

**COMPUTER
SCIENCE**

24- Static Analysis

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Approaches

```

graph LR
    subgraph Static_Analysis [Static Analysis]
        S1[Inspections]
        S2[Software metrics]
        S3[Symbolic execution]
        S4[Dependence Analysis]
        S5[Data flow analysis]
        S6[Software Verification]
    end
    subgraph Dynamic_Analysis [Dynamic Analysis]
        D1[Assertions]
        D2[Error seeding, mutation testing]
        D3[Coverage criteria]
        D4[Fault-based testing]
        D5[Specification-based testing]
        D6[Object-oriented testing]
        D7[Regression testing]
    end
    Static_Analysis --> Dynamic_Analysis
    Dynamic_Analysis --> Static_Analysis
    
```

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Review methods

- **Fagan inspections**
 - formal, multi-stage process
 - significant background & preparation
 - led by moderator
- **Active design reviews**
 - also called "phased inspections"
 - several brief reviews rather than one large review
 - guided by questions from the author
- **Cleanroom**
 - more than reviews, but reviews important component
 - we'll come back to this
- **N-fold**
 - parallel reviews controlled by moderator
 - focuses on user requirements

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Why are inspections effective

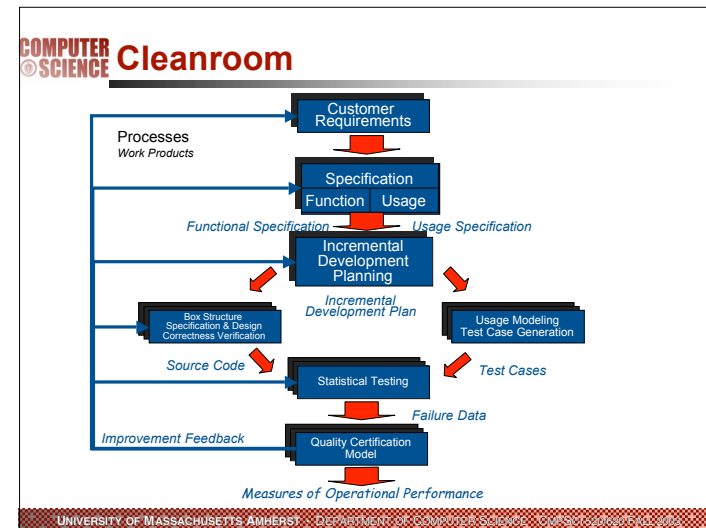
- knowing the product will be scrutinized causes developers to produce a better product
- having others scrutinize a product increases the probability that faults will be found
- walkthroughs and reviews are not as formal as inspections, but appear to also be effective
 - hard to get empirical results

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COMPUTER SCIENCE **What are the deficiencies?**

- **focus on error detection**
 - what about other "ilities" -- maintenance, portability, etc.
- **not applied consistently & rigorously**
 - inspection shows statistical improvement, but cannot ensure quality
 - inspection should have the same results without regard to the product to which it is applied or the inspection team
- **range of errors not addressed**
 - team expertise limited
 - one property may have many error modalities
- **human intensive and often makes ineffective use of human resources**
 - e.g., skilled software engineer reviewing coding standards, comments spelling, etc.
- **no automated support**
 - again inefficient of human resources
- **aspects of review not used appropriately**
 - e.g., in Fagan process, overview often covers what should be described if documentation is adequate

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COMPUTER SCIENCE **Cleanroom**

```

{F}
do  [g]
od  [h]

For all
inputs, does
[g] followed
by [h] do
[f]
  
```

- **Verification as Review Process**
 - team verification of correctness takes the place of individual unit testing; correctness is established by **group consensus** if it is obvious
 - by formal proof techniques if it is not.
- **benefits**
 - intellectual control of the process
 - motivates developers to deliver error-free code
 - verification is a form of peer review
 - each person assumes responsibility for and derives a sense of ownership in the evolving product
- **every person must agree that the work is correct before it is accepted -> successes are ultimately team successes, and failures are team failures.**
- **Markov Analysis**
- **Factors**
 - number of statistically typical (i.e., likely) usage paths through the software
- **Steps**
 - focus verification efforts,
 - identify the likelihood of given events,
 - project the test schedule, and
 - ascertain the (affordable) upper bound on inferences about reliability
- **Stopping Criterion for Testing**
 - goals (e.g., target level of estimated reliability) are achieved
 - or quality standards (e.g., errors/KLOC) are violated

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COMPUTER SCIENCE **Software Metrics**

- **measures that predict qualities about software**
- **can be applied to any of the products (e.g., design, code, test cases) or to the process (e.g., Capability Maturity Model)**
- **Qualities measured by software metrics**
 - **performance**
 - **user-friendliness**
 - **resources**
 - memory/storage
 - development costs
 - maintenance cost
 - **quality**
 - maintainability
 - reliability
 - completeness
 - consistency
 - complexity

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COMPUTER SCIENCE **Function Points**

- proposed by Albrecht in 1979
 - Originally applied to code
- UFP =
 - number of inputs x w1 +
 - number of outputs x w2 +
 - number of user inquiries x w3
 - number of files x w4 +
 - number of external references x w5
- function points = UFP * TCA = UFP * (.65 + 0.01 * SUM(Fi))
 - where the degree of influence, DI = SUM(Fi) is the sum of complexity adjustment values, Fi
- metrics:
 - productivity: FP/person-month
 - quality: defects/FP
 - cost: \$/FP

	Simple	Average	Complex
w1	3	4	6
w2	3	5	7
w3	3	4	6
w4	7	10	15
w5	5	7	10

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COMPUTER SCIENCE **More Quality Metrics**

- Modularity
 - cohesion metric
 - applied to unit design
 - the relationship among the elements of a module
 - best cohesion level is functional, and the worst is coincidental.
 - Cruickshank and Gaffney Cohesion Strength
 - Strength = $\sqrt{X^2 + Y^2}$
 - where:
 - X = reciprocal of the number of assignment statements in the module
 - Y = number of unique function outputs divided by number of unique function inputs

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COMPUTER SCIENCE **More Quality Metrics**

- Modularity
 - coupling
 - applied to system and unit designs
 - measure of the degree to which modules share data
 - data coupling (the sharing of data via parameter lists) is the best type of coupling, while common coupling (the sharing of data via global or common areas) is the worst.
 - a lower coupling value is better.
 - Cruickshank and Gaffney Coupling:
 - M_i = sum of the number of input and output items shared between components i & j
 - Z_i = average number of input and output items shared over m components with component i
 - n = number of components in the software product

$$\text{Coupling} = \frac{\sum_{i=1}^n Z_i}{n}$$

where:

$$Z_i = \frac{\sum_{j=1}^m M_{ij}}{m}$$

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COMPUTER SCIENCE **McCabe's cyclomatic complexity**

- Complexity measured by control flow information
 - based on a control flow graph where e is number of edges, n is number of nodes, p is number of connected components
- McCabe's Cyclomatic Complexity:
 - $v = e - n + 2$
 - where:
 - v = complexity of the graph
 - e = number of edges (program flows between nodes)
 - n = number of nodes (sequential groups of program statements)
 - if a strongly connected graph is constructed (one in which there is an edge between the exit node and entry node), the calculation is
 - $v = e - n + 1$

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COMPUTER SCIENCE **Example**

$n = 8$
 $e = 10$
 $p = 1$

$C = 10 - 8 + 2 = 4$

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COMPUTER SCIENCE **Software Science**

- Halstead applied information theory to computer science
- metrics
 - n , number of distinct operators
 - n_2 , number of distinct operands
 - N , total number of occurrences of operators
 - N_2 , total number of occurrences of operands
- program level estimator

$$\mathcal{D} = 1 / \mathcal{L} = (n_1 / 2) (N_1 / n_2)$$

$$\mathcal{L} = 1 / \mathcal{D} = (2/n_1) (n_2 / N_2)$$

difficulty increases as operators are introduced ($n_1 / 2$ increases) and as operands are used repetitively (N_2 / n_2 increases)
- programming time

$$\mathcal{T} = \mathcal{E} / \mathcal{S}$$

where \mathcal{S} is the "Stroud number"

$$5 \leq \mathcal{S} \leq 20, \text{ usually } 18$$

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COMPUTER SCIENCE **Software Science (continued)**

- language level

$$\lambda = \mathcal{L} \times \mathcal{V}^* = \mathcal{L}^2 \mathcal{V}^{**}$$

$$\lambda_{PL/I} = 1.53, \quad \lambda_{Algol} = 1.21,$$

$$\lambda_{Fortran} = 1.14, \quad \lambda_{CDC \text{ assemblr}} = 0.88$$
- predicted effort

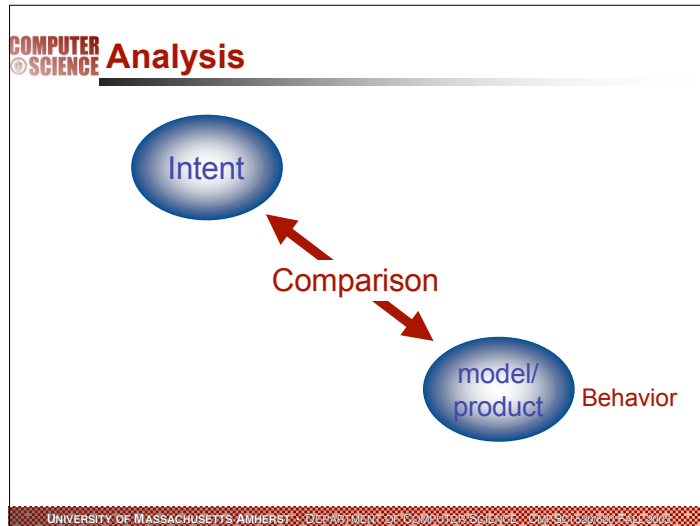
$$\mathcal{E} = \mathcal{V}^{*3} / \lambda^2$$

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COMPUTER SCIENCE **Quality Metrics for Code**

- Understandability
 - size metrics
 - lines of code
 - function points
 - function count
 - traceability metrics
 - number of comment lines per total source lines of code
 - percent comment lines of total lines
 - correctness of comments
- Predicting quality
 - LOC X domain seems to be the most reliable predictor

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COMPUTER SCIENCE Basic Verification Strategy

- analyze a system for desired properties, i.e., compare behavior to intent
 - intent
 - can be expressed as properties of a model
 - can be expressed as formulas in mathematical logic
 - behavior
 - can be observed as software executes
 - can be inferred from a model
 - can be expressed as formulas in mathematical logic
 - different representations support different sorts of inferences

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COMPUTER SCIENCE Compare behavior to intent

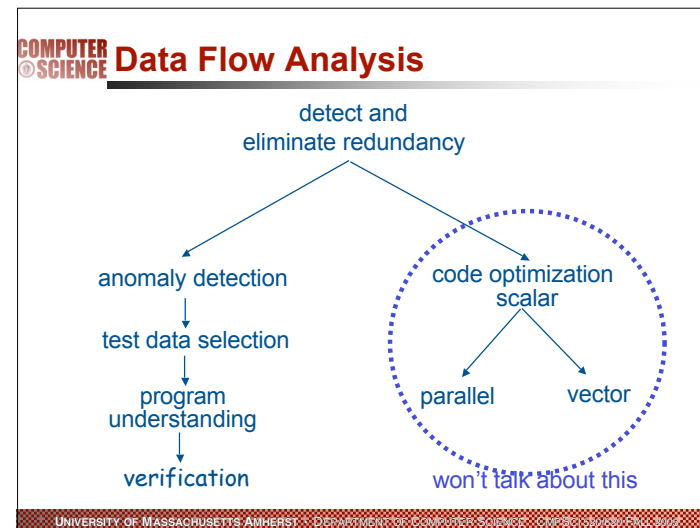
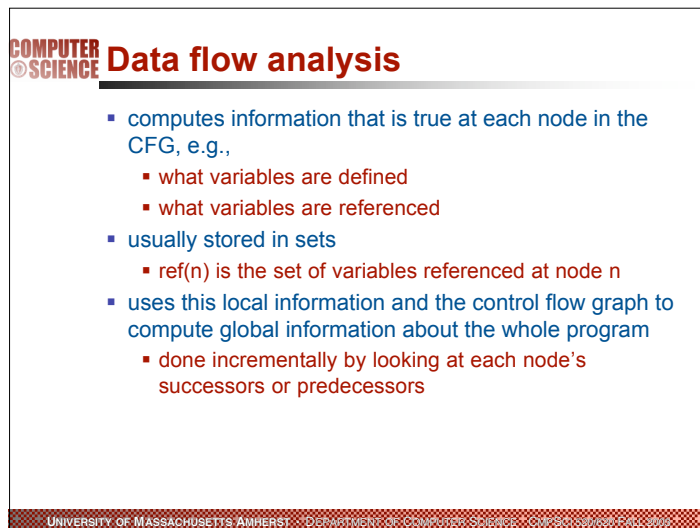
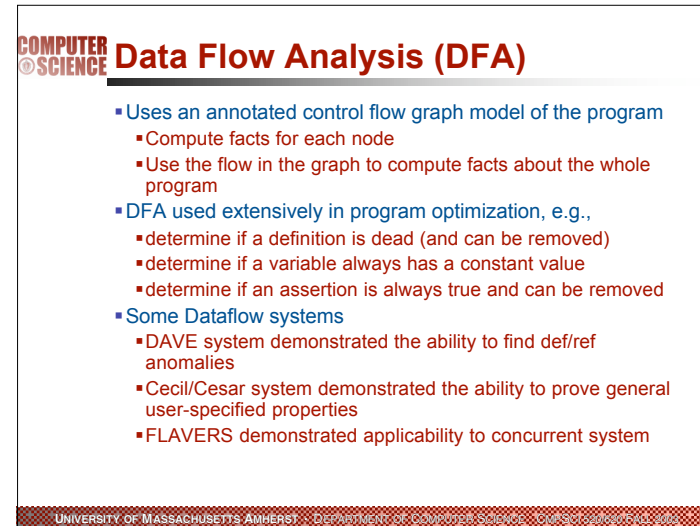
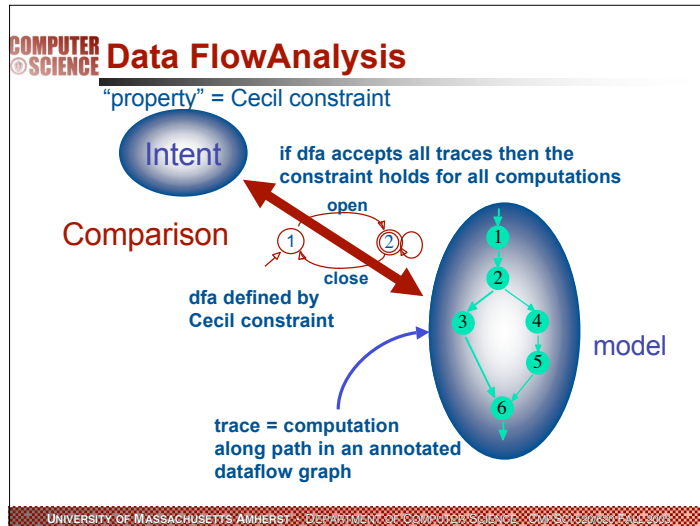
- comparison can be informal
 - done by human eye, e.g., inspection
- can be done by computers
 - comparing text strings
- can be done by model-checkers
 - such as formal machines (e.g., fsa's)
- can be done by rigorous mathematical reasoning

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COMPUTER SCIENCE Example: Dataflow Analysis

- intent:
 - stated as a property
 - captured as an event sequence
- behavior:
 - model represents some execution characteristics
 - inferred from a model: (e.g., annotated flow graph)
 - inferences based upon:
 - semantics of flow graph
 - semantics captured by annotations
- comparison:
 - done by a fsa (e.g., a property automaton)

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COMPUTER SCIENCE DFA

if ...
then ...
else ...
endif;

single-entry,
single-exit
in-line code
blocks

- compute what is true at each node
 - what variables are defined
 - what variables are referenced
- stored in sets
 - $\text{ref}(n)$ is the set of variables referenced at node n
- use local information and the control flow graph to compute global information incrementally by looking at each node's successors or predecessors

$\text{def}=\{y\}$

$\text{def}=\{x\}$
 $\text{ref}=\{y\}$

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COMPUTER SCIENCE Def-ref path expressions

- for a path P and a variable α can write a path expression describing the sequence of set memberships encountered for α , where
 - $\alpha \in \text{def}(1)$
 - $\alpha \in \text{null}(2)$
 - $\alpha \in \text{null}(3)$
 - $\alpha \in \text{ref}(4)$
- $\alpha \in \text{def}(n)$ or
 - $\alpha \in \text{ref}(n)$ or
 - $\alpha \in \text{null}(n)$
- for each node n on the path
- write (and simplify) a path expression
 - $P(n_1, n_1, \dots, n_1; \alpha)$

$P(1,2,3,4; \alpha) = d11r = dr$

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COMPUTER SCIENCE Anomalous pairs of ref/defs

d - defined, r - referenced, u - undefined

dd	bug?	du	bug?	← unreferenced definition
dr	normal	ud	normal	
uu	harmless?	rr	normal	← undefined reference
rd	normal	ru	normal	
ur	bug			

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COMPUTER SCIENCE Consider unreferenced definition

- Want to know if a def is not going to be referenced
 - dd or du
- At the point of a definition of a , want to know if there is some path where a is defined or undefined before being used
 - May be indicative of a problem if the path is executable
 - Usually just a programming convenience and not a problem
- At the point of a definition of a , want to know if on all paths a is defined or undefined before being used
 - May be indicative of a problem
 - Or could just be wasteful

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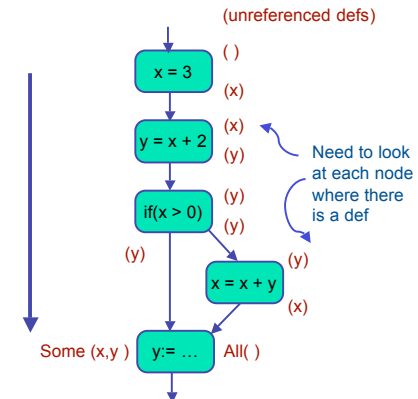
COMPUTER SCIENCE global dataflow analysis

- **classes**
 - **forward flow problems** (e.g., available expressions)
 - what definitions can affect computations at a given point in a program
 - **backward flow problems** (e.g., live variables)
 - what uses that follow a given point in the program can be affected by computations up to that point
- **paths**
 - any path
 - all path

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COMPUTER SCIENCE Unreferenced definitions

```
int x,y;  
...  
x := 3;  
y := x + 2;  
if x > 0 then  
  x := x + y;  
end if;  
y := ...
```



Forward flow,
all paths problem

Some (x,y)

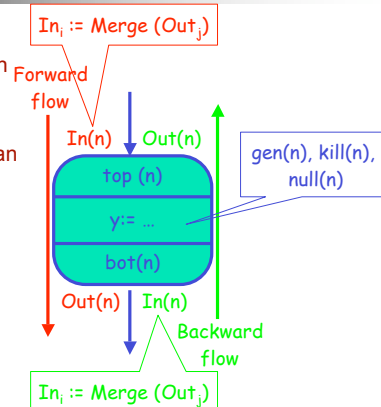
(unreferenced defs)

- Need to look at each node where there is a def

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COMPUTER SCIENCE General Approach

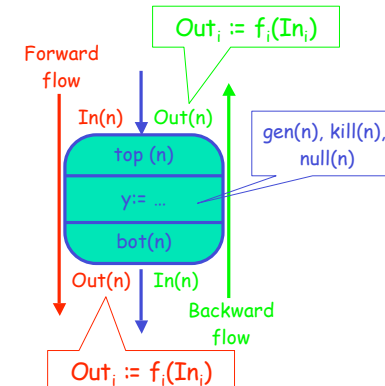
- **Initial values**
 - for each node define gen_i and kill information
- **Input Equations**
 - for each node we have an equation of the form:
 $In_i := Merge(Out_j)$
 - “Merge” operation over the “predecessors” of n_i



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COMPUTER SCIENCE General Approach

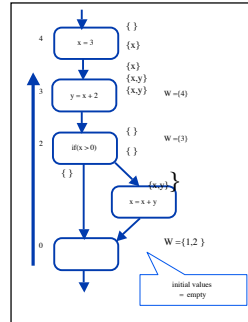
- **Transfer Equations**
 - for each node we have an equation of the form:
Out_i := f(In_i)
 - Transfer functions usually depend on Gen/Kill information that is computed for each node
 - Usually:
Out := (In - kill)U gen
- We can view the set of variables, transfer functions, and flow graph as a system of equations



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COMPUTER SCIENCE worklist algorithm

1. Start at initial node (entry for forward; exit for reverse), label IN_0 with pertinent "facts" (initial values)
2. Compute $OUT_0 = F(IN_0)$ (label OUT_0 with the computed facts)
3. Propagate OUT_0 to IN_1 (label edge $N_0 \rightarrow N_1$ with OUT_0) where N_1 are successor nodes (forward) or predecessor nodes (reverse) of N_0
4. Compute $OUT_1 = F(IN_1)$, place all N_1 on a "worklist" W , and for all N_1 label OUT_1 with the computed facts.
5. While W is not empty,
 1. pick N_i from W and propagate OUT_i to IN_k (label edges $N_i \rightarrow N_k$ with OUT_i) where N_k are successor nodes (forward) or predecessor nodes (reverse) for N_i ; delete N_i from W
 2. Compute $OUT_k = F(IN_k)$ for all N_k where $IN_k = \text{MERGE}$ all input edge labels ($\text{MERGE} = \cup$ for "some paths" and \cap for "all paths"), label OUT_k with the computed facts; and if for N_k , OUT_k changes put N_k on W
6. If W is not empty, then $W = W'$ and go to 5



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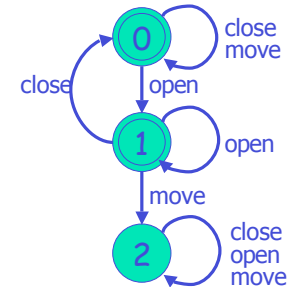
COMPUTER SCIENCE Using Quantified Regular Expressions

- Alphabet, quantification, regular expression

- For the events {open, close, move}

show that for all paths:

$$((\text{close} \vee \text{move})^*, (\text{open}^* \vee \text{open}^* \text{close})^*)^*$$



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COMPUTER SCIENCE Cecil: Olender and Osterweil

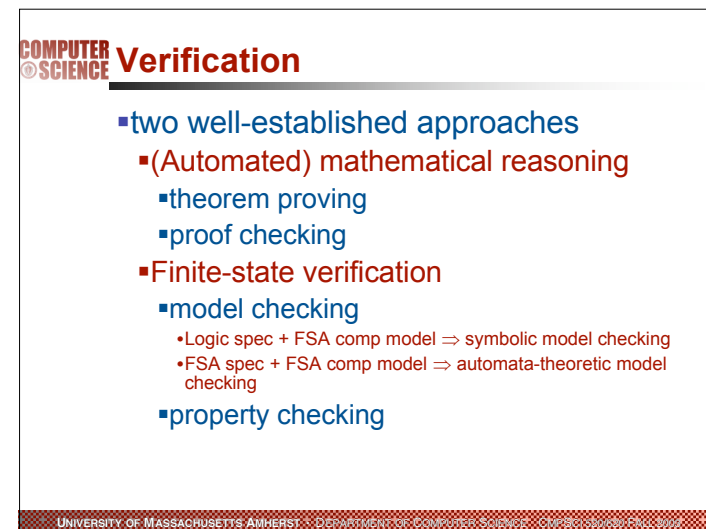
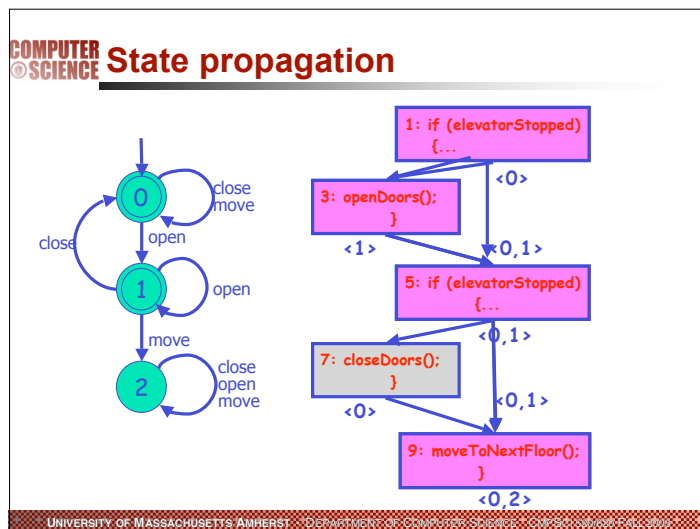
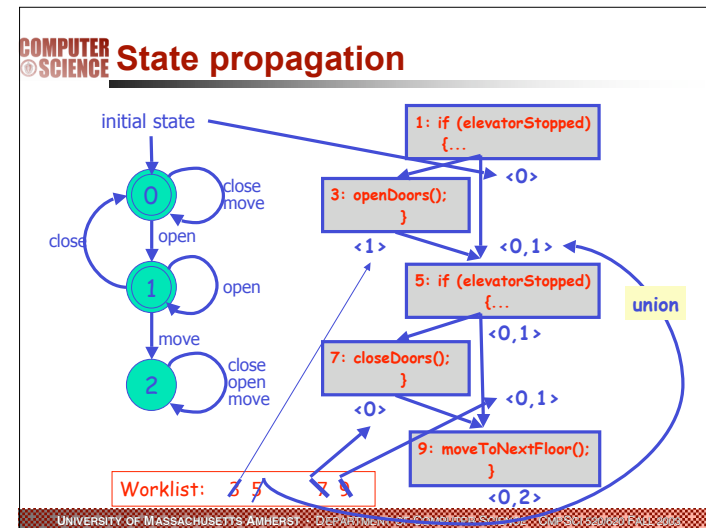
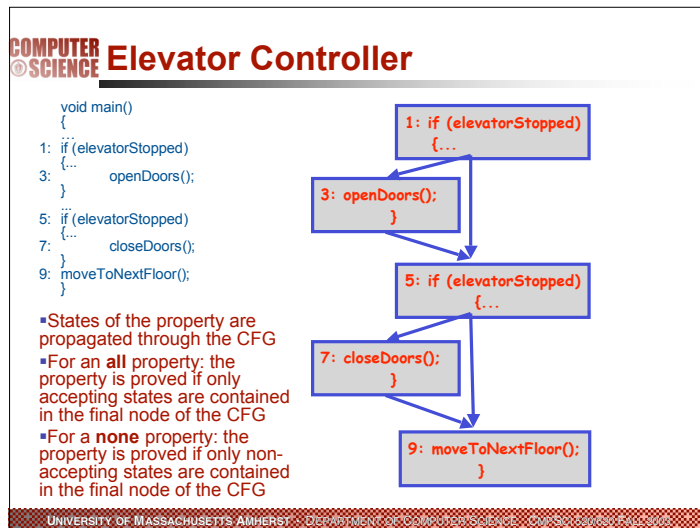
- Instead of implicitly defined facts, let the user define application-specific facts
 - Represented as a Deterministic Finite State Automaton (DFSA) or as a Quantified Regular Expression (QRE)
 - Events
 - Recognizable events
 - Method calls
 - Can reason about sequences of method calls
 - E.g., Push must be called before Pop
 - Thread interactions
 - Join or Fork
 - Arbitrary operations
 - a+b
 - Need to be able to treat events as indivisible actions
 - E.g., can treat pop and push as atomic as long as they do not contain any events of concern
- Propagate the states in the DFSA that can reach each node in the program

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COMPUTER SCIENCE State Propagation

- States of the property are propagated through the CFG
- The property is proved if only accepting (non-accepting) states are contained in the final node of the CFG
- Cecil DFSA \rightarrow
 - lattice $(\mathcal{P}(S), \subseteq, \cup)$
 - function space
 - $\delta : \mathcal{P}(S) \rightarrow \mathcal{P}(S)$
 - facts at nodes are elements of $\mathcal{P}(S)$
- propagate until convergence and check if terminal node in an accepting state of DFSA

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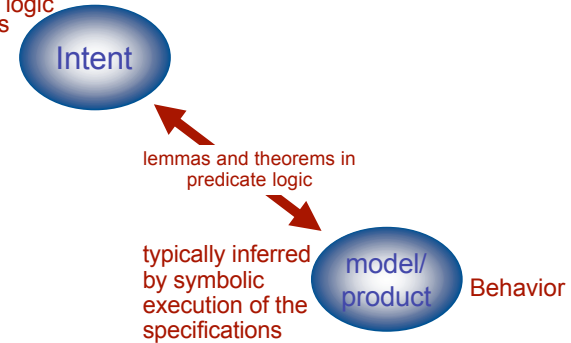


Verification

- How are they different?
 - (Automated) mathematical reasoning
 - difficult, error prone
 - decidability vs. expressiveness
 - Propositional calculus is decidable
 - Predicate calculus is semi-decidable
 - Finite-state verification
 - Reason about a finite model of the system
 - Fast, yields counterexamples, manages partial specifications, applies to concurrency
 - State explosion!

Proof

predicate logic
assertions



Floyd Method of Inductive Assertions

- Show that given the input assertions, after executing the program, program satisfies output assertions
 - show that each program fragment behaves as intended
 - use induction to prove that all fragments, including loops, behave as intended
- show that the program must terminate
- informal description
 - Place assertions at the start, final, and intermediate points in the code.
 - Any path is composed of sequences of program fragments that start with an assertion, are followed by some assertion free code, and end with an assertion
 - $A_s, C_1, A_2, C_2, A_3, \dots, A_{n-1}, C_{n-1}, A_f$
 - Show that for **every** executable path, if A_s is assumed true and the code is executed, then A_f is true

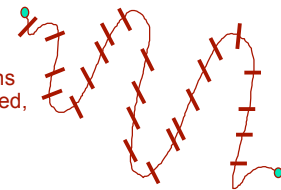
Why does this work?

- suppose P is an arbitrary path through the program
- can denote it by

$$P = A_0 C_1 A_1 C_2 A_2 \dots C_n A_n$$

- where

A_0 - Initial assertion
 A_n - Final assertion
 A_i - Intermediate assertions
 C_i - Loop free, uninterrupted, straight-line code



If it has been shown that

$$\forall i, 1 \leq i < n: A_i C_i \Rightarrow A_{i+1}$$

Then, by transitivity

$$A_0 \Rightarrow \dots \Rightarrow A_n$$



Obvious problems

- How do we do this for a path?
- How do we do this for all paths?
 - Infinite number of paths
 - Must find a way to deal with loops

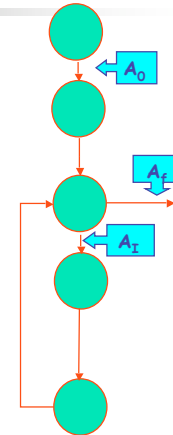
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Find loop invariant (A_I)

- subpaths to consider:
 - C_1 Initial assertion A_0 to final assertion A_f
 - C_2 Initial assertion A_0 to A_I
 - C_3 A_I to A_I
 - C_4 A_I to final assertion A_f
- Basically an inductive proof

The "Aha!" moment - finding invariants is hard!



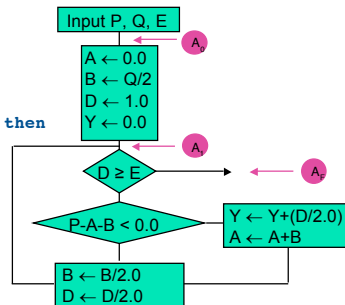
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Wensley's Algorithm

```

Procedure Wensley (P:input, Q:input, E:input, Y:output);
Declare P, Q, E, Y, A, B, D real;
A := 0.0;
B := Q/2.0;
D := 1.0;
Y := 0.0;
Do_While (D >= E)
  If ~(P - A - B >= 0.0) then
    { Y := Y+(D/2.0);
      A := A+B; }
  B := B/2.0;
  D := D/2.0;
End_do;
End Wensley;
  
```

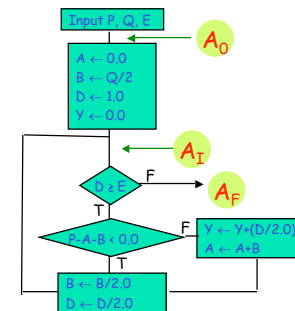


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Summary of Five Lemmas Needed

- A_0 to A_I
- A_I , true branch, to A_I
- A_I , false branch, to A_f
- A_I , true branch, to A_f
- A_I , false branch, to A_f



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COMPUTER SCIENCE **Lemma III: AI, false branch, to AI**

$A_I: \{(A=Q*Y) \wedge (B=Q*(D/2)) \wedge (k \geq 0, k \text{ integer} \wedge D=2^k) \wedge ((P/Q)-D') < Y' \leq (P/Q)\}$

code

```

D ≥ E [constraint]
P - A - B ≥ 0 [constraint]
Y' ← Y + (D/2.0)
A' ← A + B
B ← B/2
D ← D/2.0

```

$A'_I: \{(A'=Q*Y') \wedge (B'=Q*(D'/2)) \wedge (k \geq 0, k \text{ integer} \wedge D'=2^k) \wedge ((P/Q)-D') < Y' \leq (P/Q)\}$

COMPUTER SCIENCE **proof of lemma III**

$A_I \Rightarrow A'_I: \{(A'=Q*Y') \wedge (B'=Q*(D'/2)) \wedge (k \geq 0, k \text{ integer} \wedge D'=2^k) \wedge ((P/Q)-D') < Y' \leq (P/Q)\}$

we have

$A' = A + B; B' = B/2.0; D' = D/2.0; Y' = Y + D/2.0;$

1) $A' = A + B = Q*Y + Q*(D/2) = Q*(Y + (D/2));$
 $Y' = Y + (D/2); \therefore A' = Q * Y'$

2) $B' = B/2 = (Q * D/2)/2; D' = D/2$
 $\therefore B' = (Q * 2D'/2)/2 = Q * D'/2$

from A_I

and so on ... basically using symbolic evaluation

COMPUTER SCIENCE **Hoare axiomatic proof**

- assertions are preconditions and post conditions on some statement or sequence of statements
 $P\{S\}Q$
- if P is true before S is executed and S is executed then Q is true
- as in Floyd's inductive assertion method, we construct a sequence of assertions, each of which can be inferred from previously proved assertions and the rules and axioms about the statements and operations of the program
- to prove $P\{S\}Q$, we need some axioms and rules about the programming language

COMPUTER SCIENCE **Hoare axioms and proof rules**

- take a simple programming language that deals only with integers and has the following types of constructs:
 - assignment statement
 $x := f$
 - composition of a sequence of statements
 $S1, S2$
 - conditional (alternative statements)
 if B then $S1$ else $S2$
 - iteration
 while B do S

Axioms and proof rules

- axiom of assignment
 - $P \{x:=f\} Q$,
 - where Q is obtained from P by substituting f for all occurrences of x in P (symbolic execution)
- rule of composition
 - $P \{S_1, S_2\} Q \Rightarrow \exists P_1, P\{S_1\}P_1 \wedge P_1\{S_2\}Q$
- rule for the alternative statement
 - $P\{\text{if } B \text{ then } S_1 \text{ else } S_2\}Q \text{ fi}$
 $P[B \wedge S_1]Q \wedge P[\neg B \wedge S_2]Q$
- rules of consequence
 - $[P \{S\} Q \wedge Q \Rightarrow R] \Rightarrow P \{S\} R$
 - $[P \{S\} Q \wedge R \Rightarrow P] \Rightarrow R \{S\} Q$
- rule of iteration
 - $P \{\text{while } B \text{ do } S\} Q \Rightarrow P[\neg B]Q \wedge \exists I \ni P[B \wedge S]I \wedge I[B \wedge S]I \wedge I[\neg B]Q$

Proof

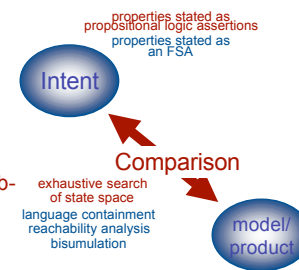
- Hoare-style and Floyd-style verification are essentially the same
 - one is based on graphical representation and the other on a textual representation.
 - In Floyd-style proof, we visualize the proof goal by annotating a CFG
 - In the other, we define the proof goal as a Hoare triple
- Mechanism for applying proof
 - may work either direction on such a proof, but because it's typically easier to work backwards, often use a technique called backwards substitution
 - we work our way from the post-condition, using the proof rules to "push formulas through" the program
 - at each point where a "pushed-through" predicate "runs into" a supplied predicate, we have a verification condition (VC) that must be proved.
 - After all VCs are proved, we need to be prove termination
 - Without a termination proof, we achieve **partial correctness**
 - With a termination proof, we achieve **total correctness**

Straightforward Observations

- Problems
 - formal proofs are long, tedious and are often hard; assertions are hard to get right; invariants are difficult to get right (need to be invariant, but also need to support overall proof strategy)
- Unsuccessful proof attempt \Rightarrow ???
 - incorrect software? assertions? placement of assertion? inept prover? although failed proofs often indicate which of the above is likely to be true (especially to an astute prover)
- Deeper Issues
 - undecidability of predicate calculus \Rightarrow no way to be sure when you have a false theorem
 - there is no sure way to know when you should quit trying to prove a theorem (and change something)
 - proofs are generally much longer than the software being verified \Rightarrow errors in the proof are more likely than errors in the software being verified

Model Checking: Overview

- properties usually expressed in
 - in a propositional logic (e.g., temporal logic)
 - as a FSA
- system represented as a (possibly "abstracted") reachability graph
- reasoning engine
 - logic \Rightarrow propagates valid sub-formulas through the graph
 - FSA \Rightarrow compares FSAs via language inclusion; reachability; or bisimulation



Conservative Analysis

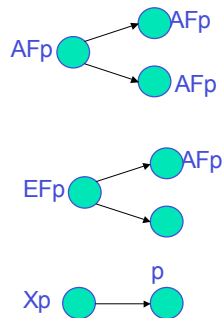
- If property is verified, property holds for all possible executions of the system
- If property is not verified:
 - an error found
 - OR
 - a spurious result
- System model abstracts information to be tractable
 - Conservative abstractions usually over-approximate behavior
 - If inconsistency relies upon over-approximations, then a spurious result
 - e.g. all counter example correspond to infeasible paths

Temporal logic

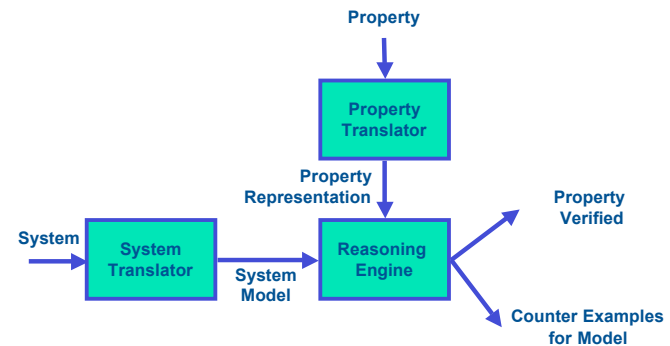
- augments the standard operators of propositional logic with “tense” operators
- “possible worlds semantics” \Rightarrow Kripke model
 - relativize the truth of a statement to temporal stages or states
 - a statement is not simply true, but true at a particular state
 - states are temporally ordered, with the type of temporal order determined by the choice of axioms.
- model of time
 - partially ordered time
 - linearly ordered time
 - linear temporal logic is typically extended by two additional operators, “until” and “since”
 - discrete time
 - branching (nondeterministic) time
 - foundation for one of the principal approaches to verifying concurrent systems = Computational Tree Logics.

Computation Tree Logics

- specification language
 - a propositional temporal logic.
- verification procedure
 - exhaustive search of the state space of the concurrent system to determine truth of specification.
- formulas constructed from path quantifiers and temporal operators:
 - path quantifier:
 - A “for every path”
 - E “there exists a path”
 - temporal operator:
 - Xp “p holds next time”
 - Fp “p holds sometime in the future”
 - Gp “p holds globally in the future”
 - pUq “p holds until q holds”



Architecture of FSV Systems



COMPUTER SCIENCE **mutual exclusion protocol**

- Example: processes can be null, trying to obtain the lock, or in a critical region ($n1, t1, c1$) or ($n2, t2, c2$)
 - TURN is a variable that indicates which process can obtain the lock (0,1,2)
- Need a reachability graph that shows that states (i.e., the values) of the variables

process1 = $n1, t1, c1$
 process2 = $n2, t2, c2$
 turn = 0,1,2

*McMillan

reachability graph

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COMPUTER SCIENCE **Example: propagation**

AG($t1 \Rightarrow AF c1$)

- $a \Rightarrow b$ means (b or $\neg a$)
- $(t1 \Rightarrow AF c1)$ means ($AF c1 \vee \neg t1$)

<process1, process2, turn>

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COMPUTER SCIENCE **Automata-Theoretic Model Checking**

properties stated as an FSA

model/product FSA

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