Data Streaming Algorithms

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Motivation

- Data arrives in a stream or streams
- If not processed immediately or stored, then data is lost forever.
- Data arrives so rapidly that it is not feasible to store it all in active storage.
- We need new algorithmic paradigm to handle data streams.
Example of Data Streams

Sensor Data.

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- Suppose the sensor senses surface height information which changes rapidly. Now the sensor is sending data back every tenth of a second. If it sends a 4-byte real number each time, then it produces

\[4 \times 10 \times 3600 \times 24 = 3456000 \text{ bytes} = 3.5 \text{ Megabyte/ per day.} \text{—Still ok.}\]
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- We may need to employ a million sensors to learn about ocean behavior.—3.5 terabytes of data per day, million of data arriving every tenth of a second.
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Image Data.

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- Surveillance cameras may produce images at every second. London is said to have six millions of such cameras.
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- Google receives several hundred million search queries per day.
- Yahoo! accepts billions of clicks per day on its various sites.
- Many interesting things can be learnt from these streams. An increase in queries like "sore throat" may help to track the spread of viruses. A sudden increase in the click rate for a link could indicate some news connected to that page etc.
Which industries are deploying stream processors?

- Smart Cities - real-time traffic analytics, congestion prediction and travel time apps.
- Oil & Gas - real-time analytics and automated actions to avert potential equipment failures.
- Security intelligence for fraud detection and cybersecurity alerts. For example, detecting Smart Grid consumption issues, and SIM card misuse.
- Industrial automation, offering real-time analytics and predictive actions for patterns of manufacturing plant issues and quality problems.
- For Telecoms, real-time call rating, fraud detection and QoS monitoring from CDR (call detail record) and network performance data.
- Cloud infrastructure and web clickstream analysis for IT Operations.
Few Stream Processing Systems

- Spark Streaming: to build streaming applications in Apache Spark. Apache Spark is a general framework for large-scale data processing that supports concepts such as MapReduce, stream processing, graph processing or machine learning.
- IBM InfoSphere Streams: IBM’s flagship product for stream processing.
- Apache Storm: an open source framework that provides massively scalable event collection.
Developing Streaming Algorithms

• The main hurdle is the space.

• Often it is much more efficient to get an approximate answer than an exact answer.

• Often the algorithm uses randomization like hashing and sampling.
Heavy Hitter Problem

Problem. Given an array $A$ of length $m$, and a parameter $k$, find those values that occur at least $\frac{m}{k}$ times.

Applications:

1. Computing popular products. $A$ could be all of the page views of products on amazon.com yesterday. The heavy hitters correspond to frequently viewed items.

2. Computing frequent search queries. For example, $A$ could be all of the searches on Google yesterday. The heavy hitters are then searches made most often.

3. Identifying heavy TCP flows. Here, $A$ is a list of data packets passing through a network switch, each annotated with a source-destination pair of IP addresses. The heavy hitters are then the flows that are sending the most traffic. This is useful for, among other things, to identify denial-of-service attacks.

4. Identifying volatile stocks. Here, $A$ is a list of stock trades.
Finding Majority

- **Input.** An array $A$ of length $m$ with the promise that it has a majority element—a value that is repeated strictly more than $\frac{m}{2}$ times.
- **Problem.** Find the Majority element in linear time.
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- **Compute median of $A$.**
Finding Majority

- **Input.** An array $A$ of length $m$ with the promise that it has a majority element—a value that is repeated strictly more than $\frac{m}{2}$ times.

- **Problem.** Find the Majority element in linear time in a single left to right pass in “constant” space.
Finding Majority

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- **Algorithm.**
  1. Set \( \text{count} = 1, \text{current} = A(1). \)
  2. For \( i = 2, 3, \ldots \)
     2.1 If \( \text{count} == 0 \), set \( \text{current} = A(i), \text{count} = 1, \)
     2.2 If \( A(i) == \text{current} \), set \( \text{count} = \text{count} + 1 \)
     2.3 Else set \( \text{count} = \text{count} - 1 \)
  3. Return \( \text{current} \)
Finding Majority

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  2. For $i = 2, 3, ...$
     2.1 If $count == 0$, set $current = A(i)$, $count = 1$,
     2.2 If $A(i) == current$, set $count = count + 1$
     2.3 Else set $count = count - 1$
  3. Return current

- **Exercise.** Given there exists a majority element, show that the above algorithm correctly returns the majority.
Heavy Hitter Problem

- Can we solve Heavy Hitter Problem in small space? Ideally in $\tilde{O}(k)$ space.
Heavy Hitter Problem

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- There is no algorithm that solves the Heavy Hitters problems in one pass while using a sublinear amount of auxiliary space.
ε-Approximate Heavy Hitter Problem

- **Input** is an array $A$ of length $m$ with two parameters $\epsilon$ and $k$.
- **Output**
  1. Every value that occurs at least $\frac{m}{k}$ times in $A$ is in the list.
  2. Every value in the list occurs at least $\frac{m}{k} - \epsilon m$ times in $A$. 
\(\epsilon\)-Approximate Heavy Hitter Problem

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- **Why not set \(\epsilon = 0\)?**
\( \epsilon \)-Approximate Heavy Hitter Problem

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- **Space usage grows proportionately with** \( \frac{1}{\epsilon} \).
\(\epsilon\)-Approximate Heavy Hitter Problem

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- **Output**
  1. Every value that occurs at least \(\frac{m}{k}\) times in \(A\) is in the list.
  2. Every value in the list occurs at least \(\frac{m}{k} - \epsilon m\) times in \(A\).
- **Why not set \(\epsilon = 0\)?**
- **Space usage grows proportionately with \(\frac{1}{\epsilon}\).**
- If we take \(\epsilon = \frac{1}{2k}\), space usage is \(\tilde{O}(k)\), all elements with frequency \(\frac{m}{k}\) is in the list and the elements in the list have frequency at least \(\frac{m}{2k}\).
Estimating Frequency of Elements

- **Input** Stream of $m$ elements from a universe $[1, n]$: $A(1), A(2), ..., A(m)$.
- **Frequency of an element** $i \in [1, n]$ in the stream is $f_i = |\{t \mid A(t) = i\}|$.
- **Problem**
  - For $i \in [n]$, estimate $f_i$ (Point Query)
  - For $\phi \in [0, 1]$, find all $i$ with $f_i \geq \phi m$. (Heavy Hitter)
Count-Min Sketch

- Select an $\epsilon > 0$ and $\delta > 0$: $\epsilon$ denotes the error-parameter, and $\delta$ denotes our confidence.
- Select $d = \ln \frac{1}{\delta}$ hash functions $h_1, h_2, ..., h_d$ independently and randomly from a pair-wise independent hash family. Each $h_i : \{1, 2, ..., n\} \rightarrow \{1, 2, ..., w\}$ where $w = \frac{e}{\epsilon}$.
- Initialize a table $T$ of dimension $d \times w$ all with 0.
- Update: At time $t$, when $A(t)$ arrives, do the following.
  - $T(1, h_1(A(t))) = T(1, h_1(A(t))) + 1$
  - $T(2, h_2(A(t))) = T(2, h_2(A(t))) + 1$
  - ...
  - $T(d, h_d(A(t))) = T(d, h_d(A(t))) + 1$

http://research.neustar.biz/tag/count-min-sketch/
Count-Min Sketch: Point Query

- **Problem** For $i \in [n]$, estimate $f_i$
- **Output** An estimate $\hat{f}_i$ such that $f_i \leq \hat{f}_i \leq f_i + \epsilon \|f\|_1$
- **Algorithm** Construct Count-Min sketch. Return

$$\min_{l=1}^{d} T(l, h_l(i))$$
Count-Min Sketch: Point Query

- **Algorithm** Construct Count-Min sketch. Return
  \[
  \min_{l=1}^{d} T(l, h_l(i))
  \]

- Each \( T(l, h_l(i)) \geq f_i \). Hence \( \min_{l=1}^{d} T(l, h_l(i)) \geq f_i \).
- Define an indicator random variable \( X_j^l, j = 1, 2, \ldots, n \) and \( l = 1, 2, \ldots, d \).

  \[
  X_j^l = 1 \text{ if } h_l(j) = h_l(i), \text{ else } X_j^l = 0
  \]

- Define \( Y = \sum_{j \neq i} f_j X_j^l \). Then \( T(l, h_l(i)) = f_i + Y \).
Count-Min Sketch: Point Query

\[ E[Y] = \sum_{j \neq i} E[f_j X_j] = \sum_{j \neq i} f_j E[X_j] \]
\[ = \sum_{j \neq i} f_j \text{Prob}(h_i(j) = h_i(i)) \]
\[ = \sum_{j \neq i} \frac{f_j}{w} \] (h is picked from a pair-wise family)
\[ \leq \frac{\|f\|_1}{w} \]
Count-Min Sketch: Point Query

\[ \text{Prob} \left( T(l, h_l(i)) \right) > f_i + \epsilon \|f\|_1 = \text{Prob} \left( Y \geq \epsilon \|f\|_1 \right) \]
\[ = \text{Prob} \left( Y > w\epsilon E[Y] \right) \]
\[ \leq \frac{1}{w\epsilon} \quad (\text{By Markov Inequality}) \]
\[ = \frac{1}{e} \quad (\text{since } w = \frac{e}{\epsilon}) \]
Count-Min Sketch: Point Query

\[
\begin{align*}
\text{Prob} \left( \min_{l=1}^{d} T(l, h_l(i)) \right) & > f_i + \epsilon \|f\|_1 \\
= \text{Prob} \left( \bigcap_{l=1}^{d} T(l, h_l(i)) \right) & > f_i + \epsilon \|f\|_1 \\
= \prod_{l=1}^{d} \text{Prob} \left( T(l, h_l(i)) \right) & > f_i + \epsilon \|f\|_1 \\
& \leq \left( \frac{1}{e} \right)^{\ln \frac{1}{\delta}} = \delta
\end{align*}
\]

- Hence \(\text{Prob} \left( \min_{l=1}^{d} T(l, h_l(i)) \right) \leq f_i + \epsilon \|f\|_1 \geq 1 - \delta\).
- Therefore \(f_i \leq \hat{f}_i \leq f_i + \epsilon \|f\|_1\) with probability \( \geq 1 - \delta\).
- Space \(= O(wd) = O\left(\frac{1}{\epsilon} \ln \frac{1}{\delta}\right)\).
Count-Min Sketch: Heavy Hitter

Set $\delta' = \frac{\delta}{n}$, using space $O\left(\frac{1}{\epsilon} \ln \frac{n}{\delta}\right)$ obtain estimates such that "For All is $f_i \leq \hat{f}_i \leq f_i + \epsilon m$.

Use a min-heap to store the heavy-hitters.
1. Keep a count on the total number of elements $m$ arrived so far.
2. When item $A(i)$ arrives, compute its estimated frequency from the count-min sketch data structure.
3. If the count is above $\frac{m}{k}$, insert it in the heap with key $\text{Count}(A(i))$, and delete any previous occurrence of $A(i)$ from the heap.
4. If any element in the heap has count less than $\frac{m}{k}$ delete it through operations such as $\text{Find-Min}$ and $\text{Extract-Min}$.
5. Assuming no large error happens in the Count-Min sketch, the heap size is bounded by $2k$. Why? Therefore extra work per item to process the heap is $O(\log k)$.
6. At the end, scan the heap, and for every item whose estimated frequency is $\geq \frac{m}{k}$ return it as a heavy hitter.
Count-Min Sketch: Heavy Hitter

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- Set $\delta' = \frac{\delta}{m*n}$, using space $O\left(\frac{1}{\varepsilon} \ln \frac{m*n}{\delta}\right) = O\left(\frac{1}{\varepsilon} \ln \frac{m}{\delta}\right)$ obtain estimates such that “For All $t = 1, 2, .., ms$ the estimated frequency is within the error-range.
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Miscellaneous

- Implementation: http://www.cs.rutgers.edu/~muthu/massdal-code-index.html

- Twitter’s algebird and ClearSpring’s stream-lib offer implementations of Count-Min sketch and various other data structures applicable for stream mining applications.

- Application: Mostly a list of papers that use CM-sketch
  - http://sites.google.com/site/countminsketch/cm-eclectics
  - http://sites.google.com/site/countminsketch/compressed-sensing
  - http://sites.google.com/site/countminsketch/databases
Mini Exercise [Due Oct 31st]

• Implement Count Min Sketch and plot the frequency of all elements as reported by the Count Min sketch data structure as well as their true frequencies using $\varepsilon=0.01$ and number of hash functions=25.

  – Data: consider a stream of size 1000000 where each element in the stream arrives from [1,1000] chosen uniformly at random.