CMPSCI 370: Intro to Computer Vision

Image processing: #4
Corner detection

University of Massachusetts, Amherst
February 22/24, 2015

Instructor: Subhransu Maji

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Administrivia

- Homework 2 due today
- Homework 3 will be posted later this week
  - will be due March 10
- No class or honors section next Tuesday, 3/1
  - I am out of town to attend a CVPR program committee meeting
- Honors section will meet today

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Overview

- Edge detection
  - Derivative filters

$$\mathbf{G_x} = \begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A}$$

$$\mathbf{G_y} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix} * \mathbf{A}$$

image

$E = \sqrt{G_x^2 + G_y^2}$

- Corner detection [today]
  - What are corners?
  - Why detect corners?
  - Harris corner detector

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Feature extraction: Corners

Slide credit: L. Lazebnik 4

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Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Why extract features?

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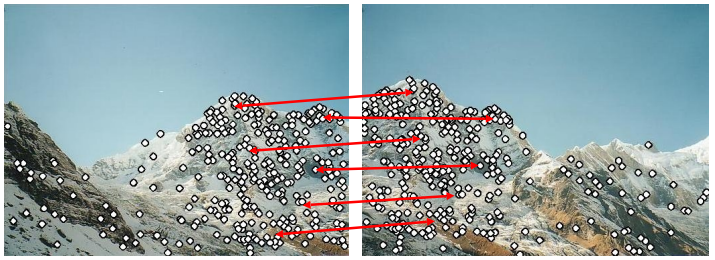


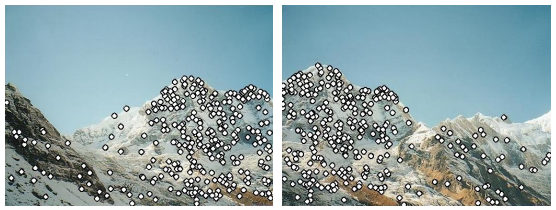
Step 1: extract features
Step 2: match features

Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Step 1: extract features
Step 2: match features
Step 3: align images

Characteristics of good features

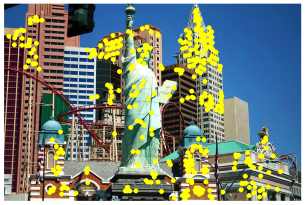


- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Applications

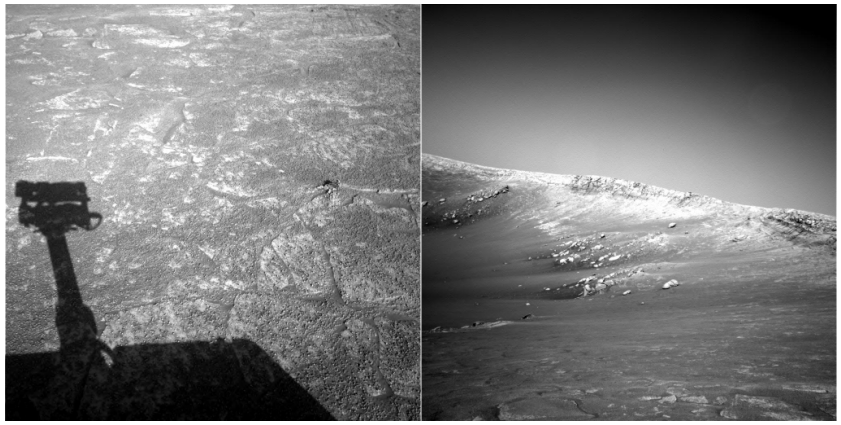
Feature points are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Indexing and database retrieval
- Object recognition



Slide credit: L. Lazebnik 9

A hard feature matching problem



NASA Mars Rover images

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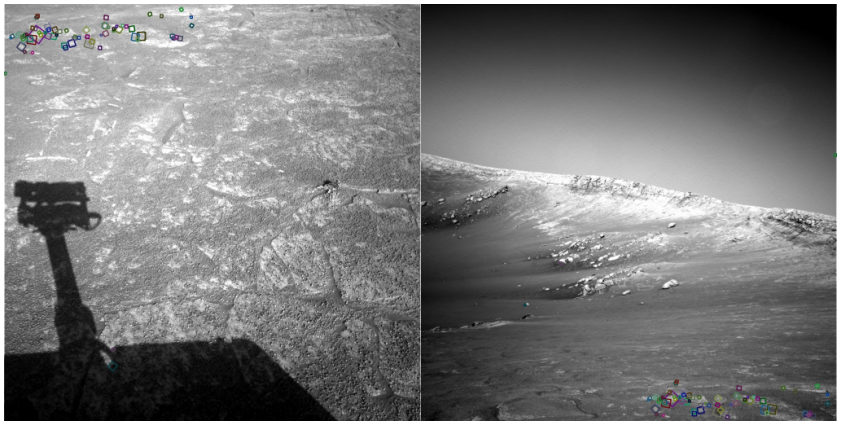
Answer below (look for tiny colored squares…)



NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

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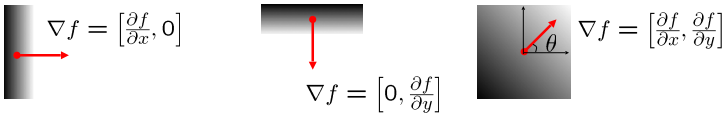
Corner detection: Attempt one

- A corner is the intersection of two edges
- We know how to detect edges

- Corner detector v1
  - Detect edges in images (G_x and G_y)
  - Find places where both G_x and G_y are high

- Problem: this also finds slanted edges



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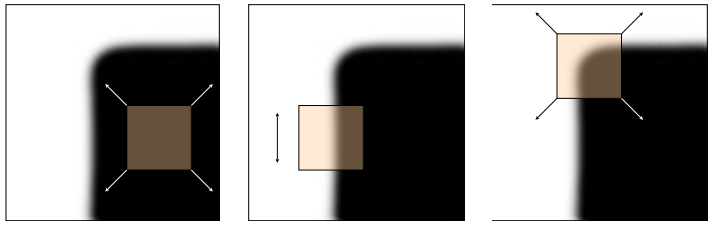
Corner detection: Attempt two

- We should easily recognize the corners by looking through a small window
- Shifting a window in ***any direction*** should give ***a large change*** in intensity at a corner



"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

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Corner detection: Attempt two

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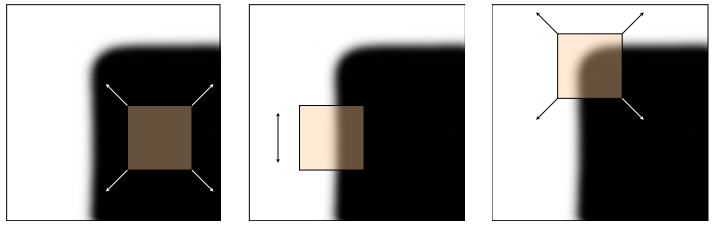

"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

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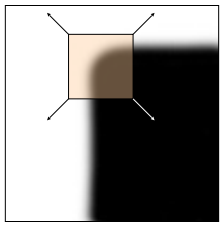
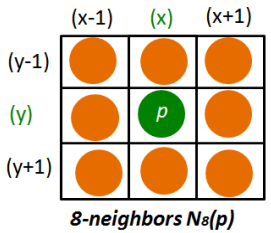
Corner detection: Attempt two

- Implementing a corner detector
  - Fix the size of the patch (window size)
    - What happens if the window is too small? or too large?
  - Consider eight directions, and measure how much does a patch change in each direction for each location of the image





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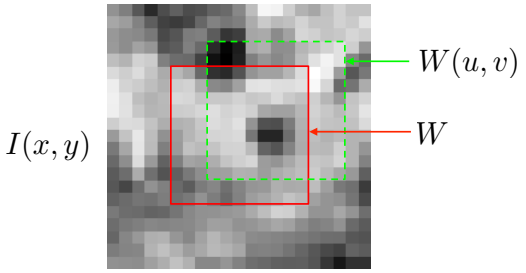
Corner detection: details

- One way to measure change is to consider the sum of squared-differences. Thus, the change for a shift *u, v* and for an image window W is

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$



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Corner detection: details

- Assume for now that the window size W is one pixel
- How can we compute E(u,v) for every pixel in the image?

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

- As an example consider E(1, 0), i.e. image shifted by one pixel to the right. This can be computed as convolution with **f** and squaring the result,

$$D(1,0) = I * \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

f

Matlab: imfilter(I, f, 'same')

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Corner detection: details

- Generalize to any window size W
- Given D(1,0) for each pixel, how can you compute the sum-of-squared-differences within a window?
- Answer: filter the output with a box kernel

$$\text{EdgeScore}(1,0) = f_{\text{box}}^{W} * D(1,0)^2$$

- Better version: Convolve with a Gaussian kernel
  - More emphasis on the center than the boundary for each window

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Corner detection: algorithm

- **Corner detector v2**
  - Input: image **I**, window size **W**
  - Output: corner score for each pixel
  - Initialize: score = zeros(size(**I**))
  - for u = -1:1 % horizontal shifts
    - for v = -1:1 % vertical shifts
      - imdiff = imfilter(**I**, **f(u,v)**, 'same'); % compute difference
      - score = score + imfilter(imdiff.^2, ones(**W**), 'same'); % weighted squared difference
  - Here, **f(u,v)** is the filter to compute difference between a pixel and another shifted by **(u,v)**

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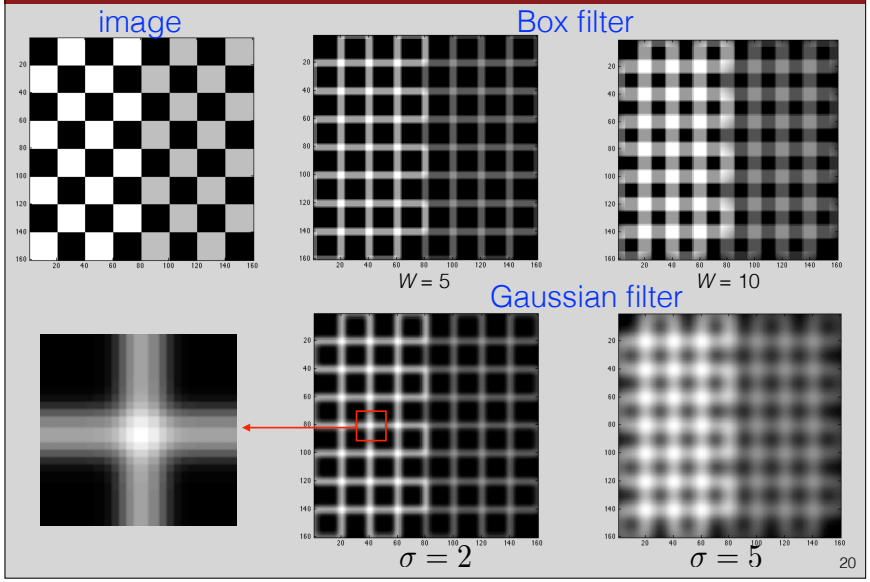
Corner score example



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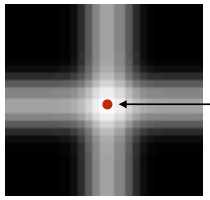
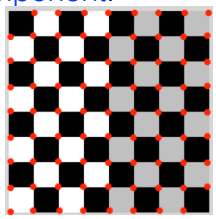
Corner detection: remaining steps

- Threshold the score (user specified)
- Find the peaks of responses: a pixel is a peak if it has a higher value than all the pixels in its neighborhood
  - 4 connectivity, 8 connectivity, etc
- The Matlab command imregionalmax computes a binary image which is 1 at the peak and 0 elsewhere
- Note that there can be multiple pixels at each peak. We can simply pick one for each connected component.



peak



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Corner detection: complete algorithm

- **Corner detector v2**
  - Input: image I, window size **W,** threshold T
  - Output: corner score for each pixel

```
• Initialize: score = zeros(size(I))
• for u = -1:1 % horizontal shifts
  • for v = -1:1 % vertical shifts
    • imdiff = imfilter(I, f(u,v), 'same'); % compute difference
    • score = score + imfilter(imdiff.^2, ones(W), 'same'); % weighted
      squared difference
• score = nms(score > T); % perform non-maximum suppression
• locs = find(score > 0);  % indices of the corners
• [cy, cx] = ind2sub(size(score), lots)); % their x, y locations
```

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Corner detection: summary

- The algorithm presented is the basis for the Harris corner detector
- One weakness is that we only considered only 8 directions for computing the differences. Why not 16? or 32?
- The Harris detector generalizes this to "all" directions
  - More directions improve performance
  - Can be thought of the limit of the earlier algorithm as the number of directions approach infinity!
  - However, it does this without explicitly enumerating directions. To understand how we need some math.

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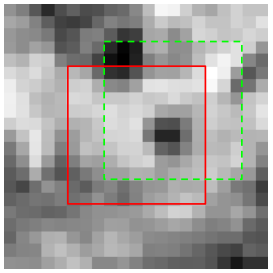
Corner detection: Mathematics

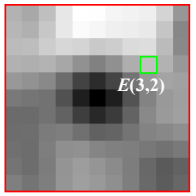
Recall that the change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x,y)$



$E(u,v)$



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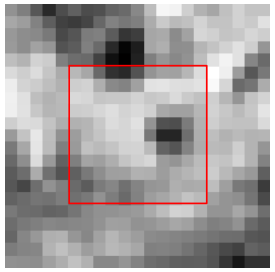
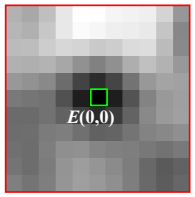


$E(u,v)$



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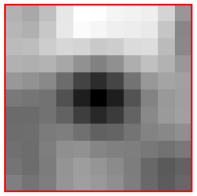
Corner detection: Mathematics

Recall that the change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

$E(u,v)$



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Corner detection: Mathematics

- First-order Taylor approximation for small motions [*u*, *v*]:

$$I(x+u, y+v) = I(x,y) + I_x u + I_y v$$

- Let's plug this into *E*(*u*,*v*)

$$\begin{aligned} E(u,v) &= \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2 \\ &\simeq \sum_{(x,y)\in W} [I(x,y) + I_x u + I_y v - I(x,y)]^2 \\ &= \sum_{(x,y)\in W} [I_x u + I_y v]^2 \\ &= \sum_{(x,y)\in W} \left[I_x^2 u^2 + I_x I_y uv + I_y I_x uv + I_y^2 v^2\right] \end{aligned}$$

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Corner Detection: Mathematics

The quadratic approximation can be written as

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a *second moment matrix* computed from image derivatives:

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

(the sums are over all the pixels in the window *W*)

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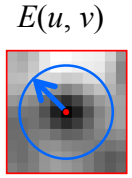
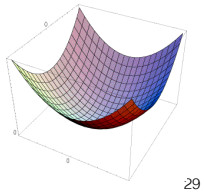
Interpreting the second moment matrix

- The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.
  - Specifically, in which directions does it have the smallest/greatest change?

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$





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Interpreting the second moment matrix

First, consider the axis-aligned case (gradients are either horizontal or vertical)

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

If either *a* or *b* is close to 0, then this is **not** a corner, so look for locations where both are large.

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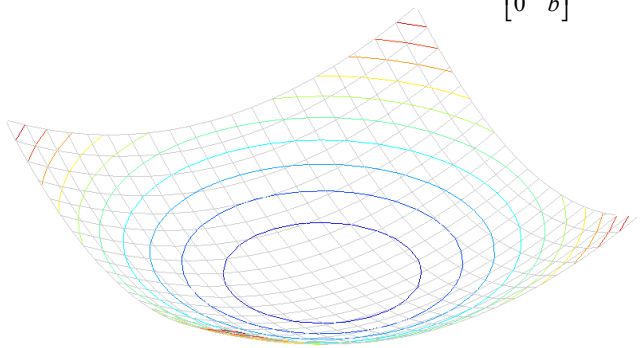
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$



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Interpreting the second moment matrix

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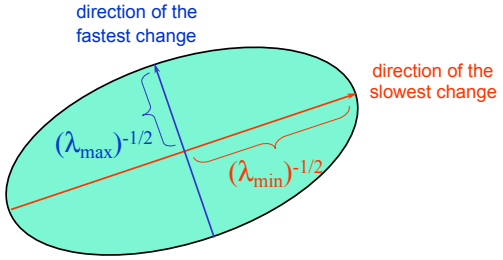
This is the equation of an ellipse.

Diagonalization of M: $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*



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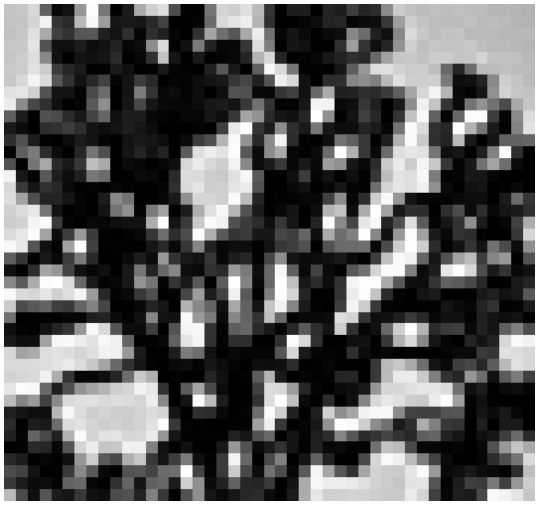
Visualization of second moment matrices



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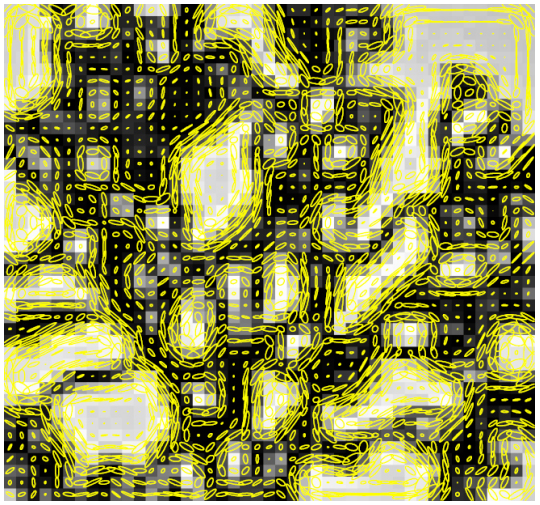
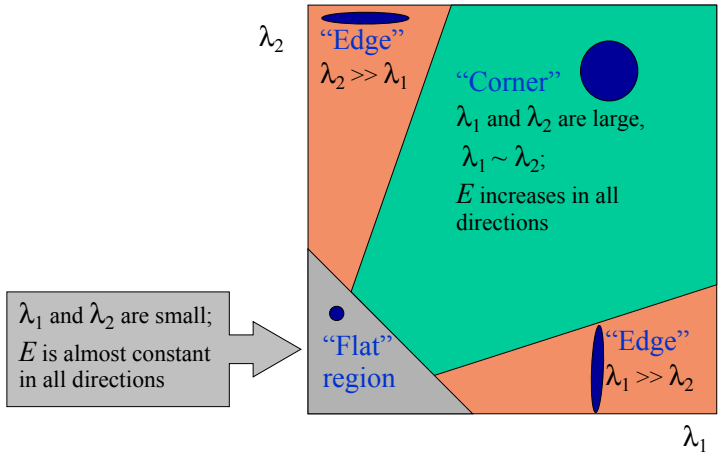


Interpreting the eigenvalues

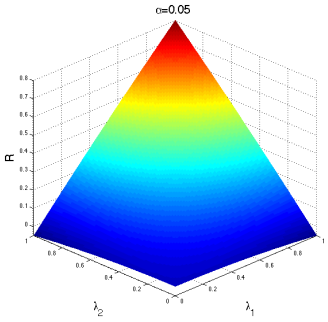
Classification of image points using eigenvalues of *M*:

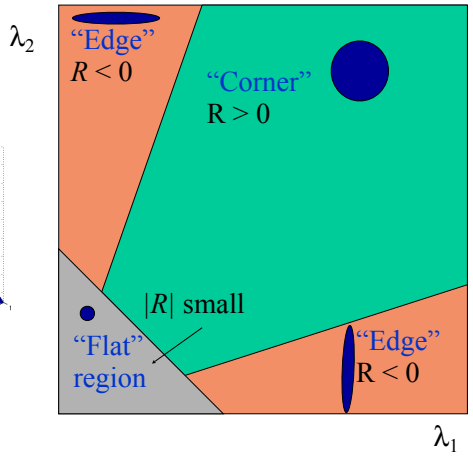


Corner response function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)





The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix ***M*** in a Gaussian window around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. **"A Combined Corner and Edge Detector."** *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

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The Harris corner detector

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Harris Detector: Steps

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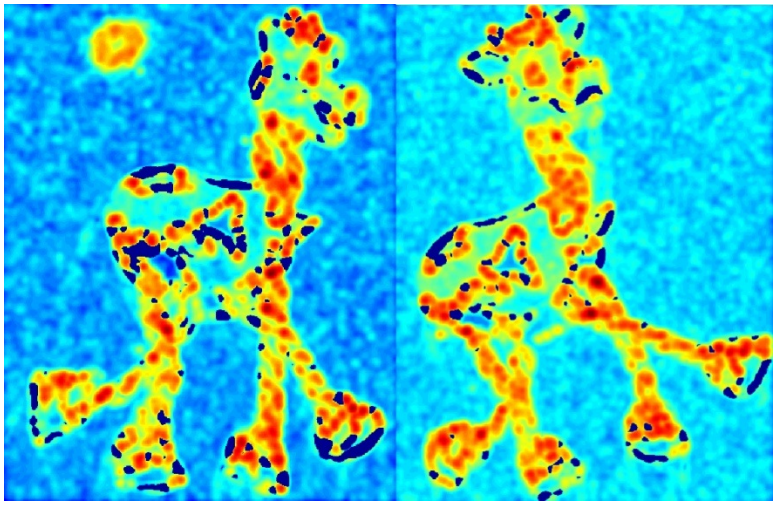
Harris Detector: Steps

Compute corner response ***R***



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The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*
4. Threshold *R*
5. Find local maxima of response function (non-maximum suppression)

C.Harris and M.Stephens. **"A Combined Corner and Edge Detector."**
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

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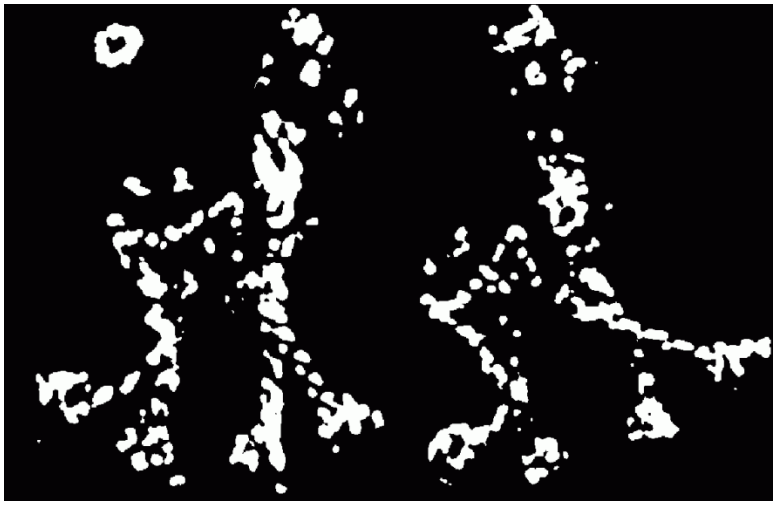
Harris Detector: Steps

Find points with large corner response: *R* > threshold



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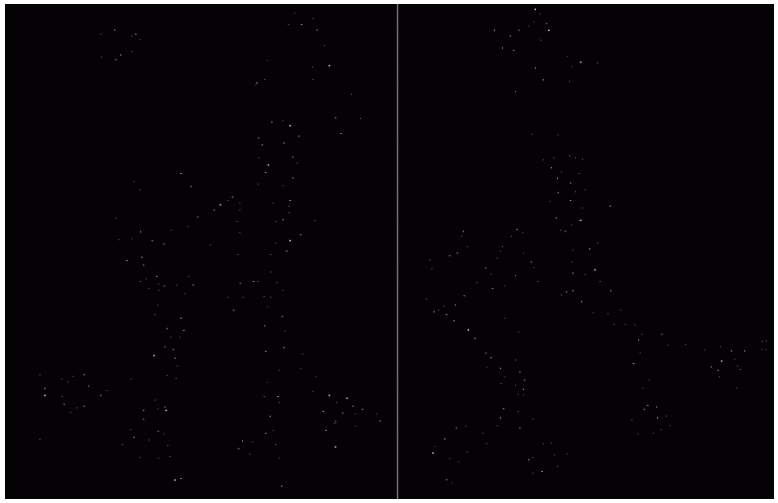
Harris Detector: Steps

Take only the points of local maxima of *R*



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Harris Detector: Steps

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Invariance and covariance

- We want corner locations to be ***invariant*** to photometric transformations and ***covariant*** to geometric transformations
  - **Invariance:** image is transformed and corner locations do not change
  - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

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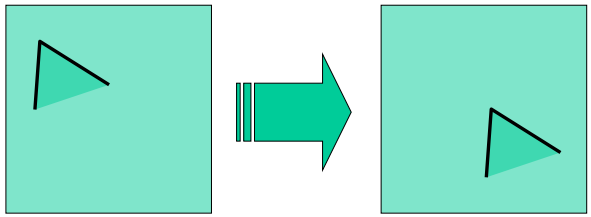
Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

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Further thoughts and readings…

- Original corner detector paper
  - C.Harris and M.Stephens, **"A Combined Corner and Edge Detector."** Proceedings of the 4th Alvey Vision Conference,1988
- Other corner functions
  - Can you think of other $f(\lambda_1, \lambda_2)$ that work for finding corners?

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