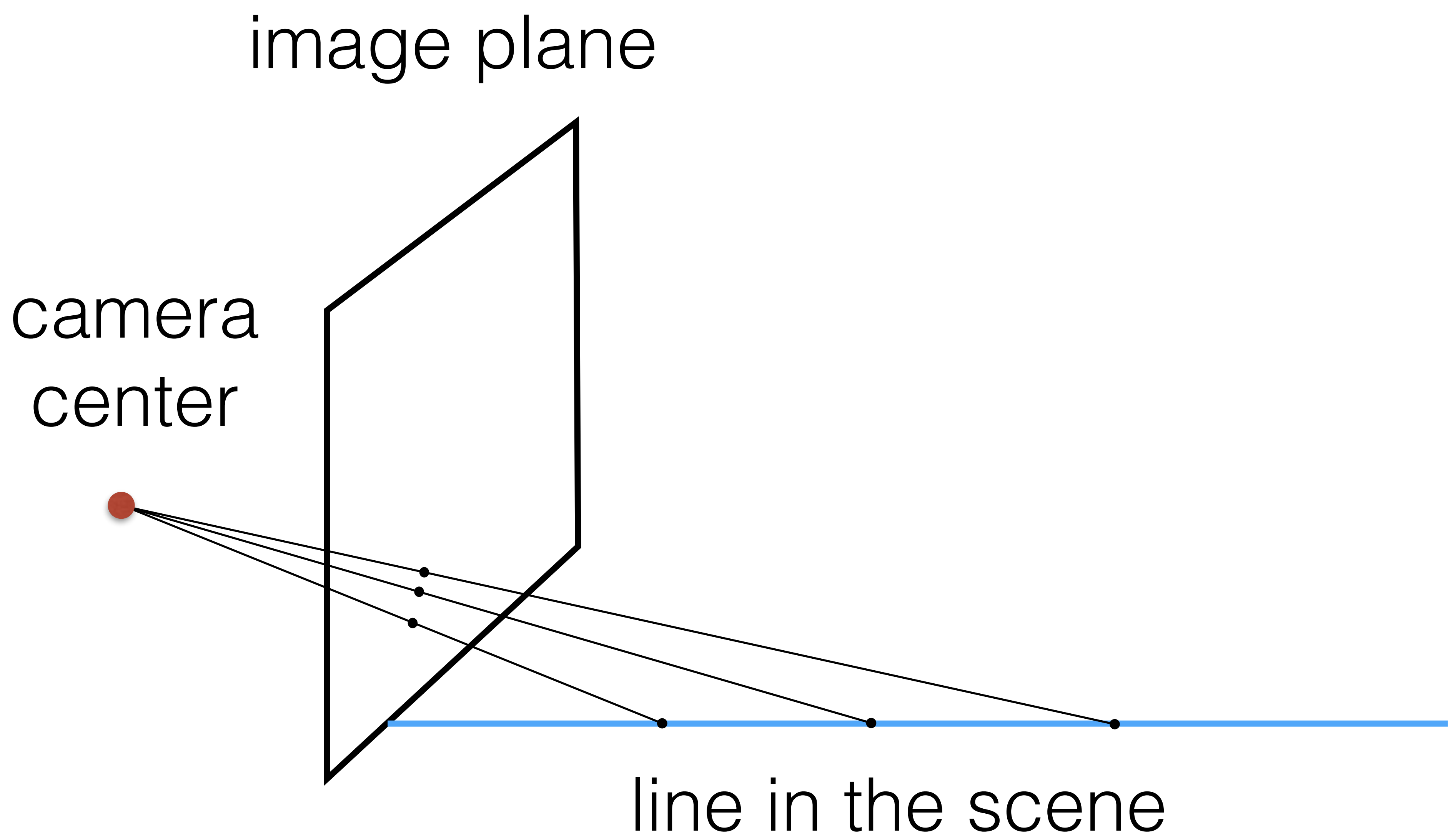

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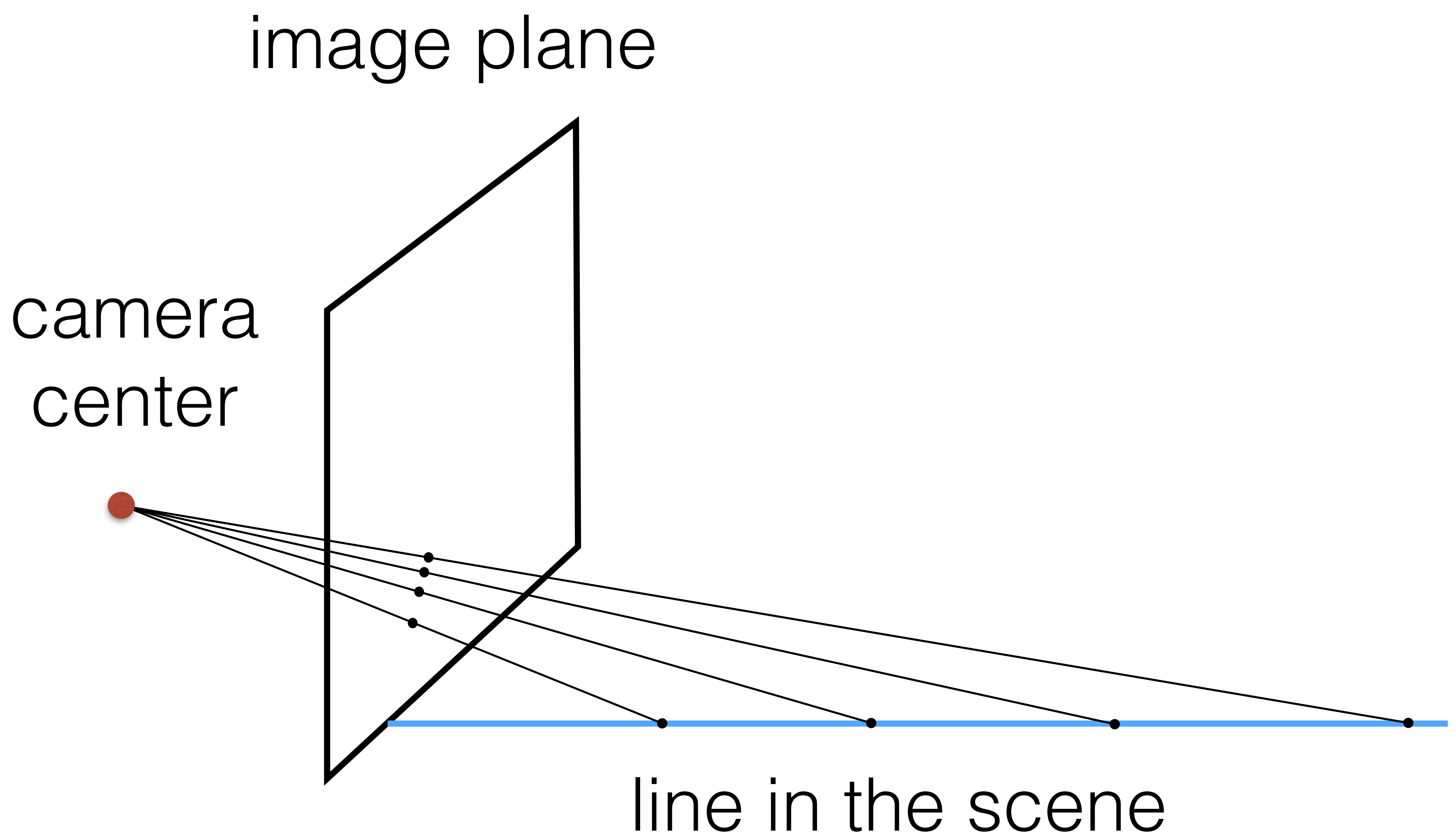

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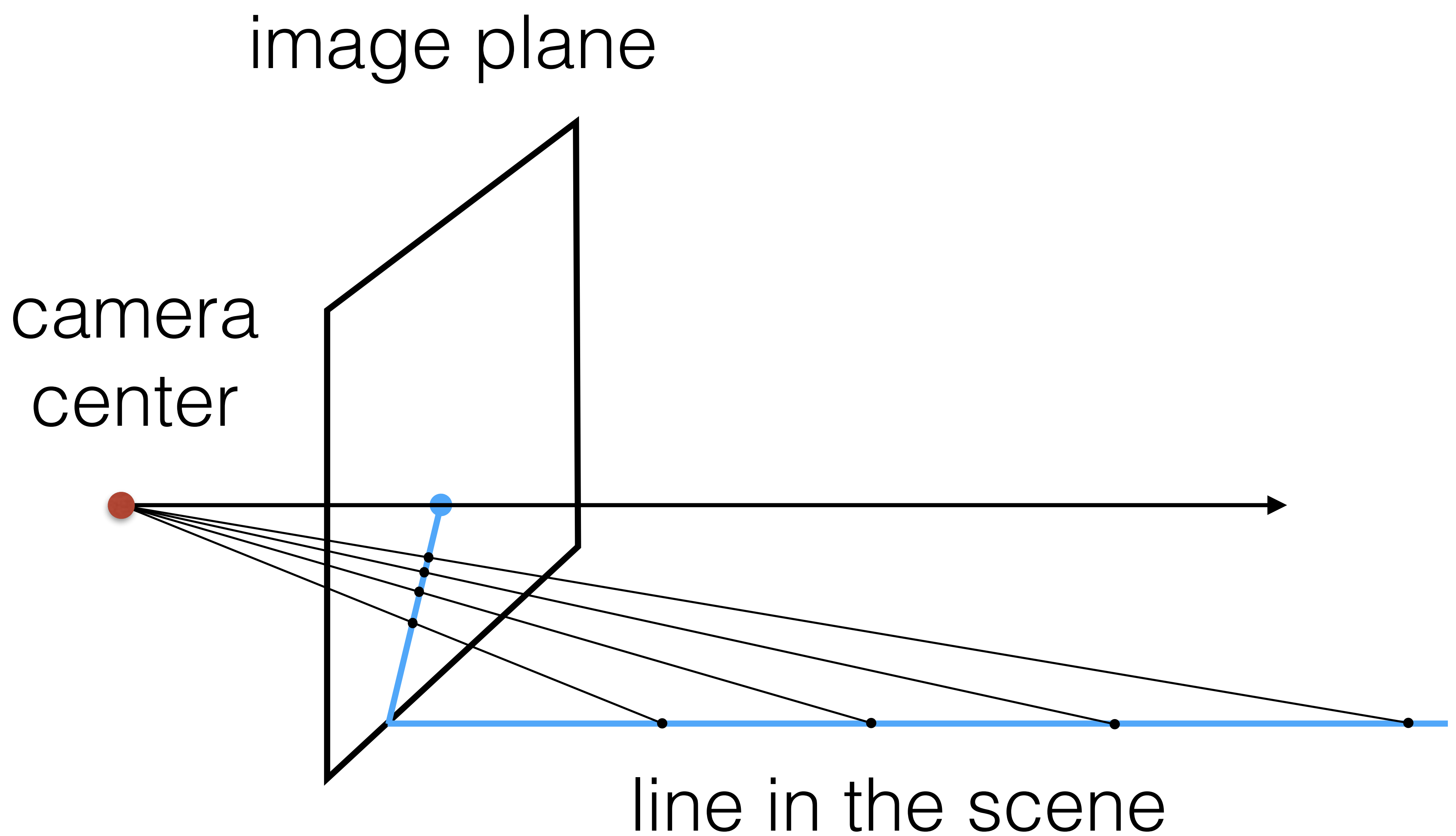

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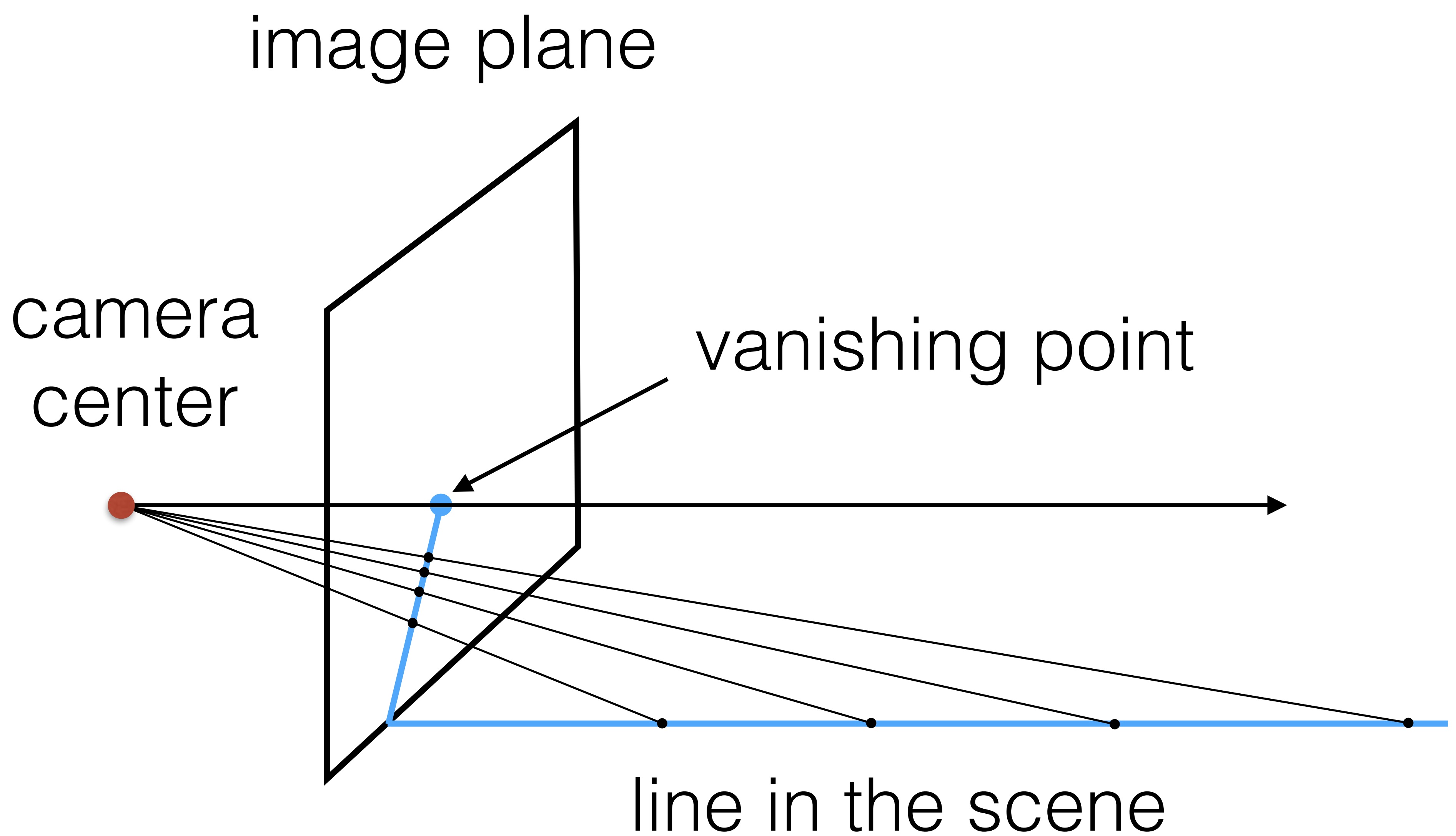

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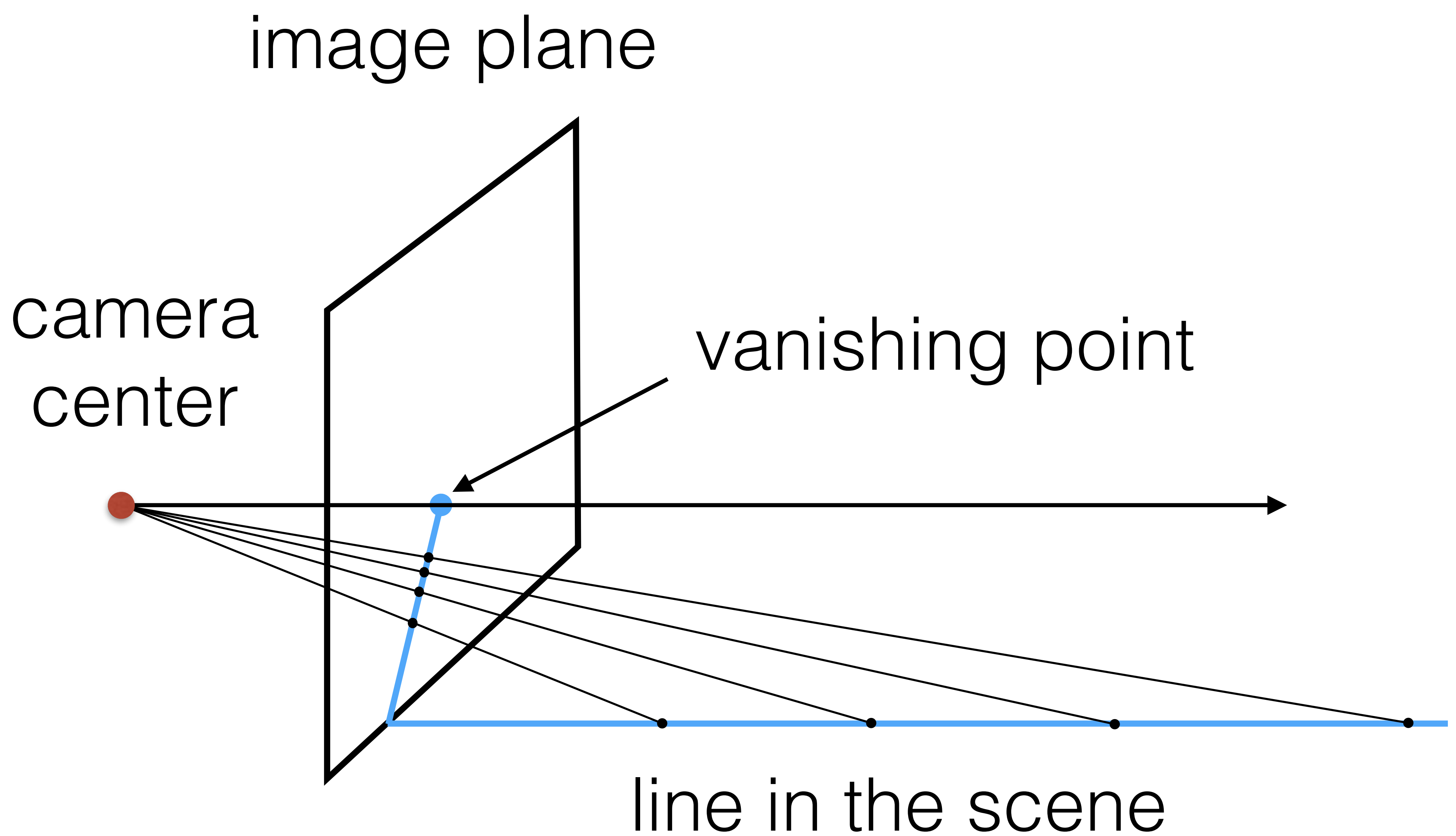
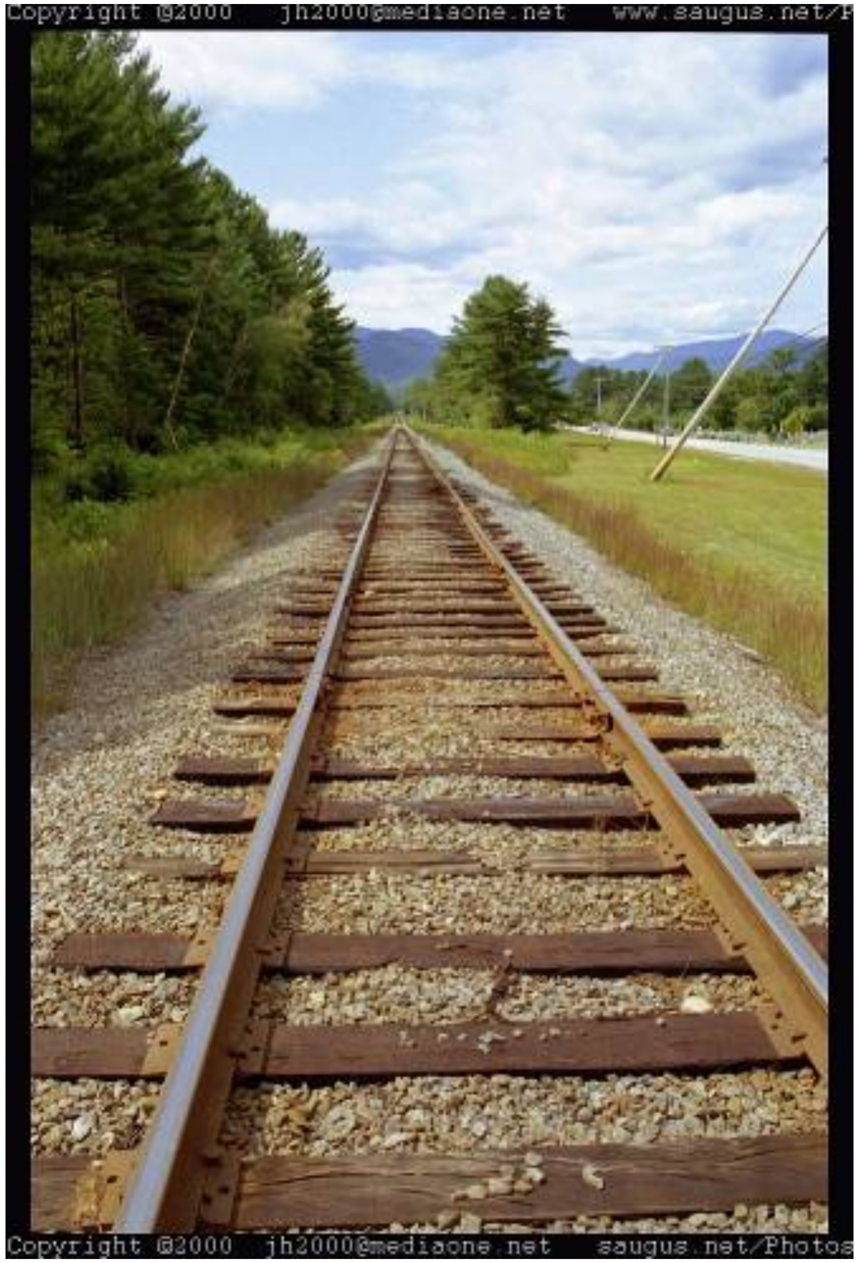


- What if we add another line parallel to the first one?

Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in the that direction converge at that point



Vanishing points

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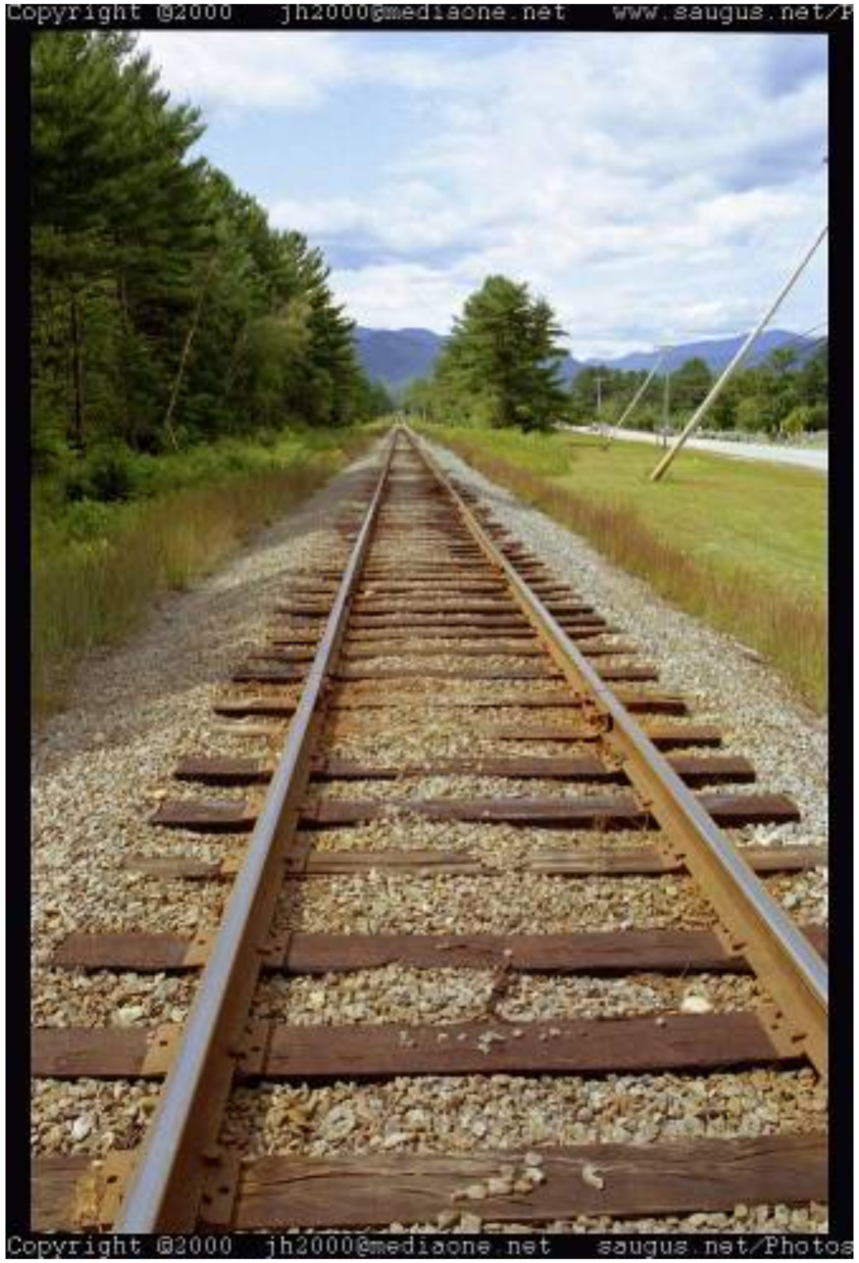
  - **Exception**: directions that are parallel to the image plane



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- What about the vanishing point of a plane?

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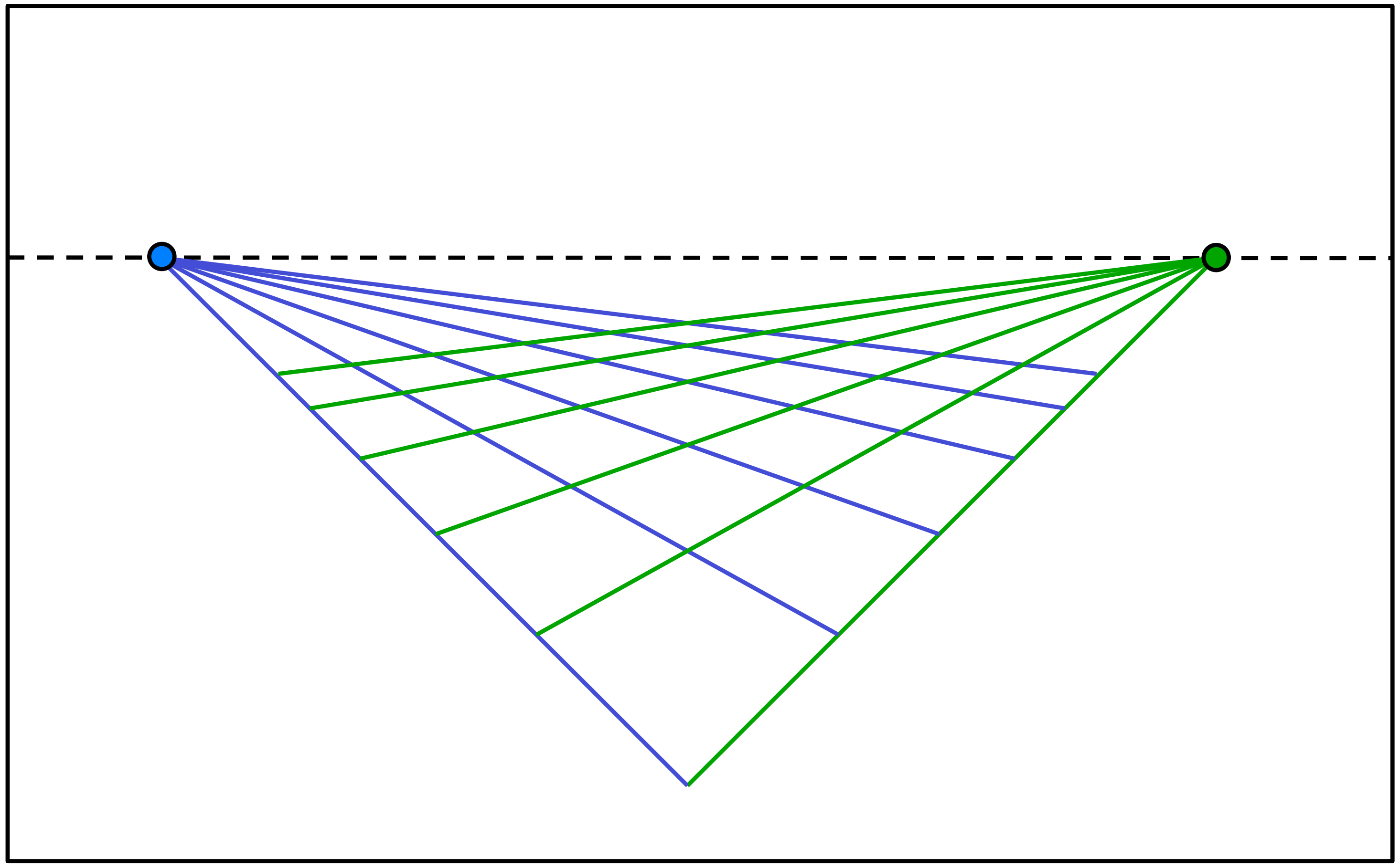
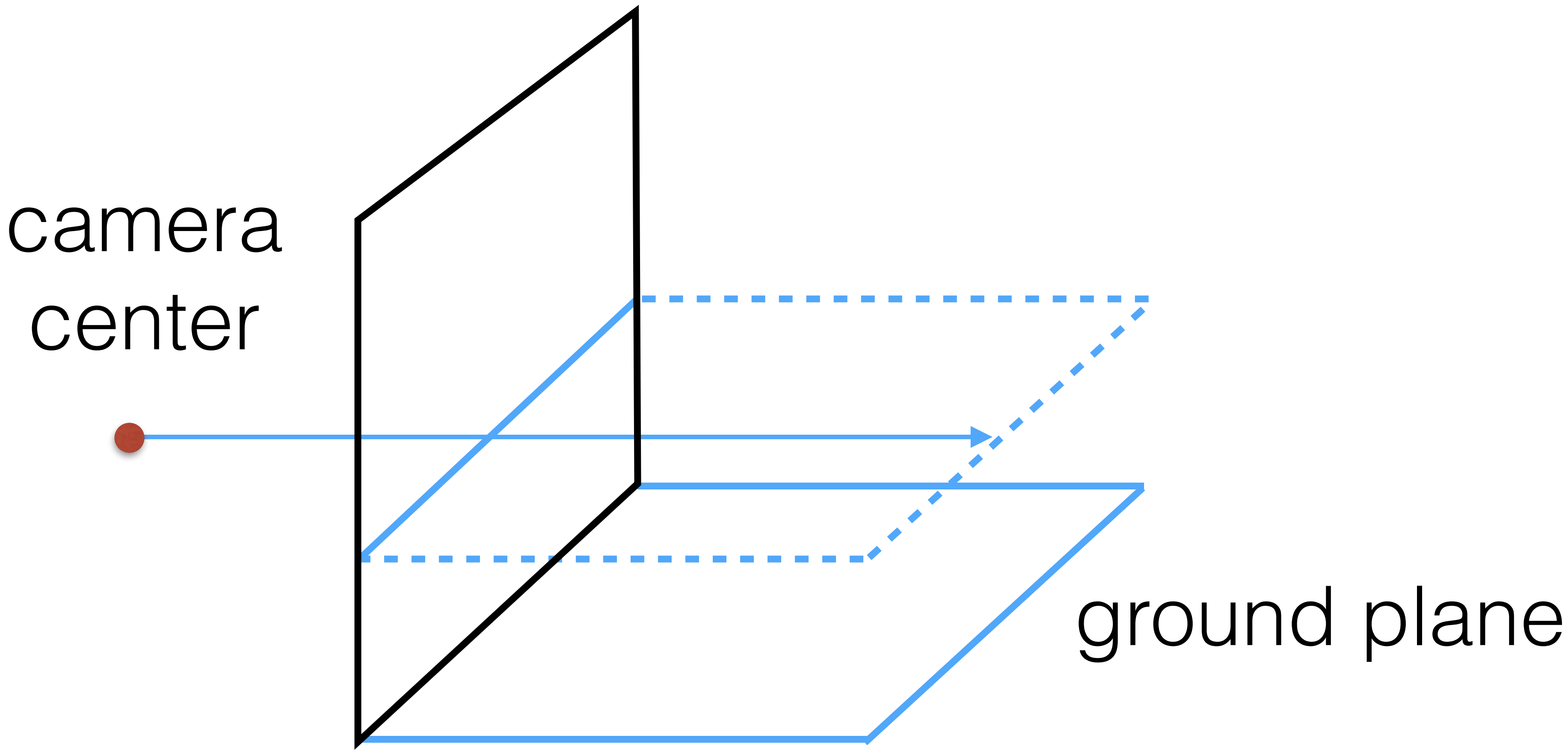


The horizon

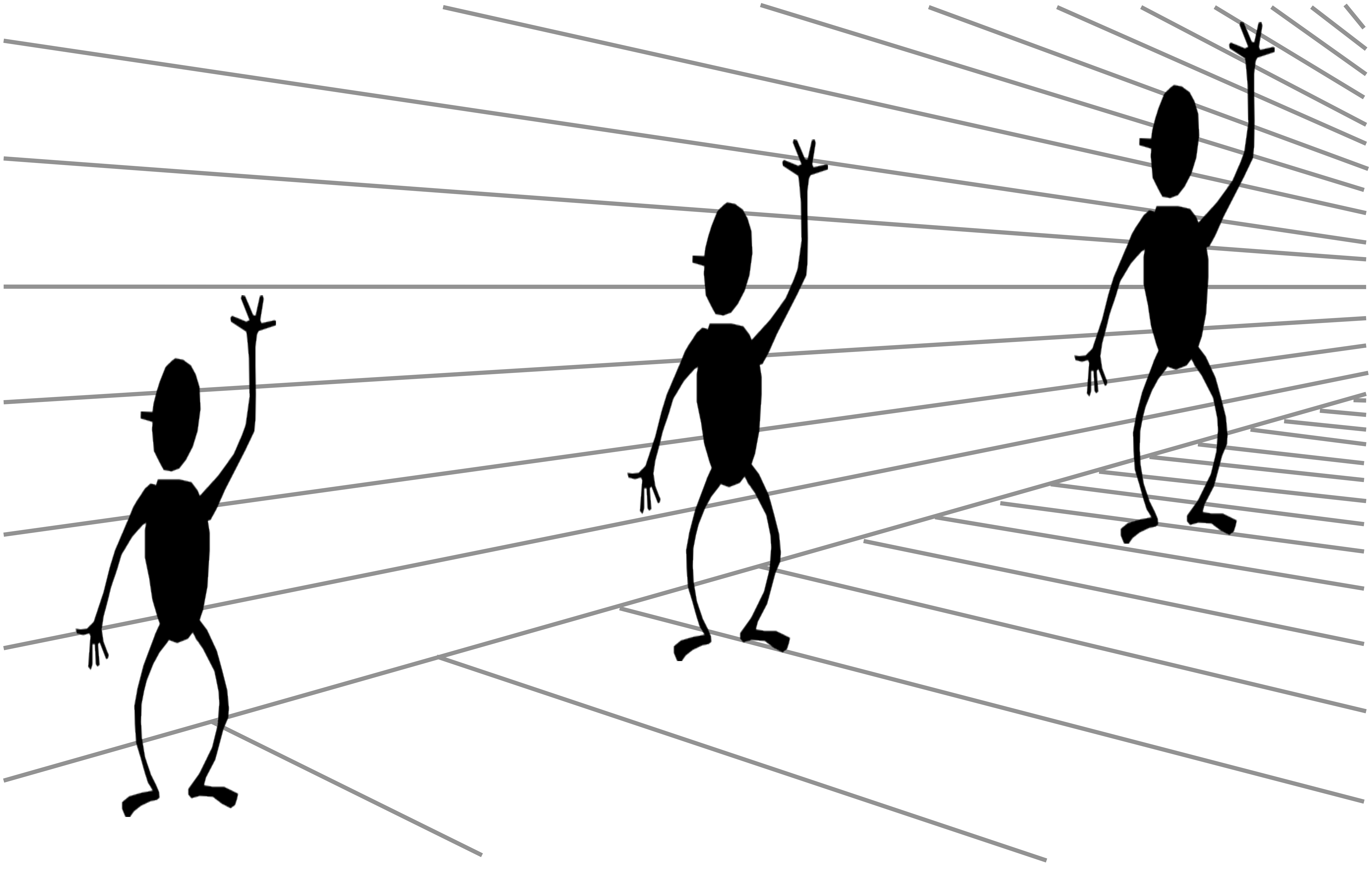


- Vanishing line of the ground plane
  - All points at the same height of the camera project to the horizon
  - Points above the camera project above the horizon
  - Provides a way of comparing heights of objects

The horizon

Is the person above or below the viewer?

Perspective cues



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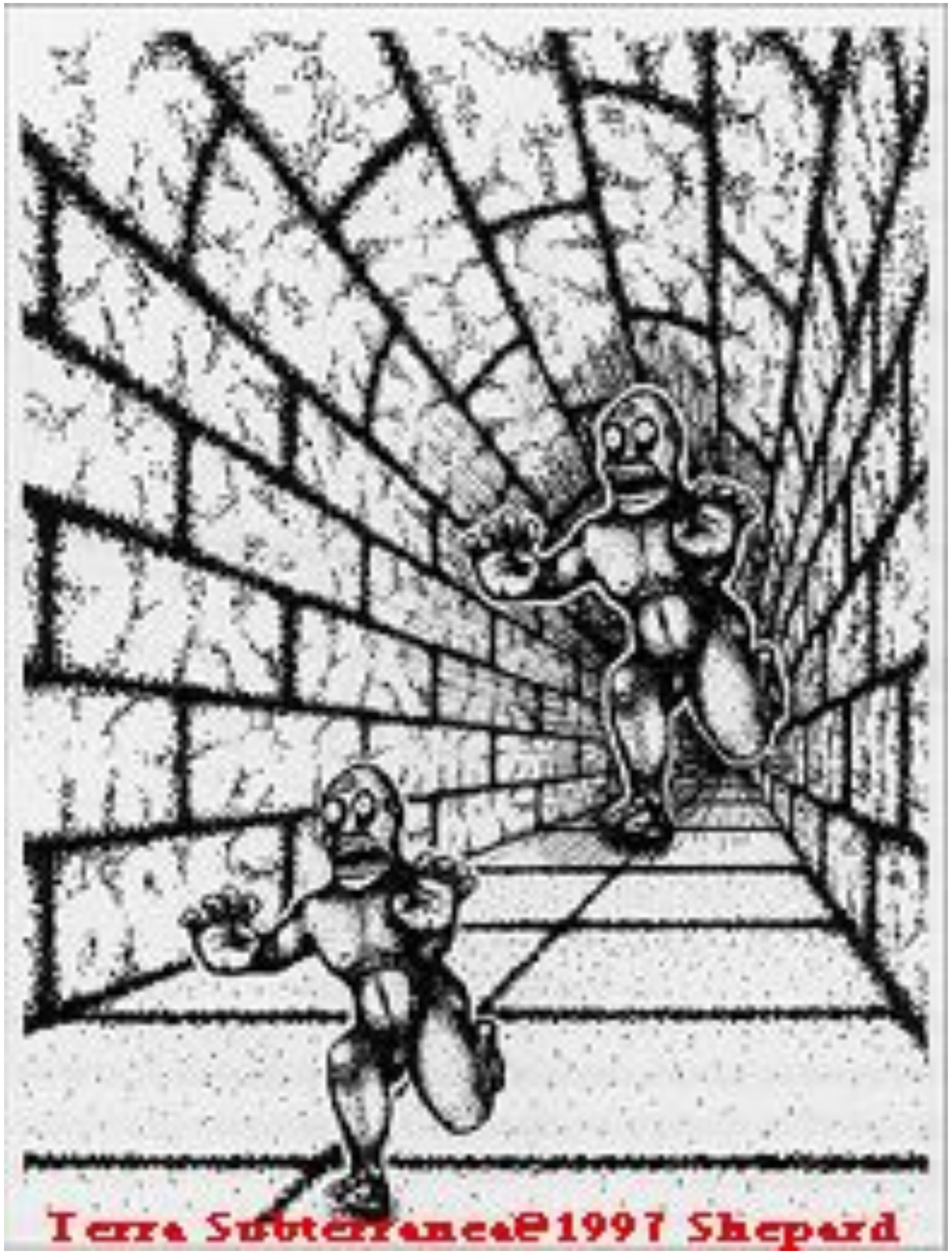

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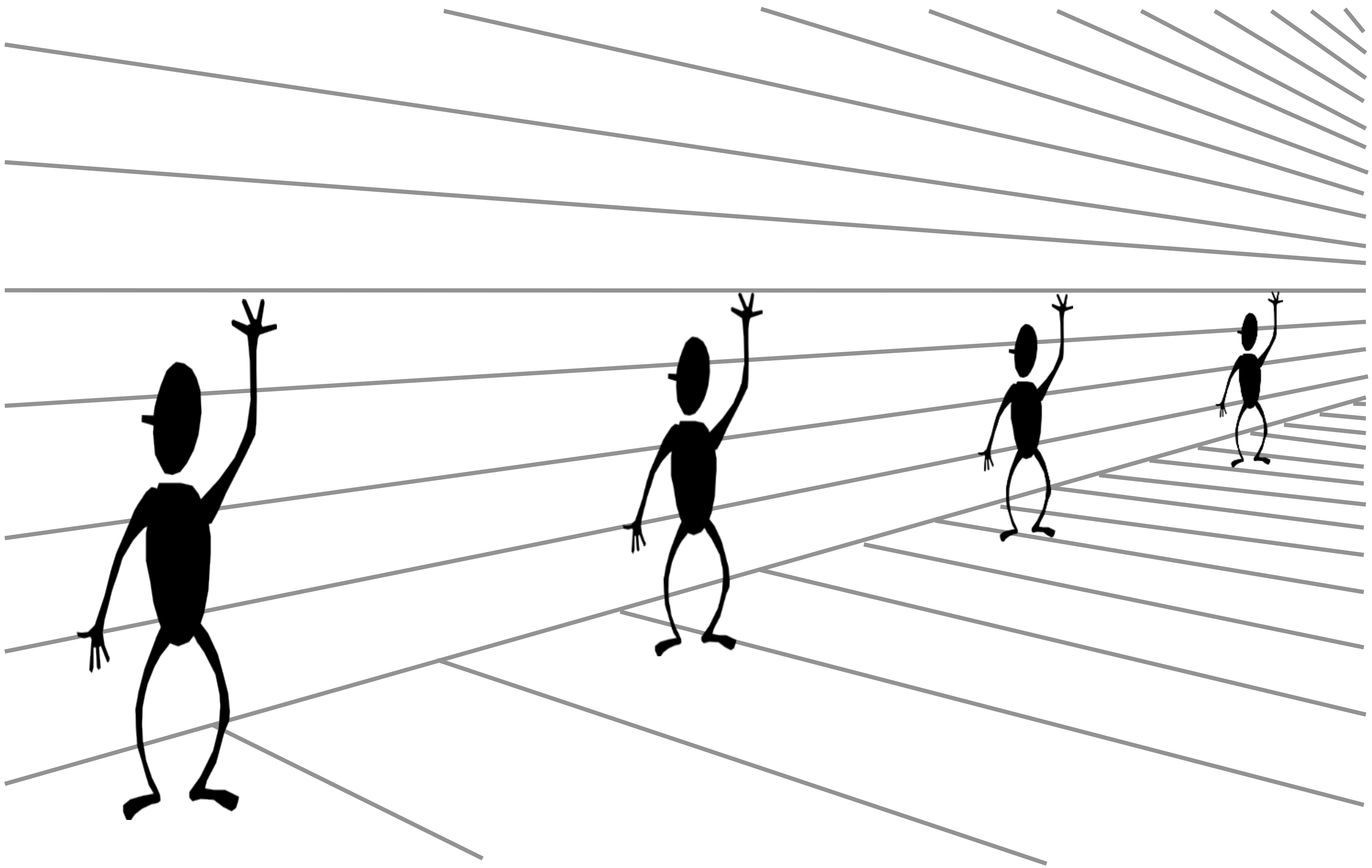
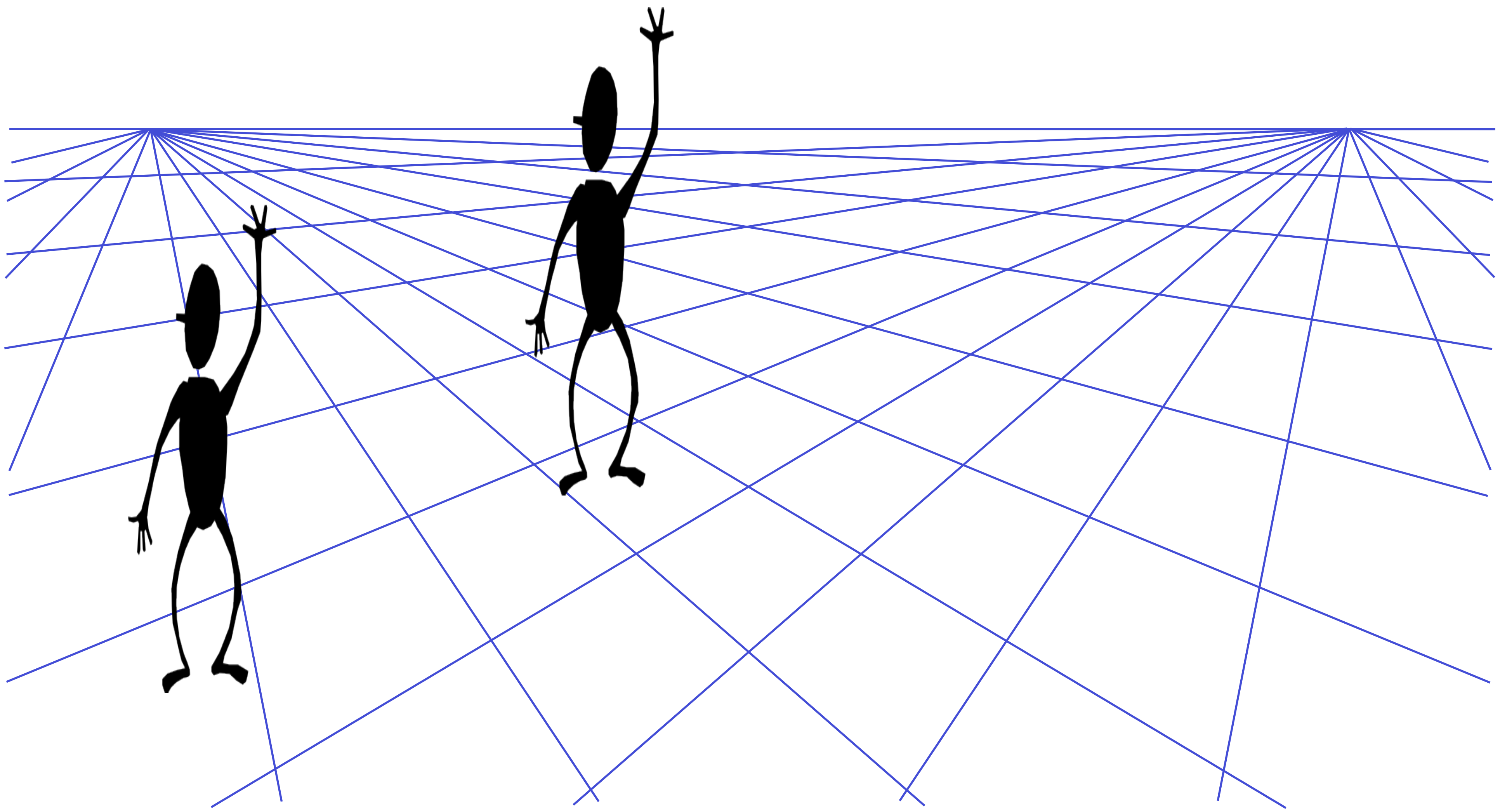


Comparing heights



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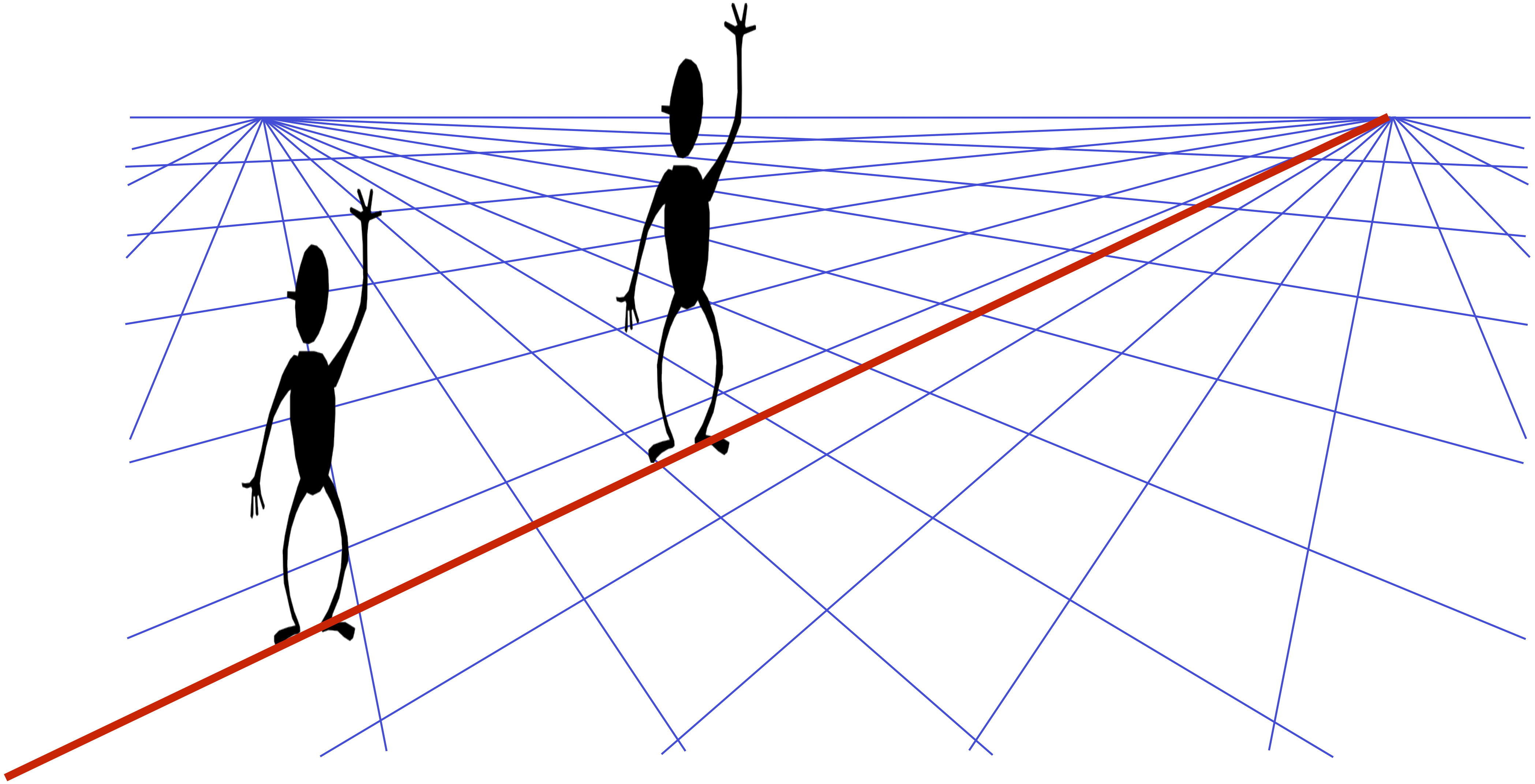

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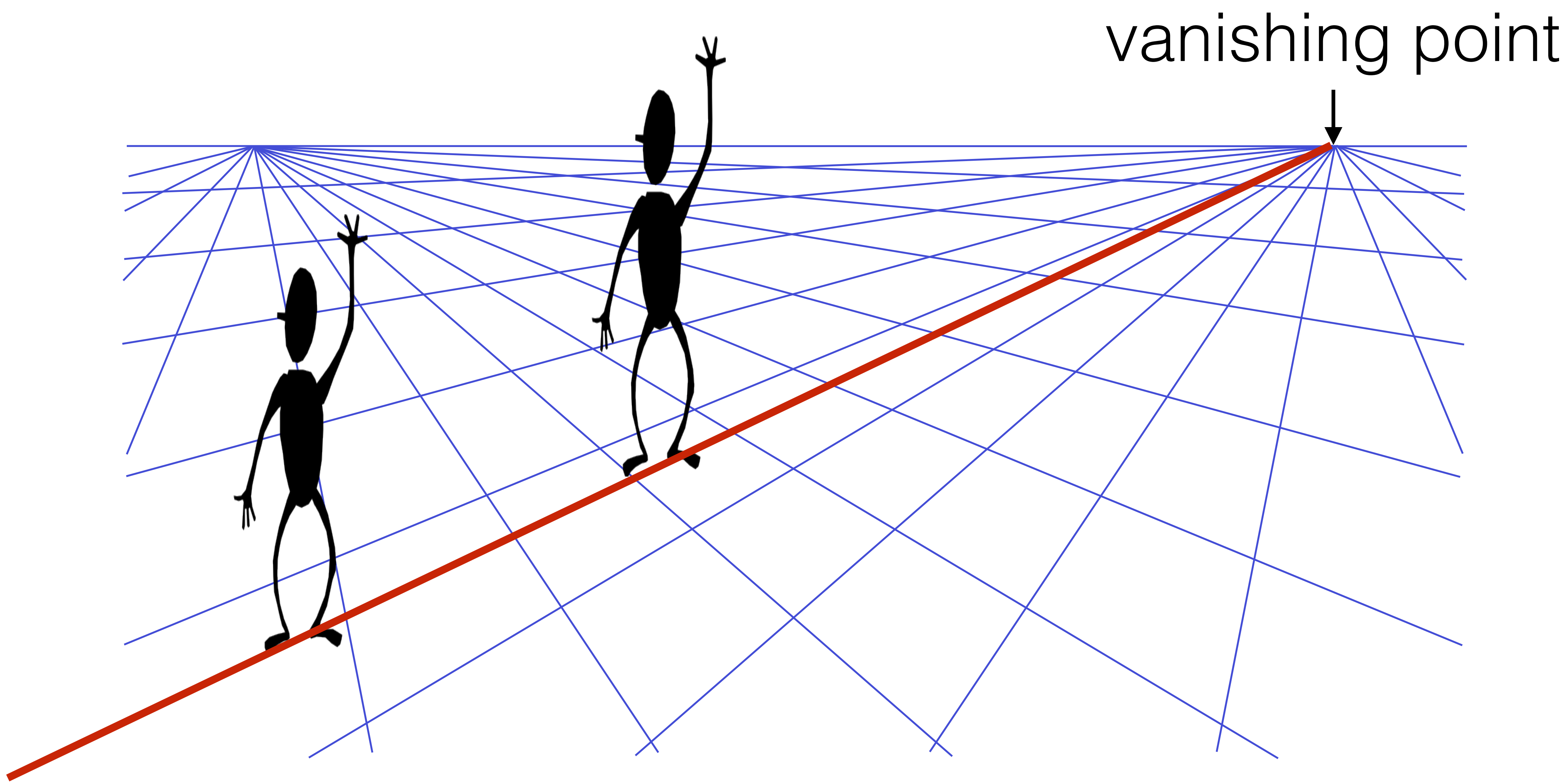

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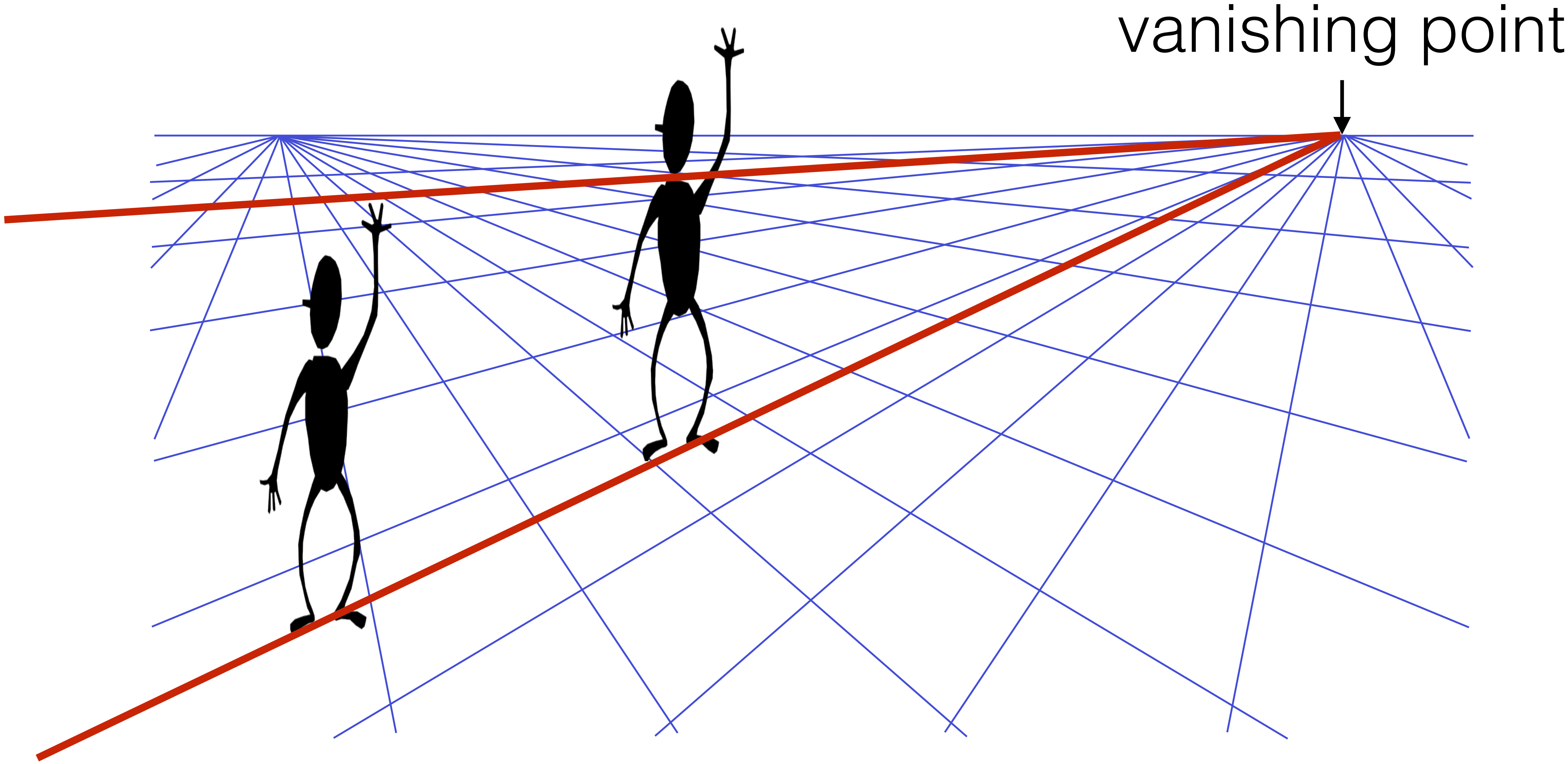
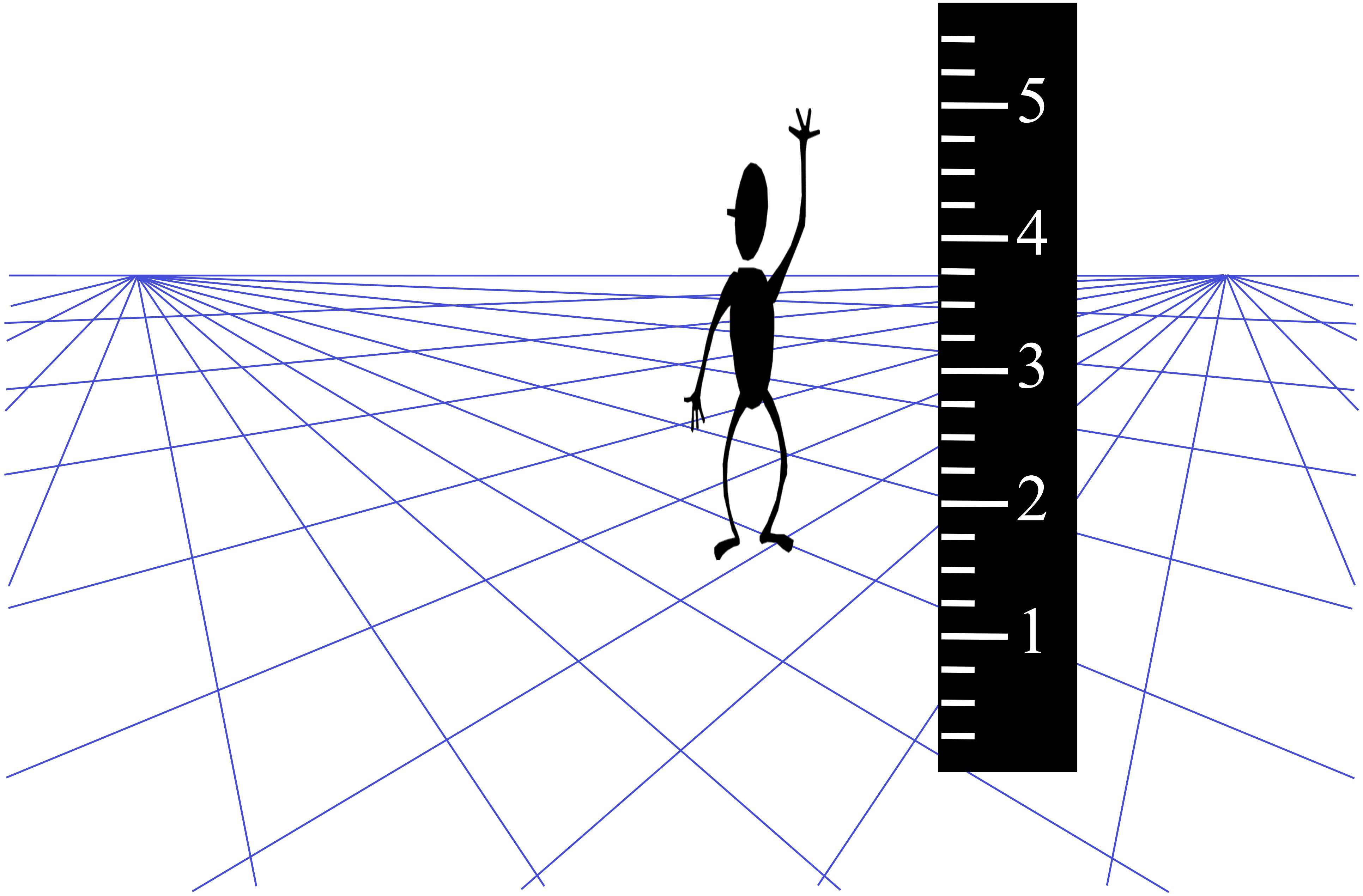


Measuring heights



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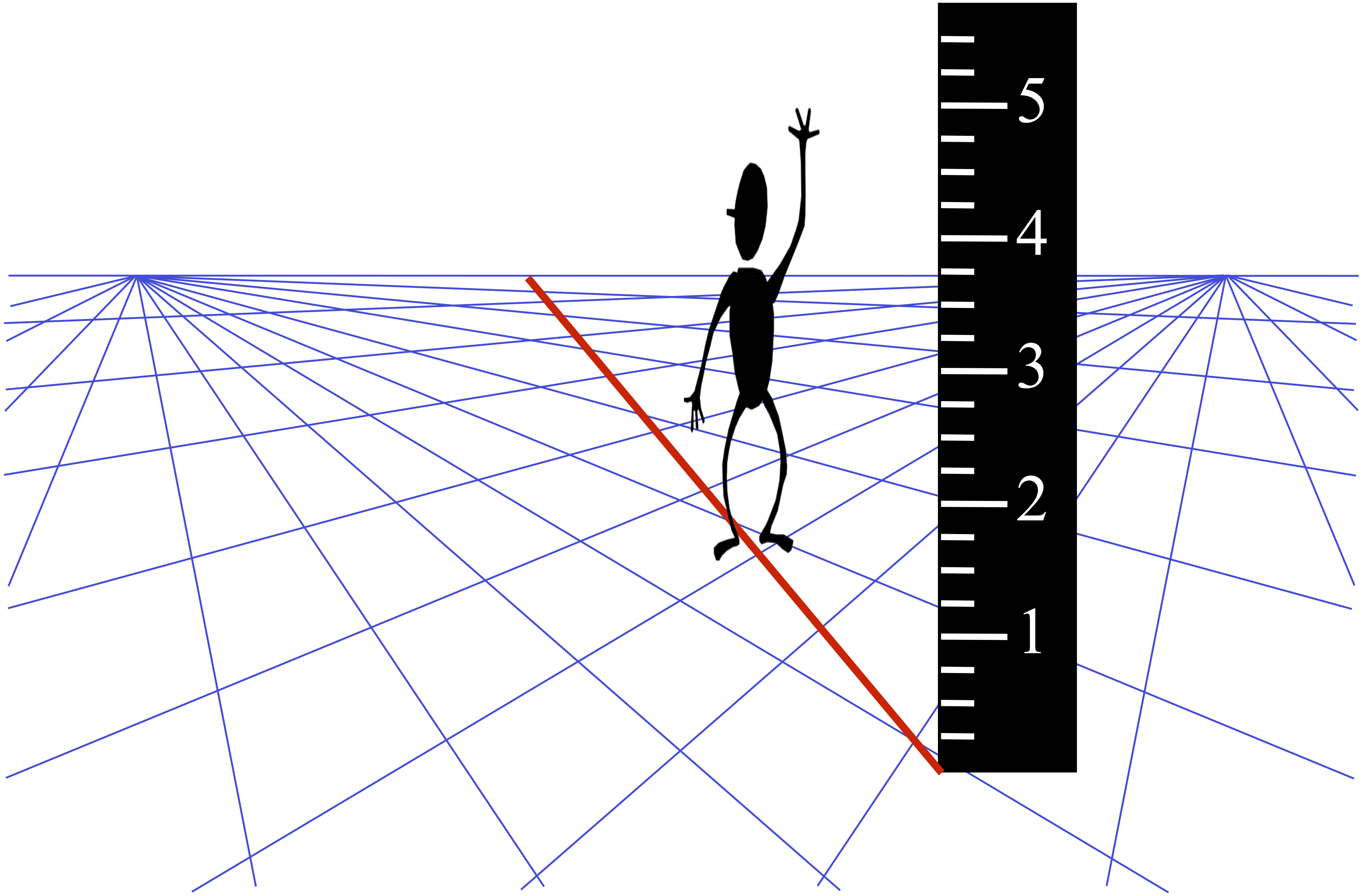

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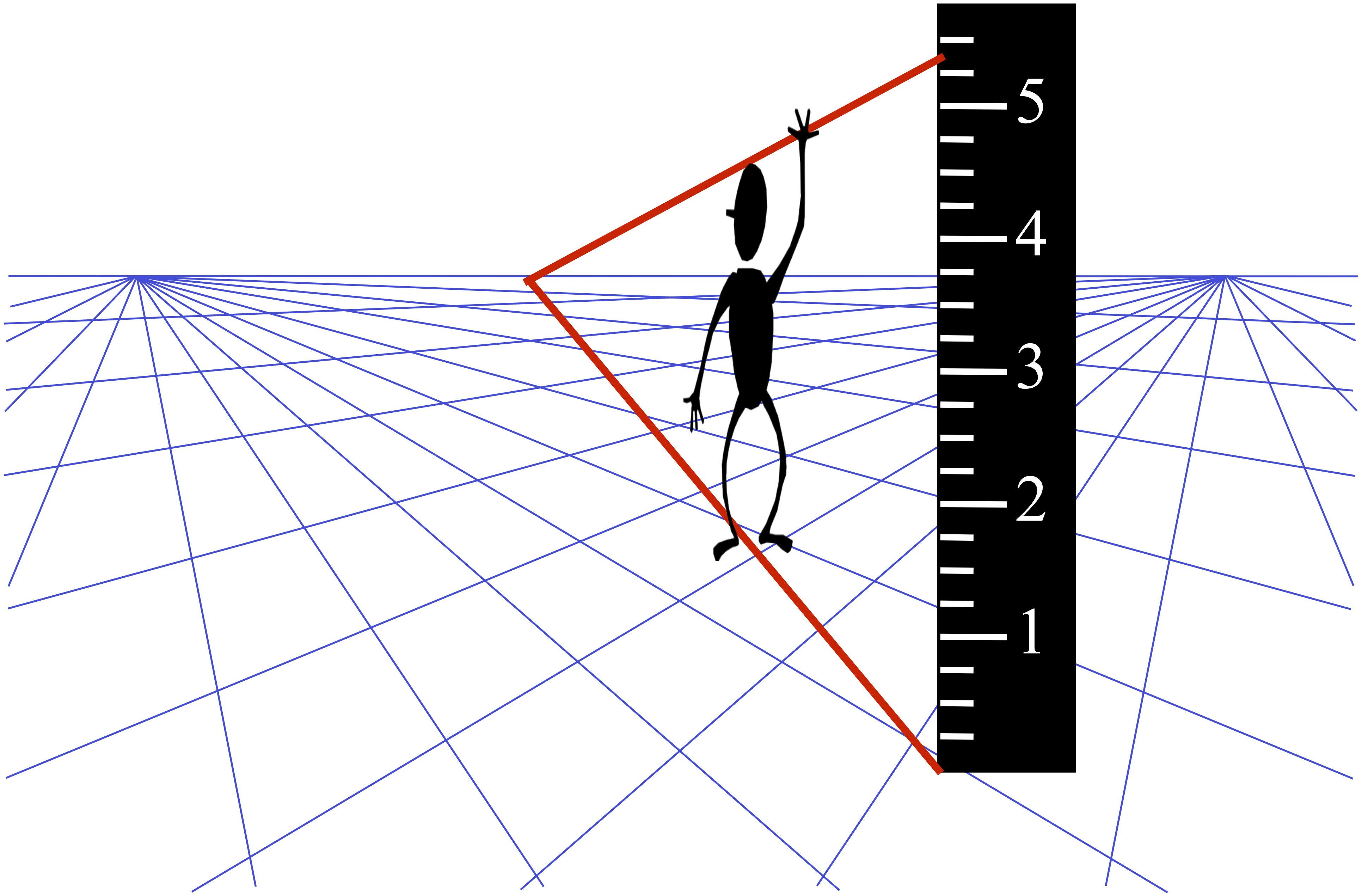

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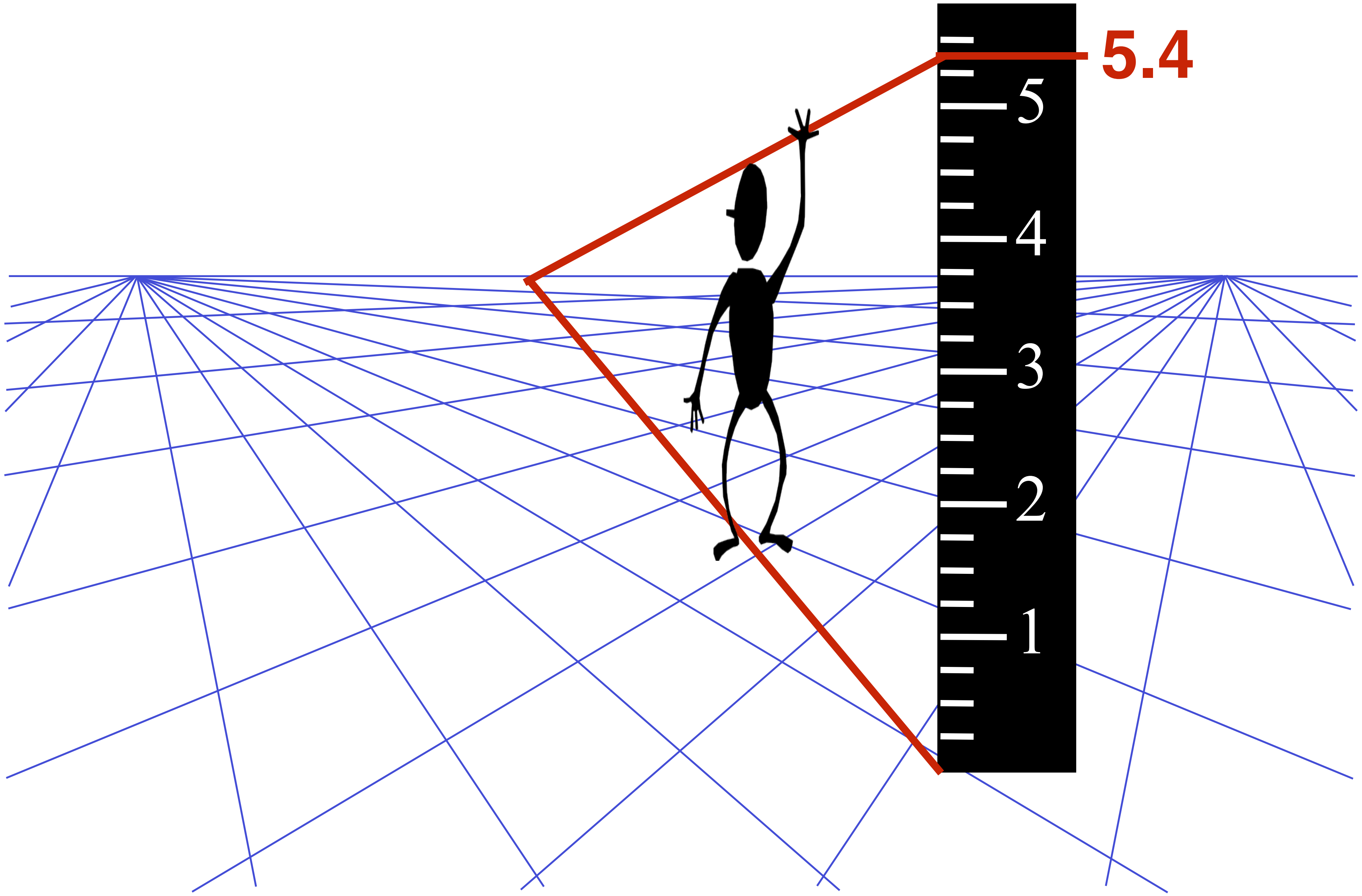

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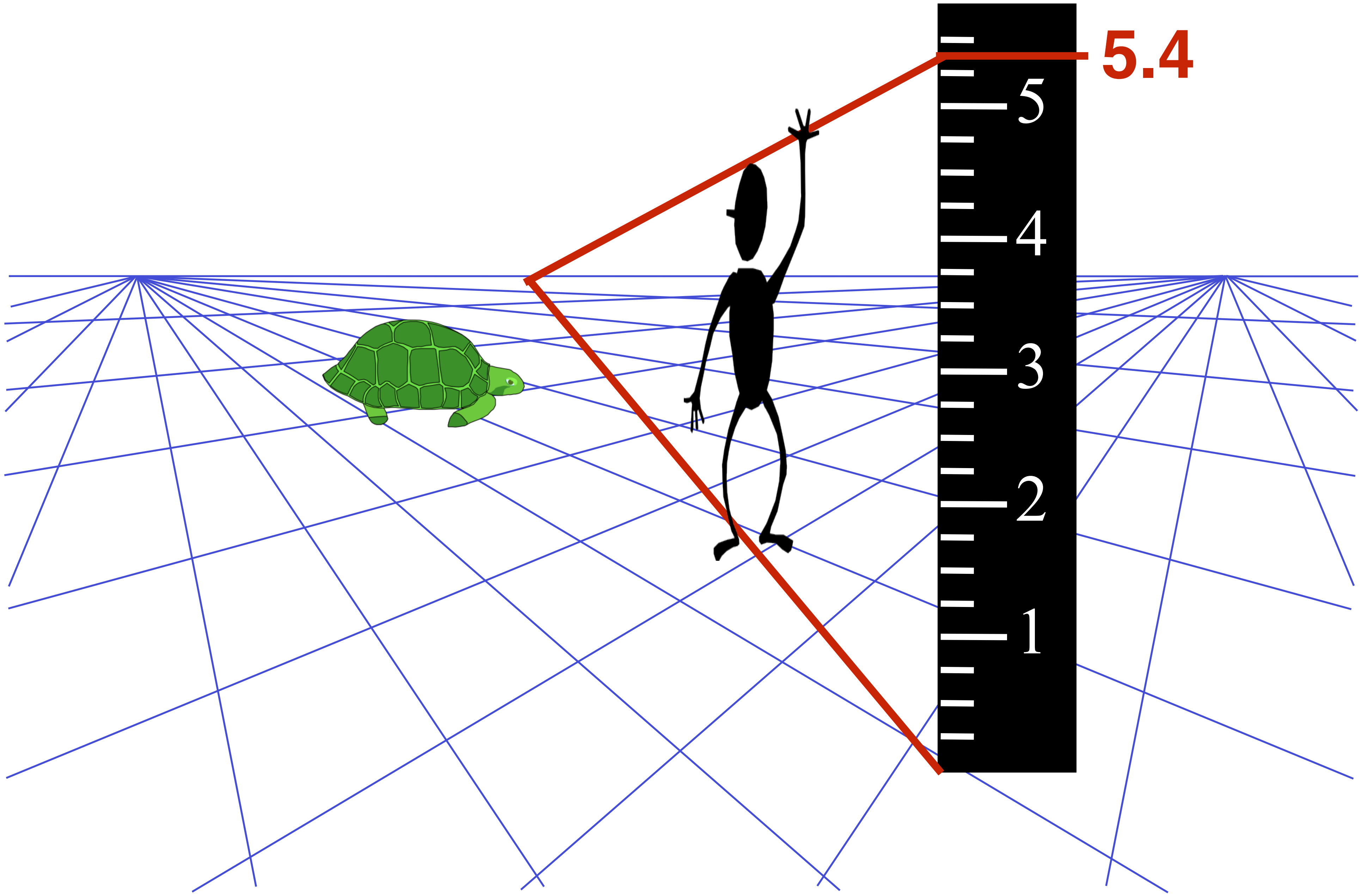

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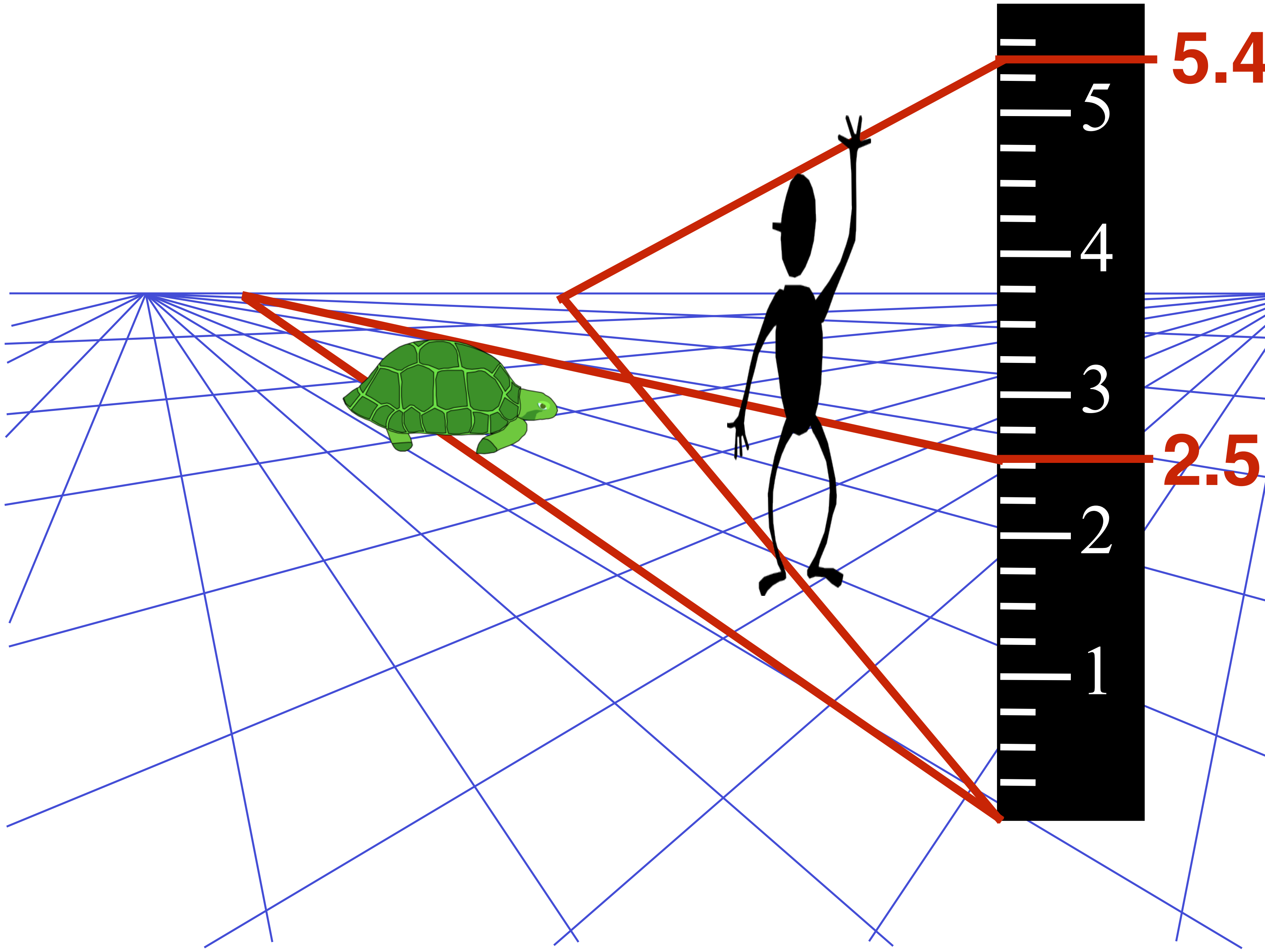

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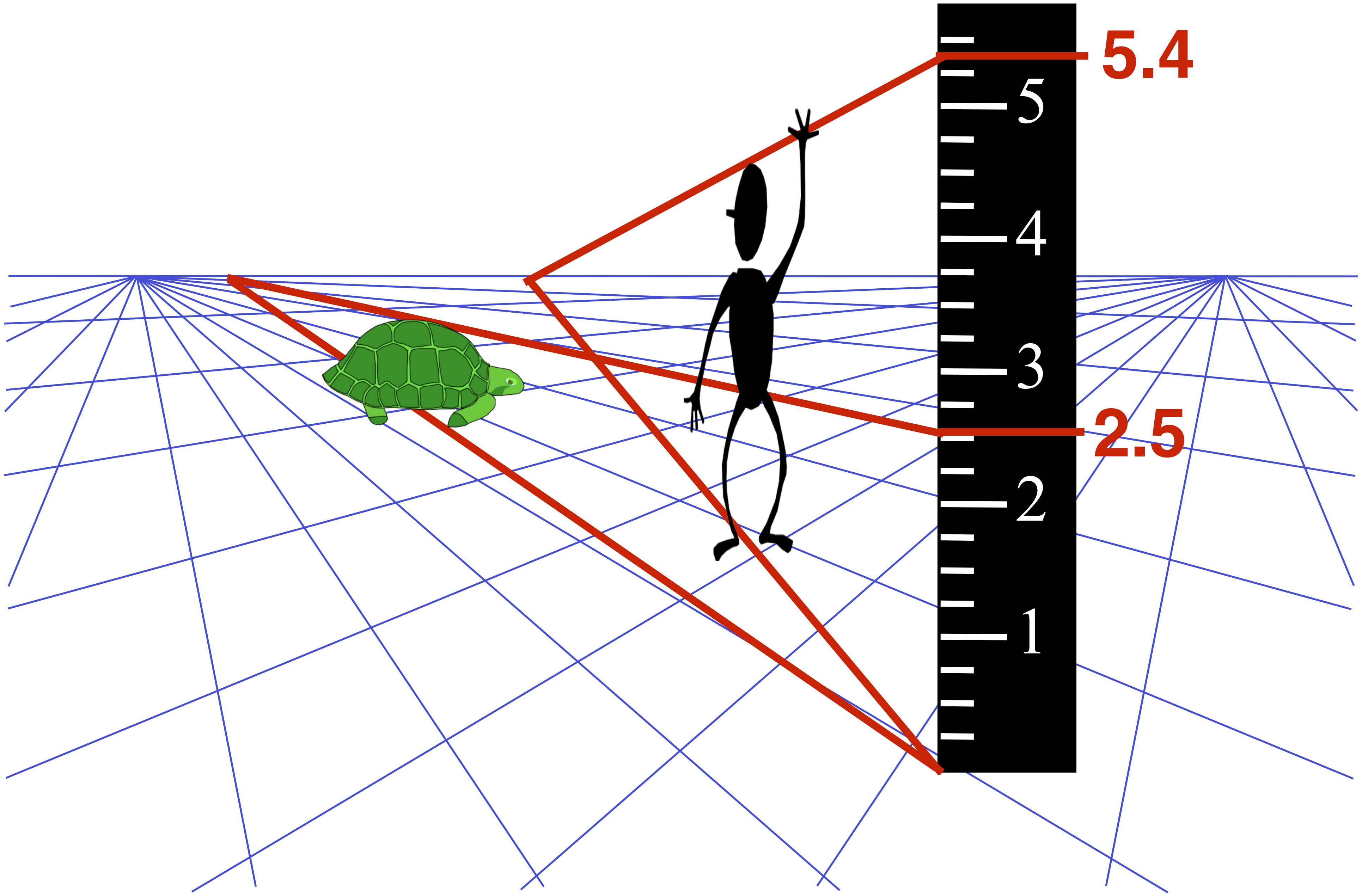
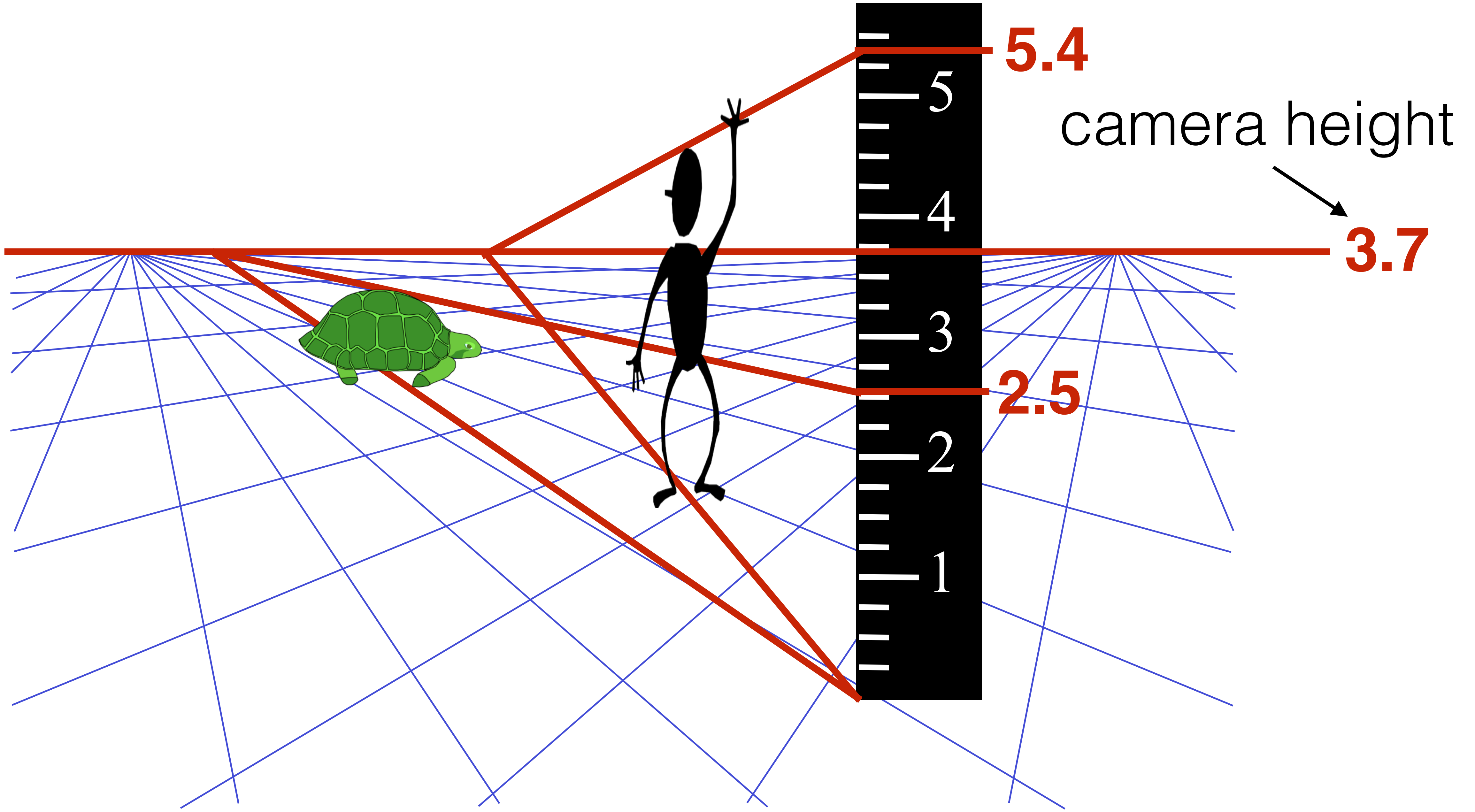


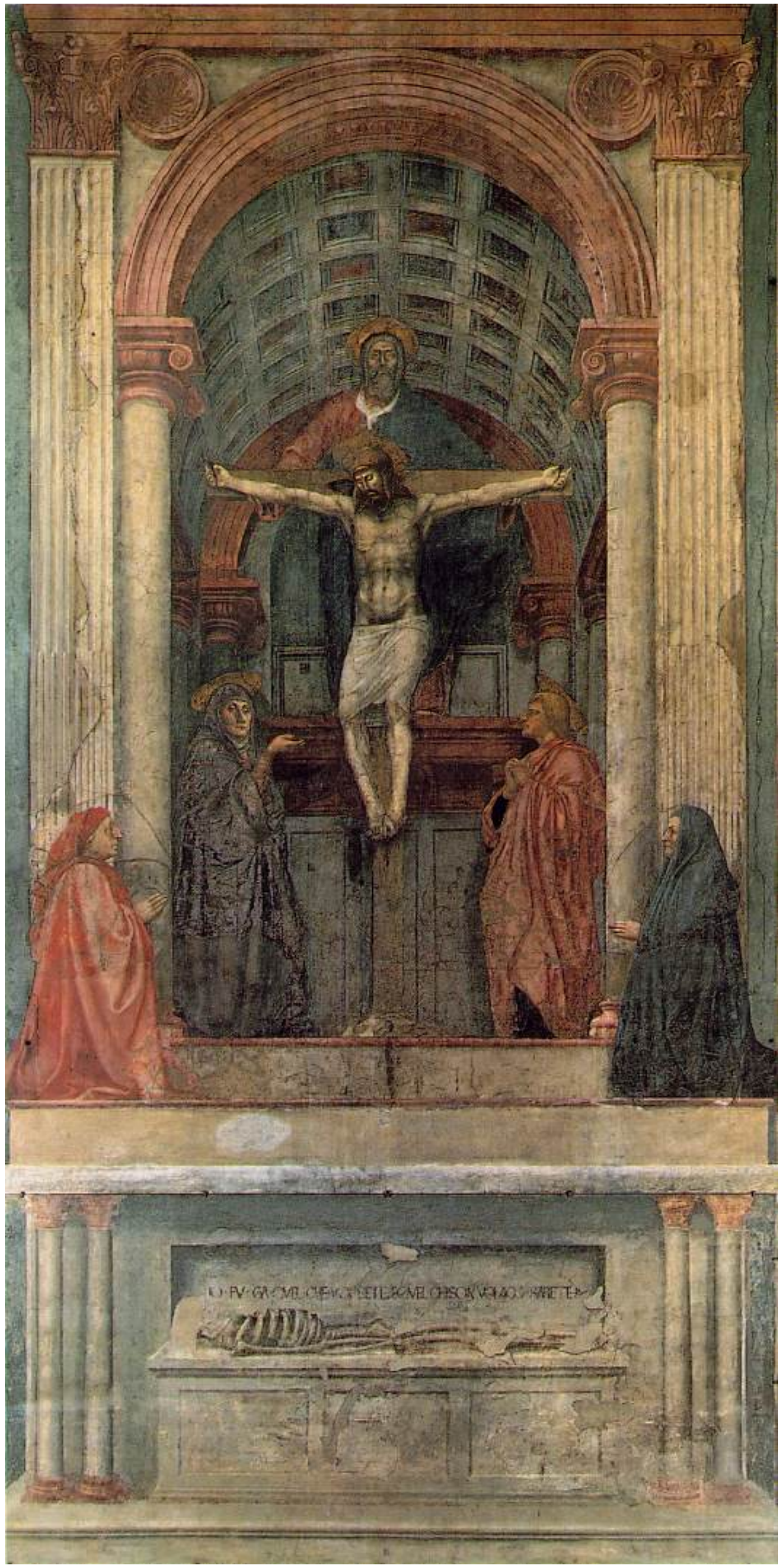
What is the height of the camera?

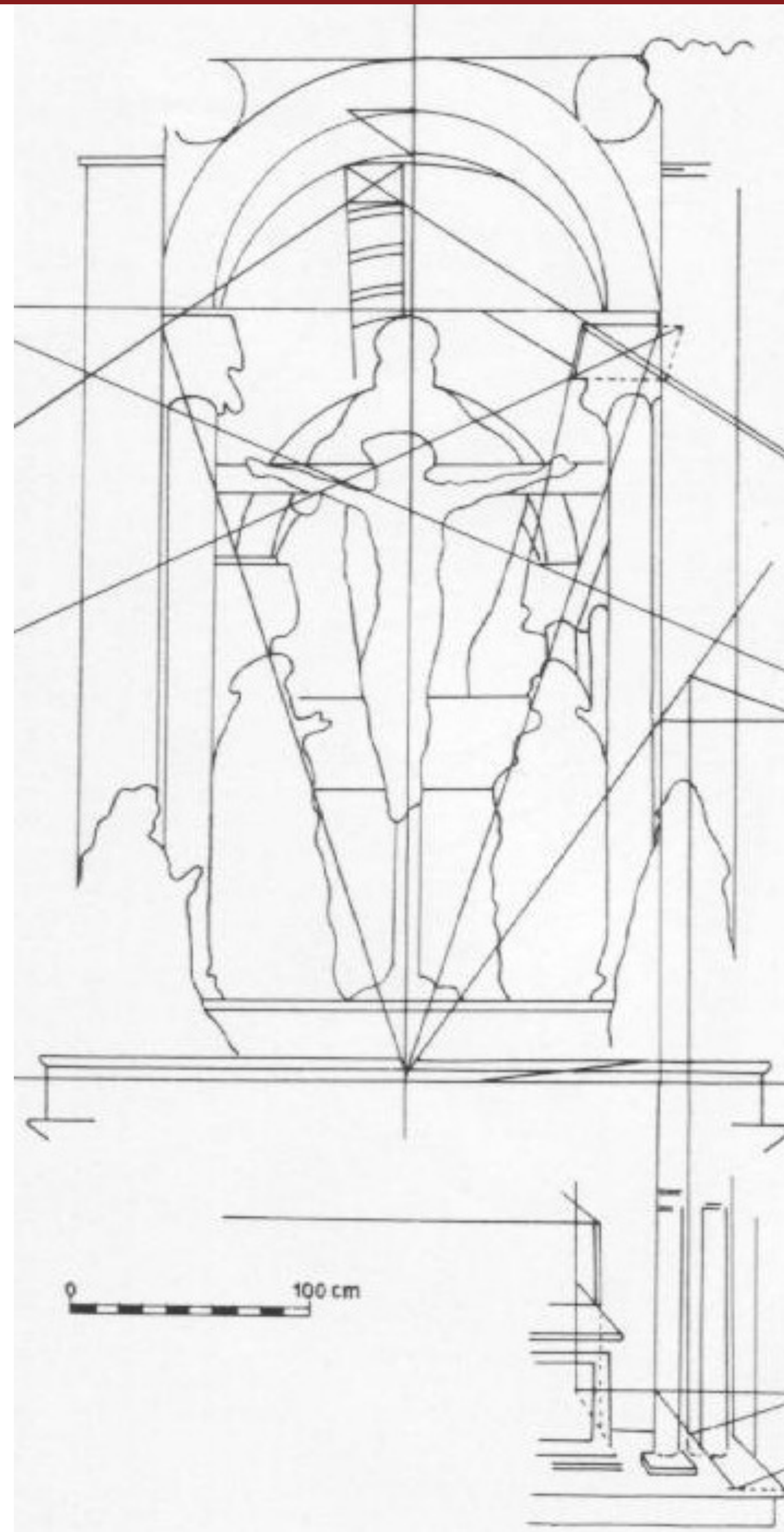
Measuring heights



What is the height of the camera?

Perspective in art





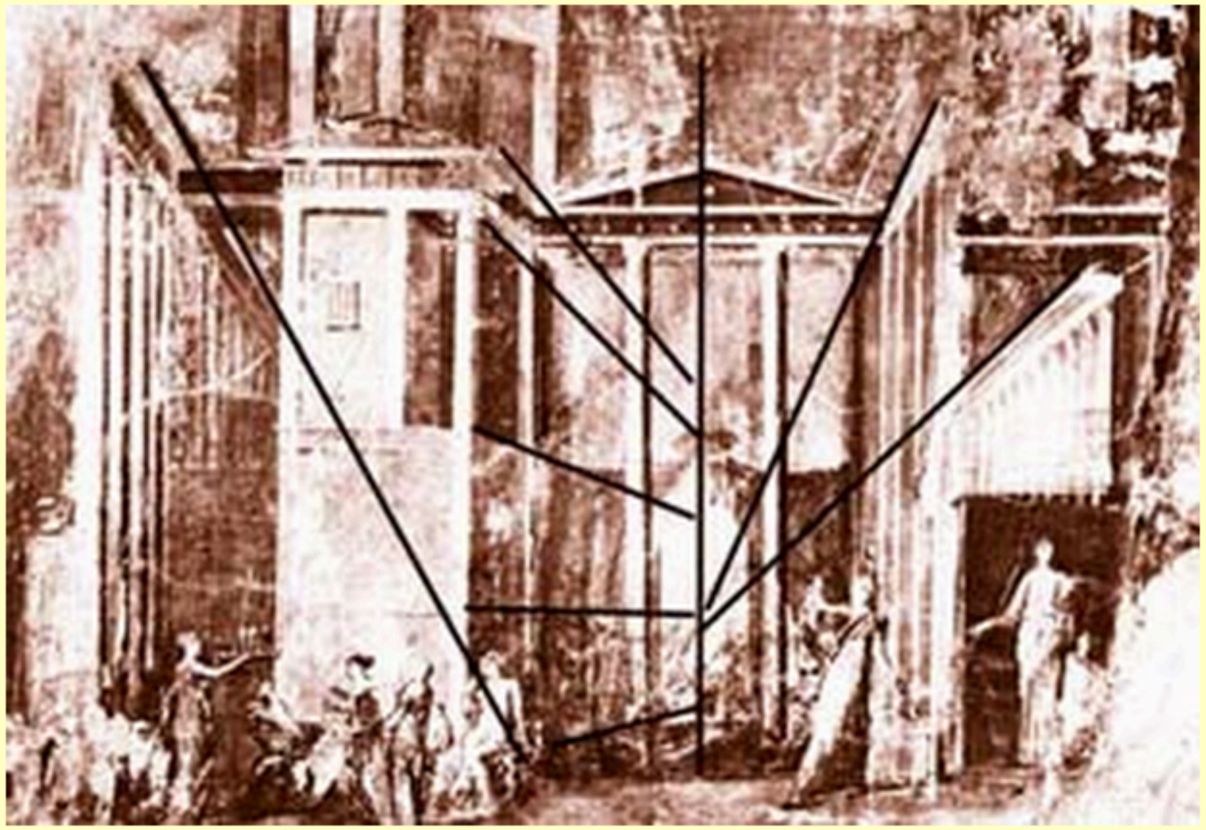
Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

One of the first consistent uses of perspective in Western art

Perspective in art

(At least partial) Perspective projections in art well before the Renaissance

Several Pompei wallpaintings show the fragmentary use of linear perspective:

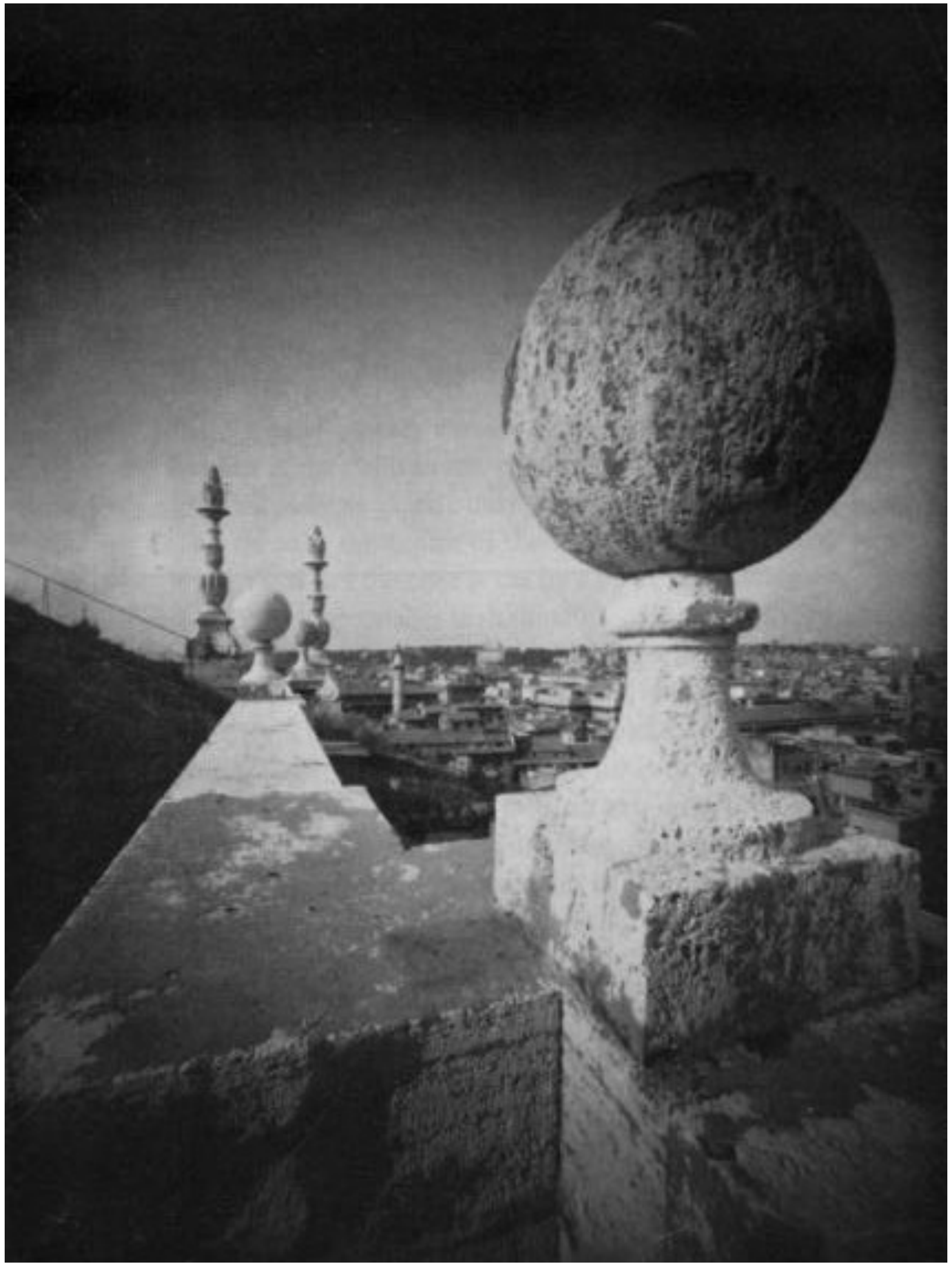


From ottobwiersma.nl

Also some Greek examples,
So apparently pre-renaissance…

Perspective distortion

- What does a sphere project to?



M. H. Pirenne

Perspective distortion

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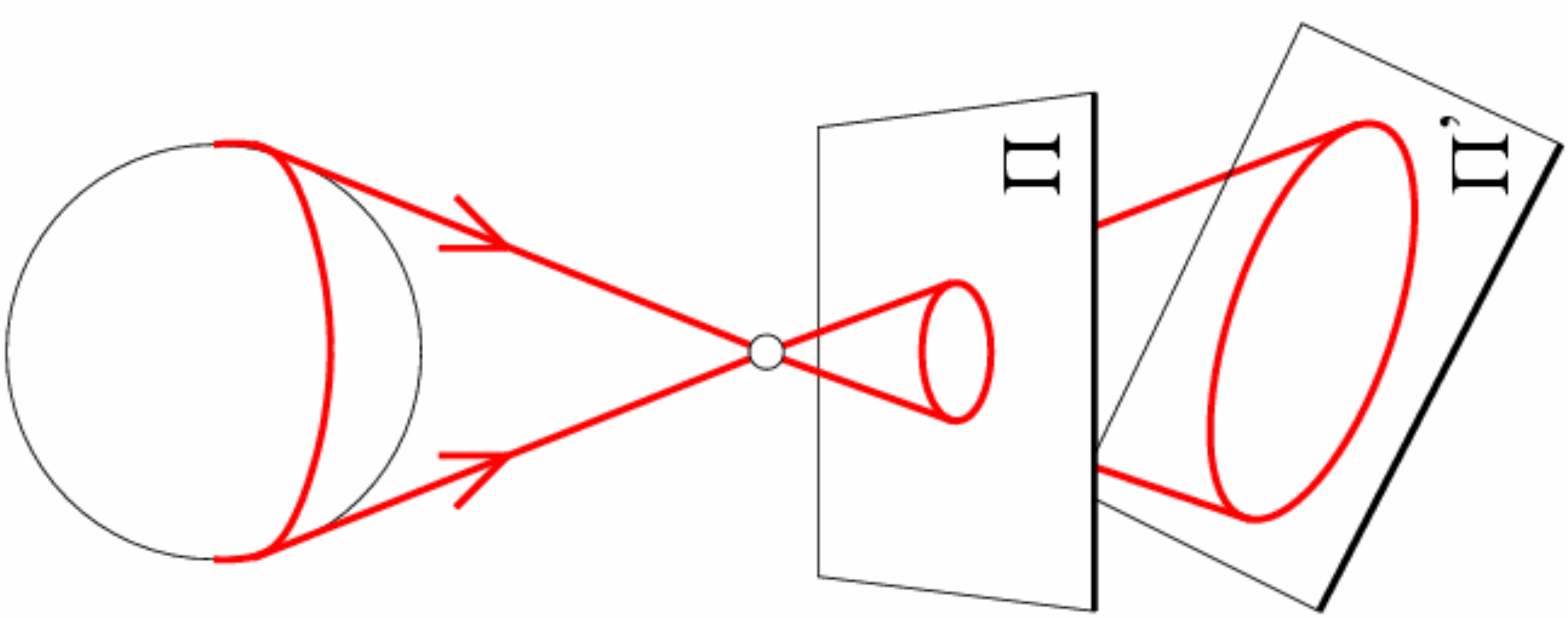
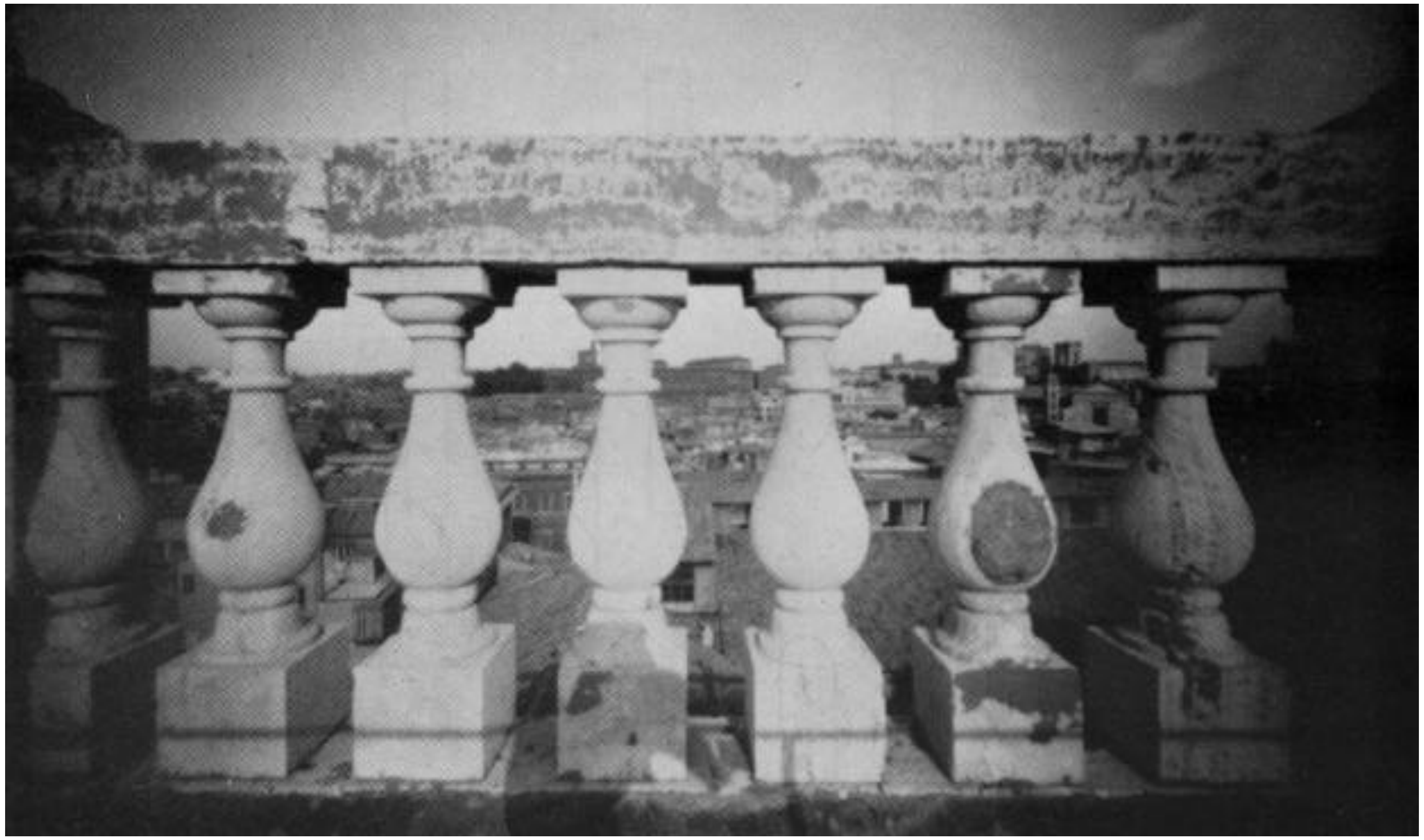
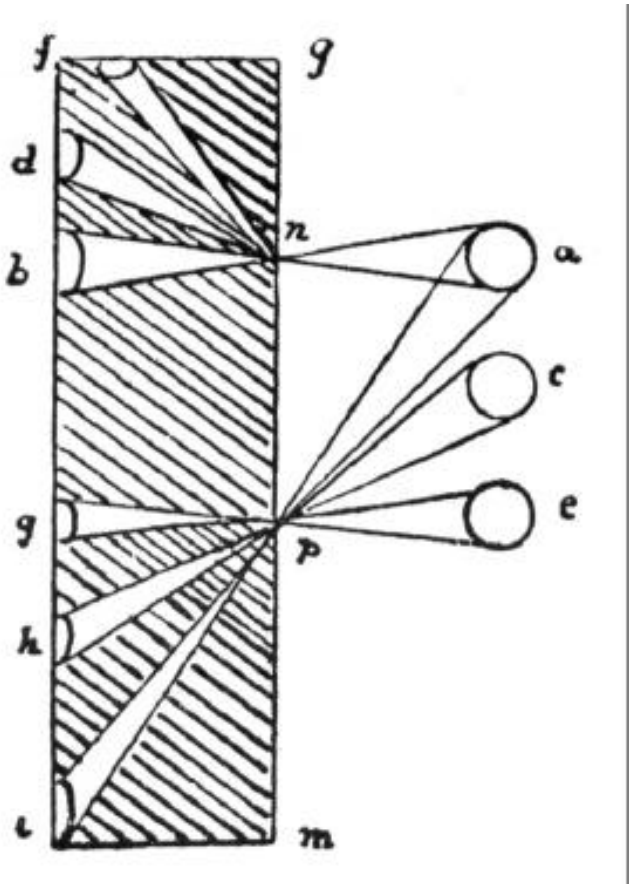


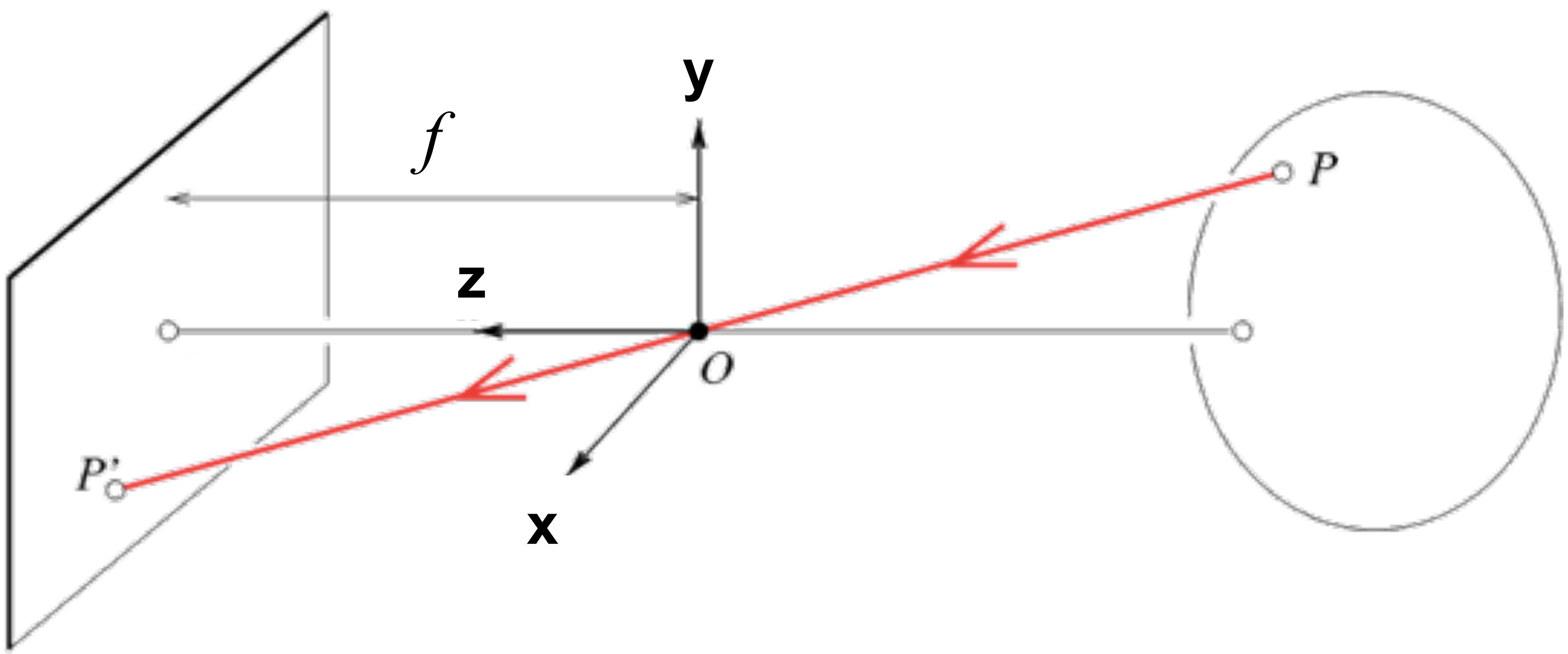
Perspective distortion

- The exterior looks bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci



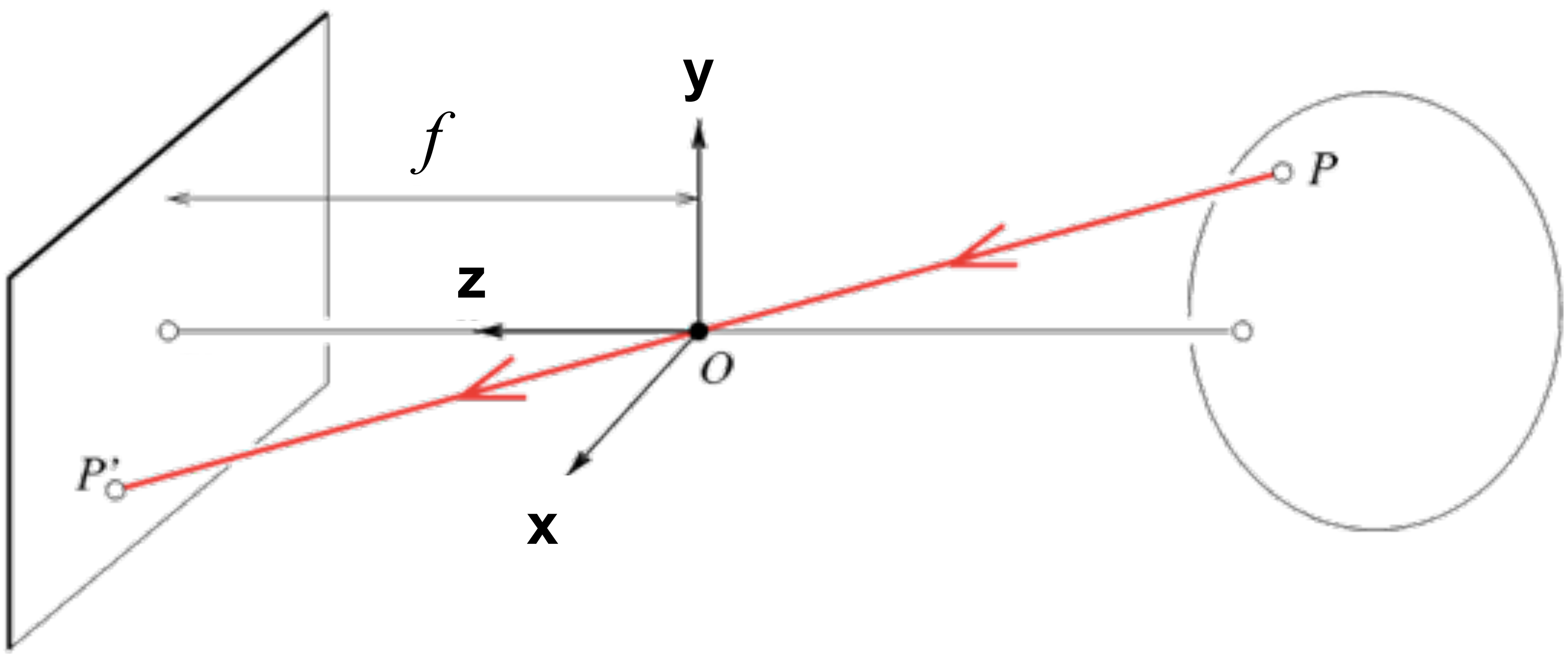


Modeling projection



- Projection equation

Modeling projection



- Projection equation $$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogeneous coordinates

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogeneous coordinates

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- Is this a linear transformation?

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  - no — division by z is not linear

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$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

- Is this a linear transformation?
  - no — division by z is not linear
- **Trick:** add one more coordinate

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Homogeneous coordinates

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

- Is this a linear transformation?
  - no — division by z is not linear
- **Trick:** add one more coordinate

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting from homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

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Perspective projection matrix

- Projection is a matrix multiplication using homogeneous coordinates (scene and image)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by the third coordinate

- In practice: lots of coordinate transforms

(2D point (3x1)) = (Camera to pixel coord. trans. matrix (3x3)) (Perspective projection matrix (3x4)) (World to camera coord. trans. matrix (4x4)) (3D point (4x1))

Whole "pipeline"

$$\begin{bmatrix} w_p p_i \\ w_p p_j \\ w_p \end{bmatrix} = \begin{bmatrix} s_x & k_1 & 0 \\ k_2 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

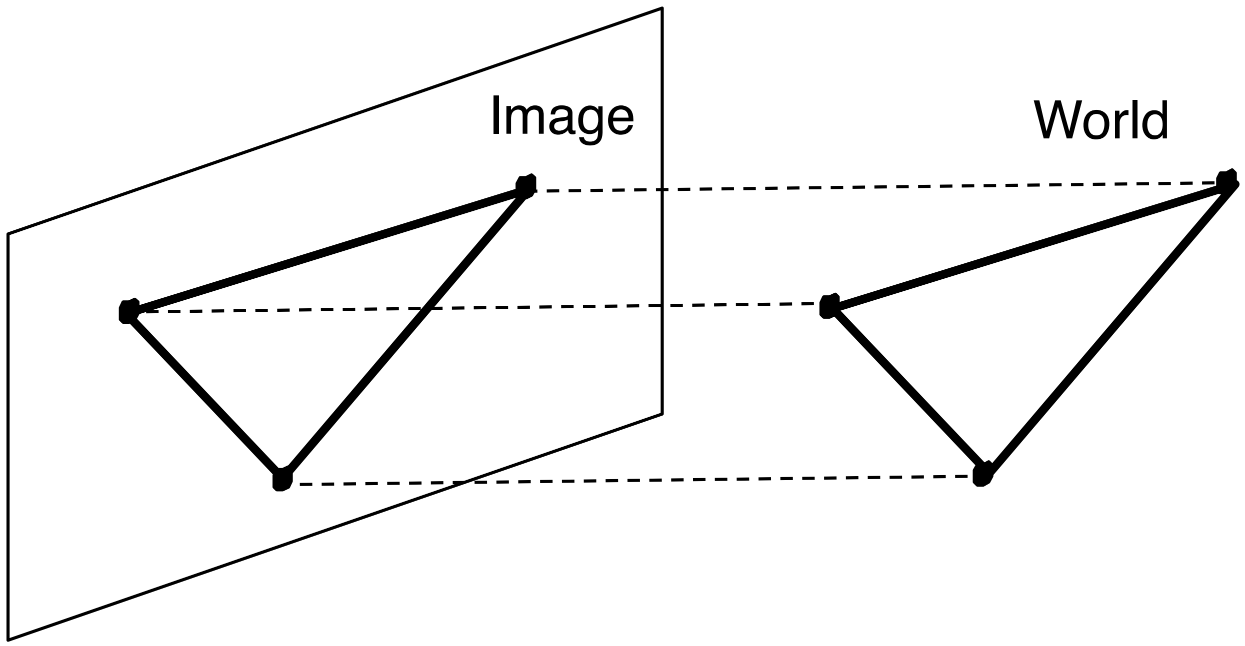
(2D point (3x1)) = (Camera to pixel coord. trans. matrix (3x3)) (Perspective projection matrix (3x4)) (World to camera coord. trans. matrix (4x4)) (3D point (4x1))

- Just one matrix with a special structure

$$\begin{bmatrix} w_p p_i \\ w_p p_j \\ w_p \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Orthographic projection

- Special case of perspective projection
  - Distance of the object from the image plane is infinite
  - Also called the “parallel projection”



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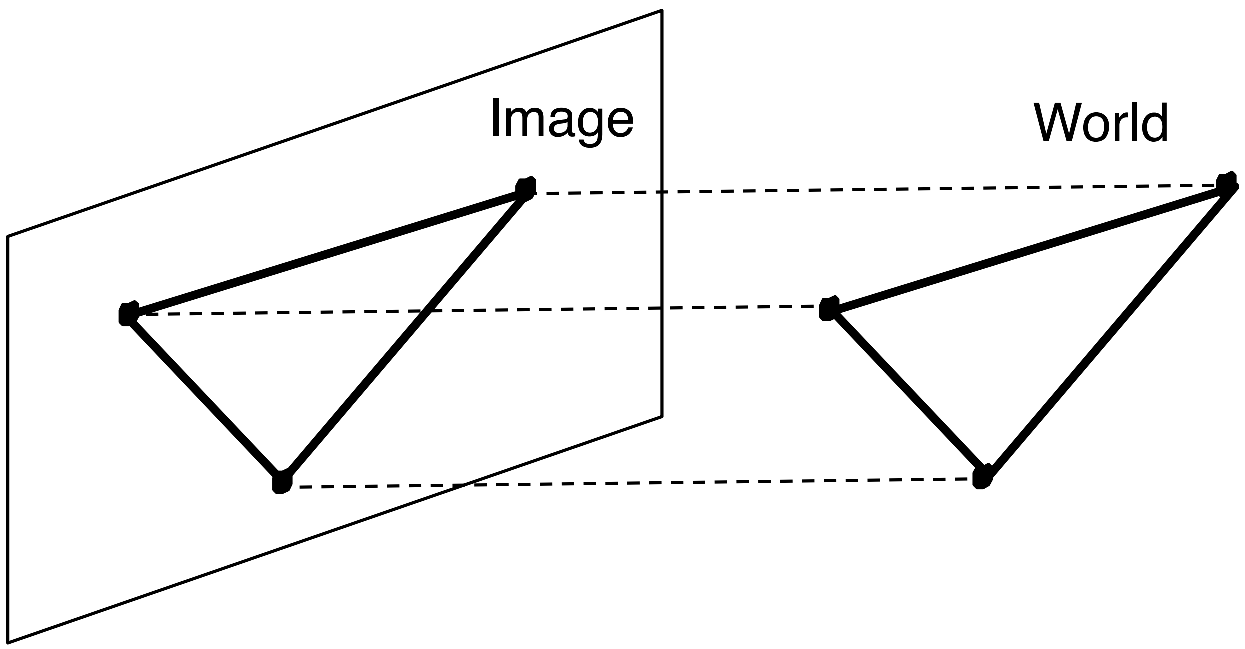

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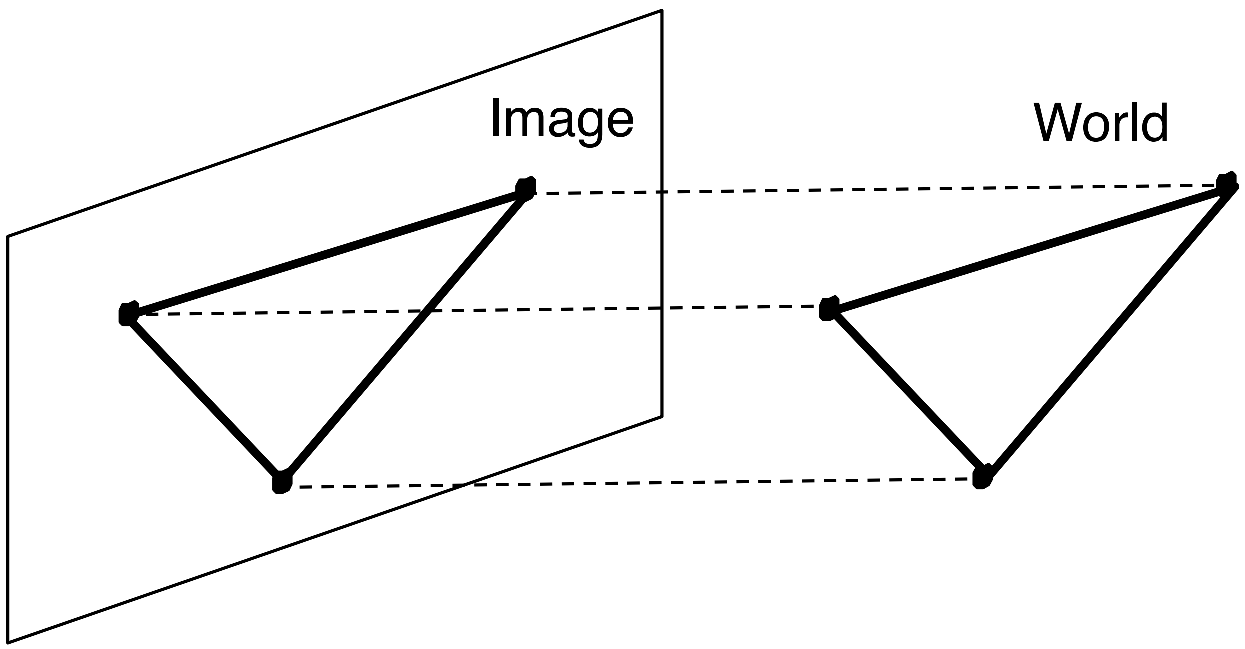

  - What’s the projection matrix?

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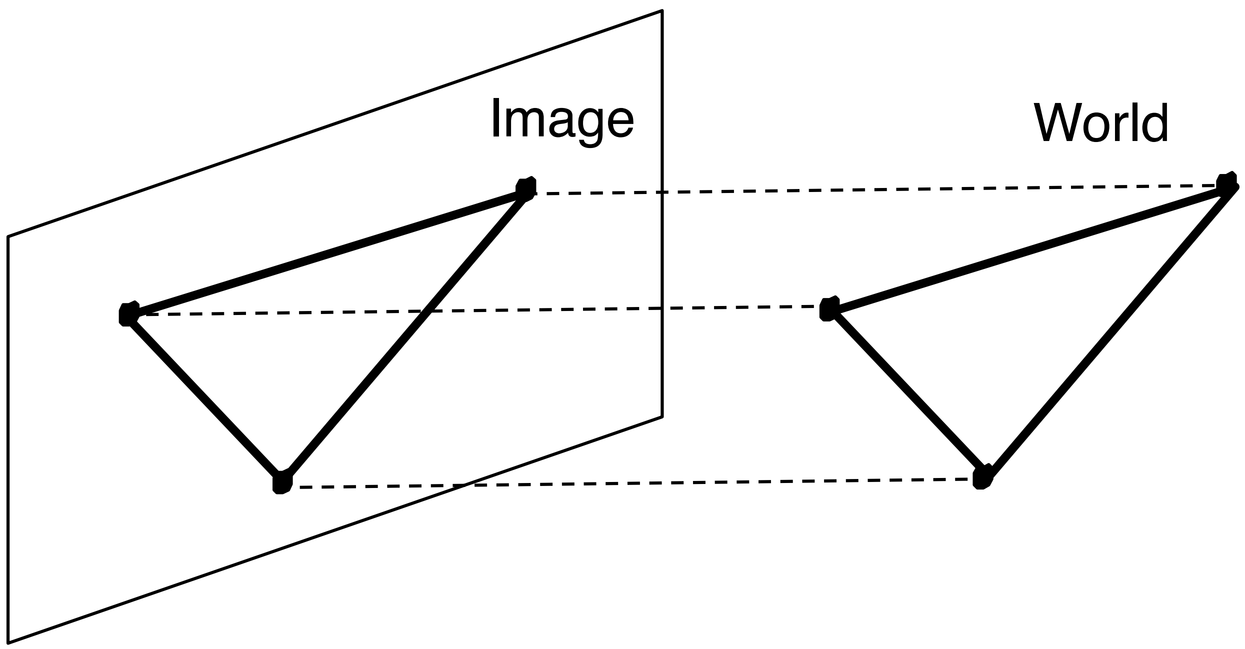

- What’s the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

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More readings and thoughts

- **History of optics**, Wikipedia
- A. Torralba and W. Freeman, **Accidental Pinhole and Pinspeck Cameras**, CVPR 2012
- DIY **http://www.pauldebevec.com/Pinhole**
- In MATLAB, compute the projection of a sphere using the perspective model and visualize the distortions