### Interactive Clustering

Barna Saha

#### Clustering



- n points k clusters
- Clustering Oracle Knows the ground truth clustering or can solve the clustering under some optimization rule
- A Query to a Clustering Oracle: Are points A and B in the same cluster?

Query Complexity?

#### Learning over Noisy Data

Learn a classifier or find clusters over noisy/uncertain data



Type-1 ‡

Noise comes from using similarity functions—add an edge between two images if they represent the same monument—clusters could be erroneous

#### Learning over Noisy Data

Learn a classifier or find clusters over noisy/uncertain data

[-] 2010 – today 🔮	Noise comes from inherent data errors/missing attributes—clustering collaboration network obtained from DBLP could be erroneous.
■ [j221] 🗎 昰 ଙ Xiao Xia, Xiaodo	ong Wang, Jian Li, Xingming Zhou: <b>Multi-objective mobile app recommendation: A system-level collaboration approach.</b>
Computers & El	ectrical Engineering 40(1): 203-215 (2014)
■ [j220] 🗎 邉 ଙ Qingjia Huang,	Kai Shuang, Peng Xu, Jian Li, Xu Liu, Sen Su: <b>Prediction-based Dynamic Resource Scheduling for Virtualized Cloud Systems.</b>
JNW 9(2): 375-3	183 (2014)
■ [j219] 🗎 邉 癸 Jingdong Wang, Neighbor Sear	Naiyan Wang, You Jia, Jian Li, Gang Zeng, Hongbin Zha, Xian-Sheng Hua: <mark>Trinary-Projection Trees for Approximate Nearest ch.</mark> IEEE Trans. Pattern Anal. Mach. Intell. 36(2): 388-403 (2014)
■ [j218] 🗎 竖 😤 George-Othon	Glentis, Kexin Zhao, Andreas Jakobsson, Habti Abeida, Jian Li: <b>SAR imaging via efficient implementations of sparse ML</b>
approaches. Si	gnal Processing 95: 15-26 (2014)
■ [c258] 🗎 显 🗟 MohammadTa	zhi Hajiaghayi, Wei Hu, Jian Li, Shi Li, Barna Saha: <mark>A Constant Factor Approximation Algorithm for Fault-Tolerant <i>k</i>-Median.</mark>
SODA 2014: 1-	12

#### **Further Applications**

- Linking Census Records
- Public Health
- Web search
- Comparison shopping
- Spam Detection
- Machine Reading

\*\*\*\*\* (1)

IP Aliasing





### Query complexity of optimal strategy?

Clustering: n points k clusters



- Sufficient: nk
  - Compare any element with all the previously formed clusters
  - Any item needs to be queried at most k times before it is assigned to a cluster

#### Query complexity of optimal strategy?

**Clustering:** *n* points *k* clusters



- Sufficient: nk
  - Compare any element with all the previously formed clusters
  - Any item needs to be queried at most k times before it is assigned to a cluster
- Necessary:  $\Omega(nk)$  Davidson, Khanna, Milo, Roy, 2014
  - Deterministic Algorithms: Needs to query Θ(n) points at least k - 1 times
  - Randomized Algorithms (find the clustering exactly whp): Same lower bound applies

#### **Faulty Oracle**



 Each query answer can be independently wrong with probability p (when the points are in same cluster) or 1 – q (when in different clusters)

#### **Faulty Oracle**



 Each query answer can be independently wrong with probability p (when the points are in same cluster) or 1 – q (when in different clusters)

Repeat the same question. Assuming p=q, repeat each question (say) 24log n/(1-2p)<sup>2</sup> times

#### **Faulty Oracle**



- Each query answer can be independently wrong with probability p (when the points are in same cluster) or 1 – q (when in different clusters)
- Resampling is not allowed

1) It is not theoretically interesting 2) Also not practical (only 20% reduction via resampling, Gruenheid et al. 2015, error increases upon aggregation Prelec et al. (Nature 2017))

#### Faulty Oracle: No Resampling

- Find seed nodes for each cluster
- If we can find 24 log n/(1-2p)<sup>2</sup> seed nodes from each cluster then we are done! [Why?]



# Faulty Oracle: How to find seed nodes?

- Let N=O(k<sup>2</sup>log n/(1-2p)<sup>4</sup>)
- Select N nodes and ask all possible pairwise queries among these nodes.
- Run correlation clustering algorithm in this small set of nodes
- Each cluster returned by the correlation clustering that has size at least 24 log n/(1-2p)<sup>2</sup> act as a seed

# Faulty Oracle: How to find seed nodes?

- Let N=O(k<sup>2</sup>log n/(1-2p)<sup>4</sup>)
- Select N nodes and ask all possible pairwise queries among these nodes.
- Run correlation clustering algorithm in this small set of nodes
- Each cluster returned by the correlation clustering that has size at least 24 log n/(1-2p)<sup>2</sup> act as a seed

Some intuition on the analysis: If we know all the query results, correlation clustering gives the maximum likelihood estimator. Moreover, it is an instance of correlation clustering where errors are random we know how to solve it!