

Separability of the Gaussian filter

$$\begin{aligned} \mathsf{G}_{\sigma}(x,y) &= \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}} \\ &= \left(\frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{x^2}{2\sigma^2}}\right) \left(\frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{y^2}{2\sigma^2}}\right) \end{aligned}$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Why is separability useful?

 Separability means that a 2D convolution can be reduced to two 1D convolutions (one among rows and one among columns)

4

- What is the complexity of filtering an n×n image with an m×m kernel?
 - O(n² m²)
- What if the kernel is separable?
 - O(n² m)

The Canny edge detector

- 1. Filter image with derivative of Gaussian
- 2. Find magnitude and orientation of gradient
- 3. Non-maximum suppression:
 - Thin wide "ridges" down to single pixel width
- 4. Linking and thresholding (hysteresis):
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

MATLAB: edge(image, `canny');

J. Canny, <u>A Computational Approach To Edge Detection</u>, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

The Canny edge detector

original image

5

7



Slide credit: Steve Seitz 6

8

The Canny edge detector



norm of the gradient

The Canny edge detector



thresholding

The Canny edge detector



Non-maximum suppression



Check if pixel is local maximum along gradient direction, select single max across width of the edge • requires checking interpolated pixels p and r

The Canny edge detector



Problem: pixels along this edge didn't survive the thresholding

11

thinning (non-maximum suppression)

Hysteresis thresholding

10

Use a high threshold to start edge curves, and a low threshold to continue them.



Hysteresis thresholding



original image



high threshold (strong edges)



low threshold (weak edges)

hysteresis threshold

Source: L. Fei-Fei13

15

Recap: Canny edge detector

- 1. Compute x and y gradient images
- 2. Find magnitude and orientation of gradient
- 3. Non-maximum suppression:
 - Thin wide "ridges" down to single pixel width
- 4. Linking and thresholding (hysteresis):
 - Define two thresholds: low and high •
 - Use the high threshold to start edge curves and the low • threshold to continue them

MATLAB: edge(image, 'canny');

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Modern edge detection

Berkeley Segmentation Dataset: Training Image #159029 [gray]

6 Gray Segmentations



Modern techniques

- "Learning to Detect Natural Image Boundaries Using Brightness and Texture" D. Martin. C. Fowlkes, and J. Malik. **NIPS 2002**
- Boundary prediction as a machine learning problem
- Lot of early work from the Berkeley vision group



14

A very fast version

• Random forest edge detector (Piotr Dollar et al., ICCV 13)

Further thoughts and readings ...

- Hybrid images project
 - <u>http://cvcl.mit.edu/hybridimage.htm</u>
- Canny edge detector
 - www.limsi.fr/Individu/vezien/PAPIERS_ACS/canny1986.pdf
- Bilateral filtering for image denoising (and other application)

18

<u>http://people.csail.mit.edu/sparis/bf_course/</u>

• If all else fails www.xkcd.com

17